Signals of critical behavior in fragmenting finite systems

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By studying the disassembly of excited drops within the framework of the classical molecular dynamics (CMD) model, a critical review of observables which can give a signature of a critical behavior is performed. In particular we look at the normalized variance of the mass of the largest fragment (NVM) and to the intermittency signal (IS). It is found the NVM displays a maximum in the critical region for CMD and percolation models, while it is not triggered in ''noncritical'' data like the one resulting from the random partition model. On the other hand, the IS displays a maximum when events with a big fragment are mixed with events composed mainly of small clusters. [S0556-2813(99)00408-2]

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I. INTRODUCTION

The possibility of observing a phase transition in nuclear collisions at intermediate energies has attracted the attention of nuclear physicists for a long time now. This possibility was initially fueled by the observation that fragment spectra resulting from heavy-ion collisions at energies around the Fermi energy followed a power law distribution of the form $Y(A)=Y_0A^{-\tau}$ [1]. This is precisely what is predicted by the Fisher's droplet model for liquid-gas phase transitions in the vicinity of the critical point [2]. Moreover, this is a common feature of processes that satisfy scaling relations, for example, percolation, Ising model, lattice gas, etc. [3]. From the theoretical point of view such a behavior would be consistent with the predicted properties of the equation of state of infinite nuclear matter which is supposed to be similar to that of van der Waals [4,5]. Nevertheless, a power law distribution of mass fragments is not enough to characterize the underlying physical process as a phase transition; for example, such a distribution is obtained when analyzing the fragment size distribution resulting from the impact of high-velocity projectiles on basalt rocks [6]. In order to properly characterize the process of a phase transition around the critical region other signals should be found. It should be kept in mind that because we are dealing with a finite system, we cannot talk about a sharp critical point, but instead of a rather fuzzy critical region. In this sense, a well-known property of such a phase transition is that fluctuations of all sizes should be present, and such a behavior gives rise to the phenomenon of critical opalescence [3]. The main efforts towards a proper characterization of the criticality of the process considered can be arbitrarily classified in two main groups.

(i) The first comprises those that explore the extraction of critical exponents from the analysis of combination of moments of the mass distribution under study [7–9].

(ii) The second comprises those approaches that aim at an analysis of the scaling properties of the other-than-statistical fluctuations of the population of the mass bins, i.e., the intermittency analysis [10–12].

Several criticisms have been put forward on both approaches. The analysis according to approach (i) relies basically on the properties of the distribution as a whole and not on the fluctuations of the population of each fragment mass around its mean value [8]. In the case of (ii) it has been shown that an intermittency signal (IS) can be obtained even for simple models (i.e., the random population of mass bins with a power law distribution) in which the only nonstatistical source of fluctuations is a conservation law, i.e., mass conservation [13]. Moreover, it has been shown that when total multiplicity is fixed the IS is washed away even for percolating networks in which fluctuations are of nontrivial origin [8]. In fact, a property of the scaled factorial moments applied to size distributions is that the signal is given mainly by the particles located in the first bin, i.e., the lightest particles [8,11], but the multiplicity distribution is also mainly determined by the lightest particles. So fixing the total multiplicity is equivalent to fixing the multiplicity of the lightest particles; thus fluctuations are severely constrained and the intermittency signal is suppressed.

In a recent paper [14] (hereafter referred as I) an extensive analysis of signals of criticality was performed on the asymptotic mass spectra resulting from the microscopic simulation of the dynamical evolution of classical excited drops. In the model used, the Hamiltonian, generator of the evolution, had been carefully tailored in order to reproduce general properties of both finite nuclei and nuclear matter [4]. In I, the behavior of the IS and moments of the resulting distributions were analyzed, finding that they were consistent with critical behavior, for systems excited to an initial temperature of about 4.5–5 MeV (see Sec. III for details). Such a value of the initial temperature for a critical behavior was also obtained from an analysis of the largest Lyapunov exponents [15].

