MSc project: The Mathieu groups

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This project is suitable for someone with an interest in Group Theory and Combinatorics. Familiarity with basic group theory and other parts of algebra is essential.

According to the Classification of Finite Simple Groups (normally abbreviated to CFSG) each finite simple group belongs to (at least) one of the following classes:

- The cyclic groups $C_p$ of prime order;
- The alternating groups $A_n$ of degree at least 5;
- Six families of classical groups, parametrised by dimension and field (size);
- Ten families of exceptional groups of Lie type, parametrised by field; and
- Some 26 or 27 sporadic groups (depending on whether you count the Tits group $2F_4(2)'$ as sporadic).

Five of the sporadic groups were discovered in the 1860s and 1870s by Émile Mathieu, and named after him; we shall by studying these groups, and their subgroups. We shall also be studying objects associated with these groups, such as various Steiner systems and certain codes. One aim is to prove the uniqueness of certain Steiner systems such as $S(5,6,12)$.

For a proper understanding of the Mathieu groups, it is also necessary to study the exceptional behaviour of certain alternating and linear ($L_n(q) = PSL_n(q)$) groups, including exceptional isomorphisms, actions, and automorphism groups. For example the symmetric group $S_6$ has an exceptional (outer) automorphism. Another aim will be to study this behaviour also (proving some exceptional isomorphisms, etc).

Bibliography


