MSc project: \( t \)-designs with repeated blocks

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This project is appropriate for someone with an interest in combinatorics. A block design is an ordered pair \((V, \mathcal{B})\), such that \(V\) is a finite non-empty set, whose elements are called points, and \(\mathcal{B}\) is a finite non-empty collection of nonempty subsets of \(V\) called blocks. In this collection, the order of blocks does not matter, but the number of times a block occurs (its multiplicity) does matter. In technical terms, the collection of blocks is a multiset.

An important class of block designs is that of \( t \)-designs (see \([1, 3]\)). For \( t \) a non-negative integer and \( v, k, \lambda \) positive integers with \( t \leq k \leq v \), a \( t-(v, k, \lambda) \) design (or simply a \( t \)-design) is a block design with exactly \( v \) points, such that each block has size \( k \) and each \( t \)-subset of the point-set is contained in exactly \( \lambda \) blocks.

Now, given positive integers \( t, v, k, \lambda \), with \( t \geq 2 \), it can be very difficult (often an open research problem) to determine whether or not a \( t-(v, k, \lambda) \) design exists. In addition, when such a design exists, it may or may not have repeated blocks.

There has been some recent work on the following problem. Given positive integers \( t, v, k, \lambda \), determine an upper bound \( m \) on the multiplicity of any block in any \( t-(v, k, \lambda) \) design (see \([4, 2]\)). It is interesting to know when such a bound is met by some block in some \( t-(v, k, \lambda) \) design. For this, there is new work on constructing \( t \)-designs with repeated blocks (see \([5, 4]\)).

This project will be based on the study of the recent papers \([5, 4, 2]\). One interesting task would be to determine, for a list of possible parameter tuples \((t, v, k, \lambda)\) (for reasonably small parameter values), what is known about the maximum multiplicity of a block in a \( t-(v, k, \lambda) \) design.

References