We first explore a signal of critical behavior that has not been fully analyzed, to our knowledge, so far. This is, the normalized variance of the size of the maximum fragment (NVM). In order to check the validity of the proposed approach when used to study the results of our classical molecular dynamics (CMD) simulations, we first perform the calculations of the NVM on two simple systems for which the existence or not of critical behavior is known. First we used a random partitioning model in which the population of

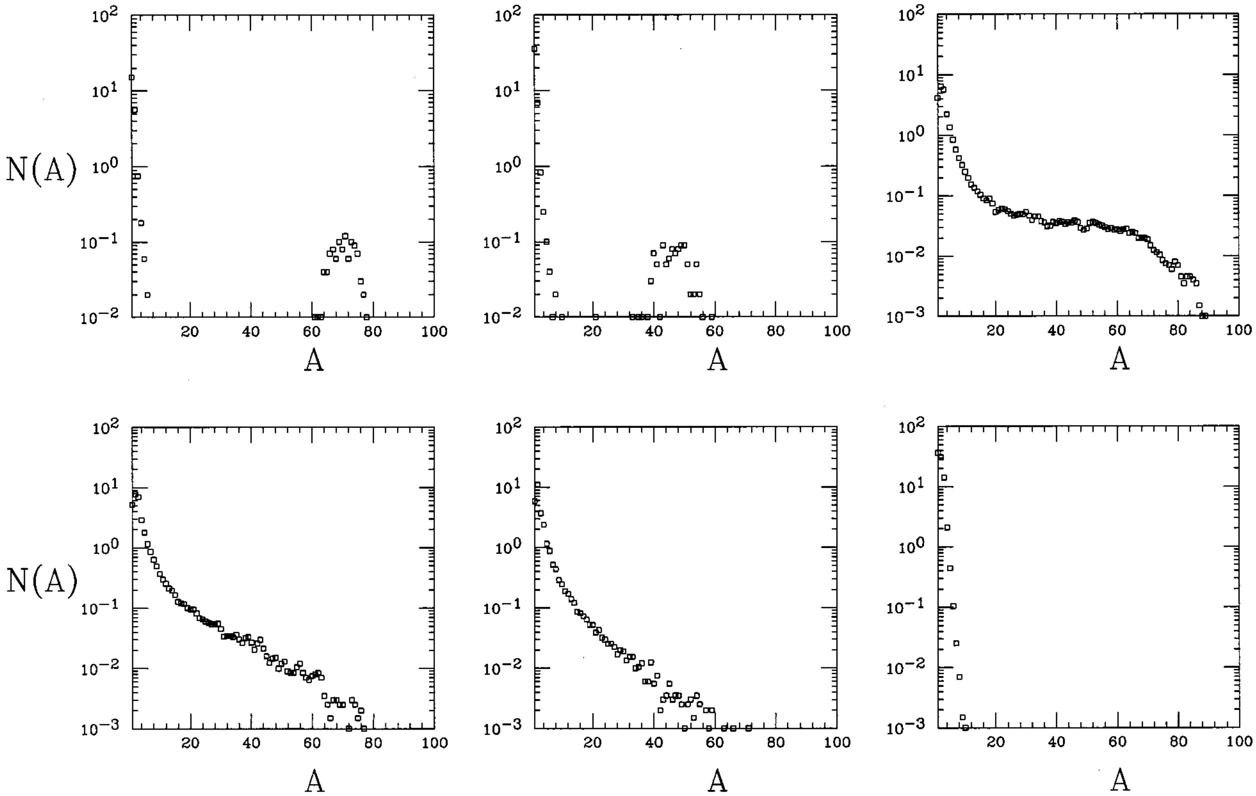

FIG. 1. Mass distributions obtained in the expansion of the system $A=100$ with six different initial energies, namely, 2, 3, 4, 5, 7, and 15 MeV g from 2 to 15 MeV (begining at the upper left corner).

the different mass numbers is obtained by randomly choosing values of A following a prescribed mass distribution [13]. In this case the fluctuations in the populations are of statistical origin or related to the fact that the total mass A_{tot} is fixed and one should not obtain a signal of criticality. Afterwards we explore bond percolation model on a finite lattice which displays true critical behavior. We find that the NVM peaks close to the critical bond activation probability corresponding to the infinite system. This is a well-known result [9], and it is usually stated that for finite systems the critical point is shifted towards higher values of the bond probability.

We then analyze the NVM from the outcome of microscopic simulations of the evolution of excited classical drops in the framework of CMD. Again, we find that NVM peaks close to the value at which the maximum IS is found.

We then reanalyze the IS in the CMD simulations. We focus on the effect of performing different selections of events according to cuts in the multiplicity distribution on the resulting IS. We demonstrate that the signal is strongest when events coming from the ''liquid'' side (events in which a rather big fragment is present) are mixed with events coming from the ''gas'' side (events which are composed by mainly small fragments) at 5 MeV. Such a property disappears when we perform the same analysis at a ''noncritical'' temperature.

In Sec. II, we briefly review the CMD model. In Sect. III, we define the NVM signal of criticality and show the results of the analysis as obtained from the simple random partition model, for percolation, and in CMD models.

