The projects listed on the following pages are suitable for MSc/MSci or PhD students. An MSc/MSci project normally requires a review of the literature and finding recent related results in the existing literature. Original research is not required. A PhD project requires, of course, original research and the projects listed below may be the starting point for a PhD thesis.

The projects cover topics in Nonlinear Dynamics and Statistical Physics. The description of the projects is rather brief and uses sometimes specialised terms. For further details, for instance the required prerequisites, please contact:
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A: Lyapunov exponents in systems with many degrees of freedom

1. Dynamics with large time delay
   It is quite well established that dynamics including time delay yields high dimensional phase spaces. Within the project such a topic is addressed on the basis of simple one dimensional map. For that purpose the time discrete dynamics given by $x_{n+1} = (1-\varepsilon)f(x_n) + \varepsilon f(x_{n-\tau})$ will be studied, with a special emphasis on Lyapunov exponents, fractal dimensions of the attractor, and corresponding entropies [25]. Analytical computations for large delay time $\tau$ can be performed for maps with constant slope, $f(x) = ax \mod 1$, while the universal character of such results can be confirmed by numerical simulations for a larger class of systems. In particular, the project will focus on how the features of the dynamical systems change qualitatively when the delay parameter $\varepsilon$ increases.

2. Co-moving Lyapunov exponents and Lyapunov spectra
   Chaotic systems with spatial degrees of freedom, like for instance partial differential equations or coupled map lattices, can be used to describe transport in random media. In simple cases such transport properties can be quantified by appropriate Lyapunov exponents which describe the sensitivity on initial conditions in a comoving frame [26]. Such co-moving Lyapunov exponents can be evaluated for maps with constant slope, e.g. Bernoulli shift maps, by analytical means. The relation between co-moving Lyapunov exponents and the whole Lyapunov spectrum should be uncovered in this project [27].
B: Linear stability of systems with time delay

1. Quasiperiodically driven linear delay systems
   Time-delayed feedback control of periodic orbits, in particular the corresponding linear stability analysis, may result in dynamical systems driven by a quasiperiodic force. Using results for quasiperiodically driven linear differential equations [10] simple linear differential-difference equations with quasiperiodic coefficients will be analysed. Results will be compared with resonant cases where Floquet theory and numerical tools can be applied [11, 12].

2. Linear stability of oscillators subjected to time delay
   Stability analysis of time-delay dynamics yields transcendental characteristic equations, for instance $z \exp(z) = c$, for the corresponding eigenvalues $z \in \mathbb{C}$. The analysis of such an equation, i.e. the dependence of $z$ on the parameter $c \in \mathbb{C}$ can be found in the literature [13, 14]. The more advanced tools described in the appendix of [15] will be applied to analyse more complicated characteristic equations, like $(z^2 + az + b) \exp(z) = c$, which govern the stability of a harmonic oscillator subjected to time delayed feedback control $\ddot{x}_t + \gamma \dot{x}_t + \omega^2 x_t = K(x_t - x_{t-\tau})$.

3. Bifurcation analysis of time-delayed feedback control
   Bifurcation analysis and numerical continuation tools for differential-difference equations [12] are used to study the properties of simple oscillators, like the driven Toda equation, subjected to time-delayed feedback control [16]. Of particular interest is the impact of different coupling schemes of the control force, and the analysis of different control methods, like unstable control loops [17], control in autonomous systems [18], or time-dependent modulations of the control loop [19].
C: Phase transitions in dynamical systems

1. Renormalisation of the Ising map
   Piecewise linear Markov maps are simple dynamical systems. The dynamical properties, like expectation values, correlations functions, or invariant measures can be analysed in terms of the Statistical Mechanics of spin chains by analytical means [1]. A simple model which is equivalent to the nearest neighbour coupled Ising chain will be investigated [2]. The link between the renormalisation by spin decimation and higher iterates of the map will be investigated based on quantities like the magnetisation $\langle m \rangle$ or the corresponding generating function $\langle \exp(qnM_n) \rangle$ (i.e. the topological pressure in formal terms [3]). A transfer of the spin decimation renormalisation group to the dynamical system, e.g., on suitable function spaces is one of the goals of the project.

2. Analytical solutions for one-dimensional probabilistic cellular automata
   Spatially one-dimensional probabilistic cellular automata are simple dynamical systems which can be analysed by analytical means [4]. Stationary distributions of models with nearest neighbour coupling will be determined, with special emphasis on asymmetric couplings and violation of detailed balance [24]. The possibility of phase transitions in models with long range coupling and the computation of the spectrum of the corresponding Master equation is of interest as well. Furthermore, the question of the equivalence between mean field theories and globally coupled models will be addressed.
D: Globally coupled dynamical systems

1. **Globally coupled Ising maps**
   Piecewise linear Markov maps with global coupling are investigated [2]. The model can be mapped to globally coupled spin models which can be analysed by analytical means and which show phase transitions [4]. Topics which are of interest for the project are: (i) Analysis of the critical behaviour using dynamical mean field equations for the magnetisation. Such equations are exact because of the global coupling and of the Markov property. (ii) Stability of the piecewise constant solution for the one particle density. (iii) Features of the dynamical system in the neighbourhood of the phase transition, e.g. the statistical weights and the number of space-time periodic patterns. Relations with microcanonical descriptions of equilibrium phase transitions may be relevant in such a context [5].

2. **Globally coupled Bernoulli maps**
   Dynamical system with global coupling can be analysed in terms of the one-particle density [6]. A model of piecewise linear shift maps will be investigated. Simple solutions of the mean field equation will be compared with phase diagrams of short ranged coupled models [7]. Stability of such simple solutions may be studied by numerical means [8]. Phase transitions of the globally coupled model may be investigated as well with regards to changes in the structure of space-time periodic patterns, e.g., pruning of such orbits [9].
E: Critical behaviour in coupled map lattices

1. Finite-size scaling of coupled Bernoulli maps
   Spatially two-dimensional arrays of coupled shift maps display a variety of phase transitions [7]. The properties of the transition is quantified by critical exponents which govern the scaling behaviour of average values. Accurate estimates for such exponents are obtained by finite-size scaling procedures [20, 4]. The universality of ferromagnetic phase transitions in such a model will be investigated as well as the scaling behaviour with regards to dynamical critical behaviour and transitions which involve time-dependent phases.

2. Finite-size scaling of the Miller-Huse model
   Spatially tow-dimensional arrays of asymmetric tent maps display phase transitions of ferromagnetic type [22]. The properties of the transition is quantified by critical exponents which govern the scaling behaviour of average values. Accurate estimates for such exponents are obtained by finite-size scaling procedures [20]. But such results have been questioned recently since computations are corrupted by finite size corrections [21]. Accurate values of the critical exponents will be obtained by numerical analysis of systems of sufficient size. The influence of the underlying lattice structure, e.g. square lattice vs. honeycomb lattice, will be investigated as well [23].
References