In Sec. IV we show the results of an intermittency analysis for simulations of finite expanding systems. We pay special attention to the sources of intermittent behavior. Finally, conclusions are drawn in Sec. V.

II. CMD MODEL

In the classical molecular dynamics model it is assumed that the nucleus is made up of A nucleons that behave classically. These particles move under the influence of a pairwise spherical interaction potential V given by [4]

$$V_{np}(r)=V_r[\exp(-\mu_r r)/r-\exp(-\mu_r r_c)/r_c] - V_a[\exp(-\mu_a r)/r-\exp(-\mu_a r_a)/r_a],$$

$$V_{nn}(r)=V_{pp}(r)=V_0[\exp(-\mu_0 r)/r-\exp(-\mu_0 r_c)/r_c], \tag{1}$$

where $r_c=5.4$ fm is a cutoff radius and V_{np} is the potential acting between a neutron and a proton while V_{nn} is the potential acting between two identical nucleons. The first interaction is attractive at large r and repulsive at small r, while the latter is purely repulsive. In this way, no bound state of identical nucleons can exist. The values of the parameters

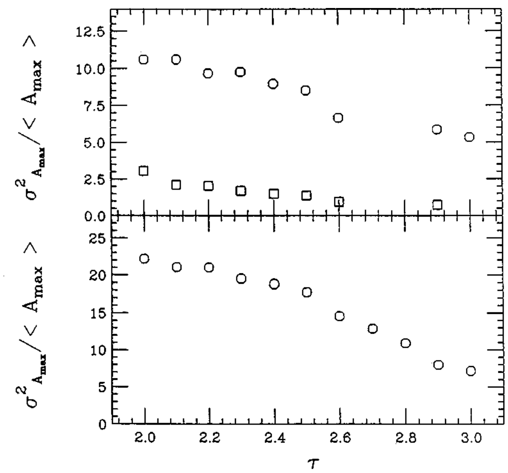

FIG. 2. Normalized variance for the random partition model when the partitioning is performed according to a power law with τ in the range $2<\tau<3$ (the physically interesting region) for, in the upper part, the system with $A_{\rm tot}=100$ with unrestricted multiplicity (open squares) and fixed multiplicity (in the most probable value, open circles), and in the lower part the same but for $A_{\rm tot}=200$ and free multiplicity. In all cases no signal is observed.

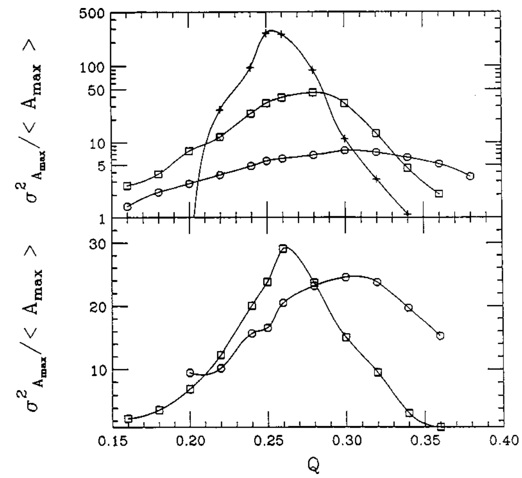

FIG. 4. Normalized variance for bond percolation in finite systems as a function of the bond activation probability Q, for, in the upper part, lattices of sizes $20\times20\times20$ (crosses), $10\times10\times10$ (squares), and $5\times5\times5$ (circles) with unrestricted multiplicity. It is readily seen that the smaller is the size of the lattice the bigger is the difference of the maximum of the curves from the percolation critical bond for infinite lattice ($Q_c=0.23$). This is a clear manifestation of finite size effects. The lower part shows the same as the upper part but for fixed multiplicity and for lattices of sizes $10\times10\times10$ (squares) and $5\times5\times5$ (circles).

entering the Yukawa potentials are given in Ref. [4] and give a corresponding equation of state (EOS) of classical matter with a compressibility of about 250 MeV (set M in Ref. [4]). This EOS strongly resembles the one expected for nuclear matter [i.e., equilibrium density $\rho_0=0.16$ fm^{-3} and energy $E(\rho_0)=-16$ MeV/nucleon]. Furthermore, in Refs. [4,16], it was shown that many experimental data on heavy-ion collisions are reasonably explained by this classical model. Of course this is not accidental but it is due to the accurate choice of the parameters of the two-body potentials.

The classical Hamilton's equations of motion are solved using the Taylor method at the order $O[(\delta t)^3]$ where δt is the integration time step [17]. Energy and momentum are well conserved. The nucleus is initialized in its ground state by using the frictional cooling method [18]. These ground state configurations are excited to a temperature T by giving a Maxwellian velocity distribution to the nucleons by means of a Metropolis sampling [17]. We have studied the disassembly of a system with $A=100$ and $Z=50$ starting from an initial density $\rho=0.125$ fm^{-3} and with different values of the initial temperature. Coulomb interaction was not taken into account.

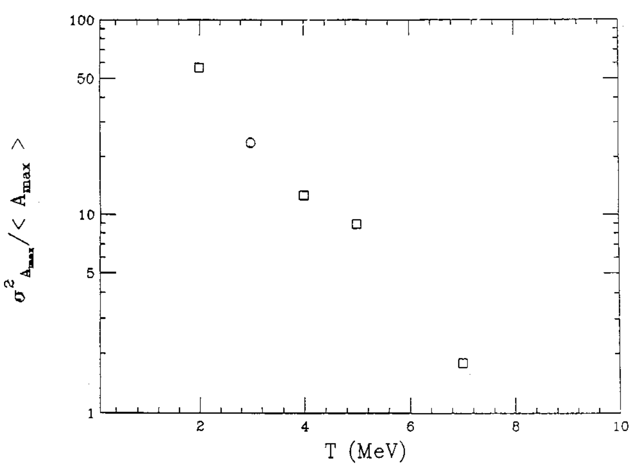

FIG. 3. Normalized variance for the random partition model when the partition is performed according to Fisher's law and the parameters are those displayed in Table I. The point for $T=3$ MeV (open circle) is generated imposing the extra condition that the mass of the biggest fragment should be smaller than 60 (see text for details).

As a reference we show in Fig. 1 the mass spectra as already depicted in I for different initial excitation energies (see figure caption for details). The case $T=5$ MeV gives a power law in the mass distribution.

III. NVM

It has been suggested that a possible signal of critical behavior could be the fluctuations in the size of the maximum fragment [7]. It is supposed that cluster size distributions show the maximum of fluctuations around the critical point where the correlation distance ξ diverges. As a result of mass conservation, the size of the largest cluster should show

TABLE I. Values of the fitting parameters Y_0, X, Y, and τ entering in the formula (4)

T(MeV)	2	3	4	5	7
Y_0	442.8	146.1	30.7	39.5	69.7
X	0.042	0.10	1.00	1.00	1.00
Y	2.01	1.83	1.012	0.995	0.87
τ	2.23	2.23	2.23	2.23	2.23

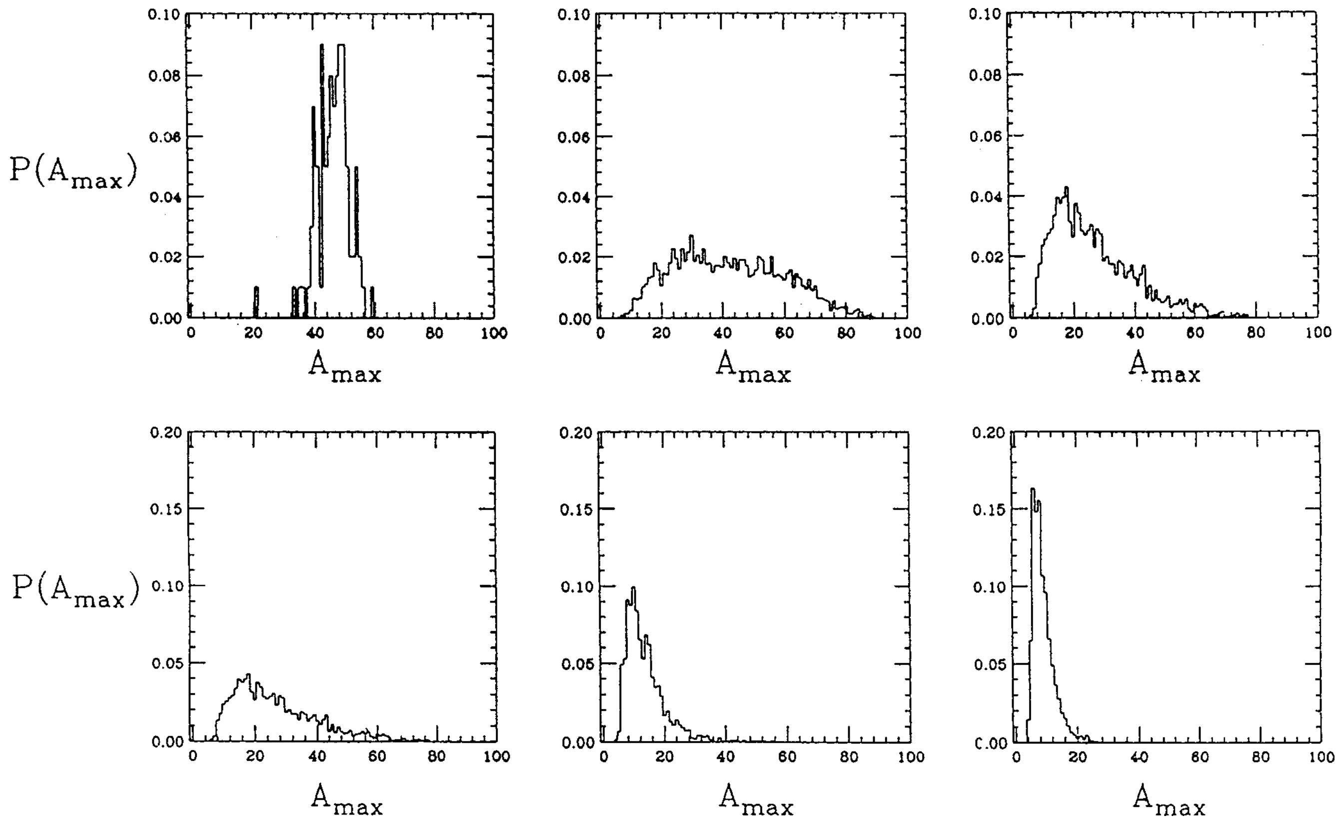

FIG. 5. We show the probability distribution of the size of the maximum fragment for six different initial temperatures for the $A=100$ fragmenting system in the CMD model. We observe that the width of the distribution attains its maximum at about $T=4$ MeV.

in this case large fluctuations [3].

For the following, we thus propose to study the normalized variance of the size of maximum fragment, NVM, as a signal of criticality. This normalized variance is defined as

$$\sigma_{\mathrm{NV}}=\frac{\sigma^2_{A_{\max}}}{\langle A_{\max}\rangle}, \tag{2}$$

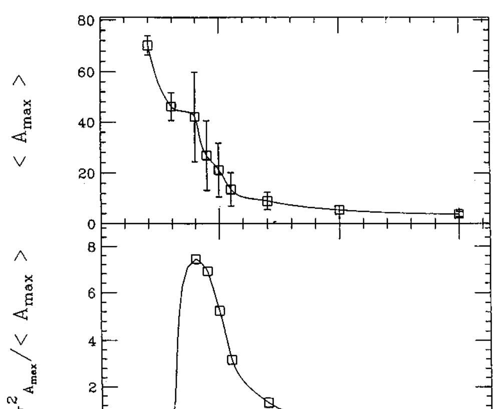

FIG. 6. The upper part shows the average mass of the maximum fragment while the lower part the NVM as a function of the initial temperatures displayed in Fig. 5. It can be seen that the NVM displays a sharp maximum for events with initial temperature of $T=4$ MeV.

where

$$\sigma^2_{A_{\max}}=\langle A^2_{\max}\rangle-\langle A_{\max}\rangle^2, \tag{3}$$

The brackets $\langle\cdot\rangle$ indicate an ensemble averaging.

As mentioned in the Introduction, one has to be sure that the signal that one is using to analyze a phenomenon assumed to be critical is not triggered by spurious fluctuations induced by conservation laws or finite size effects.

In order to test the NVM as a signal of critical behavior we adopt the following strategy.

(1) We test NVM in a simple random partition model [13]. In this model mass spectra are generated via a simple Monte Carlo procedure. Given a mass distribution of the form $Y(A)=Y_0A^{-\tau}$ where the exponent τ is bounded between 2 and 3, $2\leqslant\tau\leqslant3$ [19], we generate events with fixed total mass A_{tot}, having fragments of mass k and multiplicity m_k such that for each event $A_{\mathrm{tot}}=\Sigma_1^A km_k$. It is obvious that with this procedure the only fluctuations in the resulting spectra are of statistical type plus spurious effects related to the mass conservation. In such a process no signatures of phase transition should readily be observed.

The results of the NVM for this simple toy model are displayed in Fig. 2. In the upper part, we show the results for $2\leqslant\tau\leqslant3$, with $A_{\mathrm{tot}}=100$ and for free multiplicity (open squares) and fixed multiplicity (fixed to the most probable one for each value of τ, open circles). It can be seen that no maximum appears. The general picture is the same when we

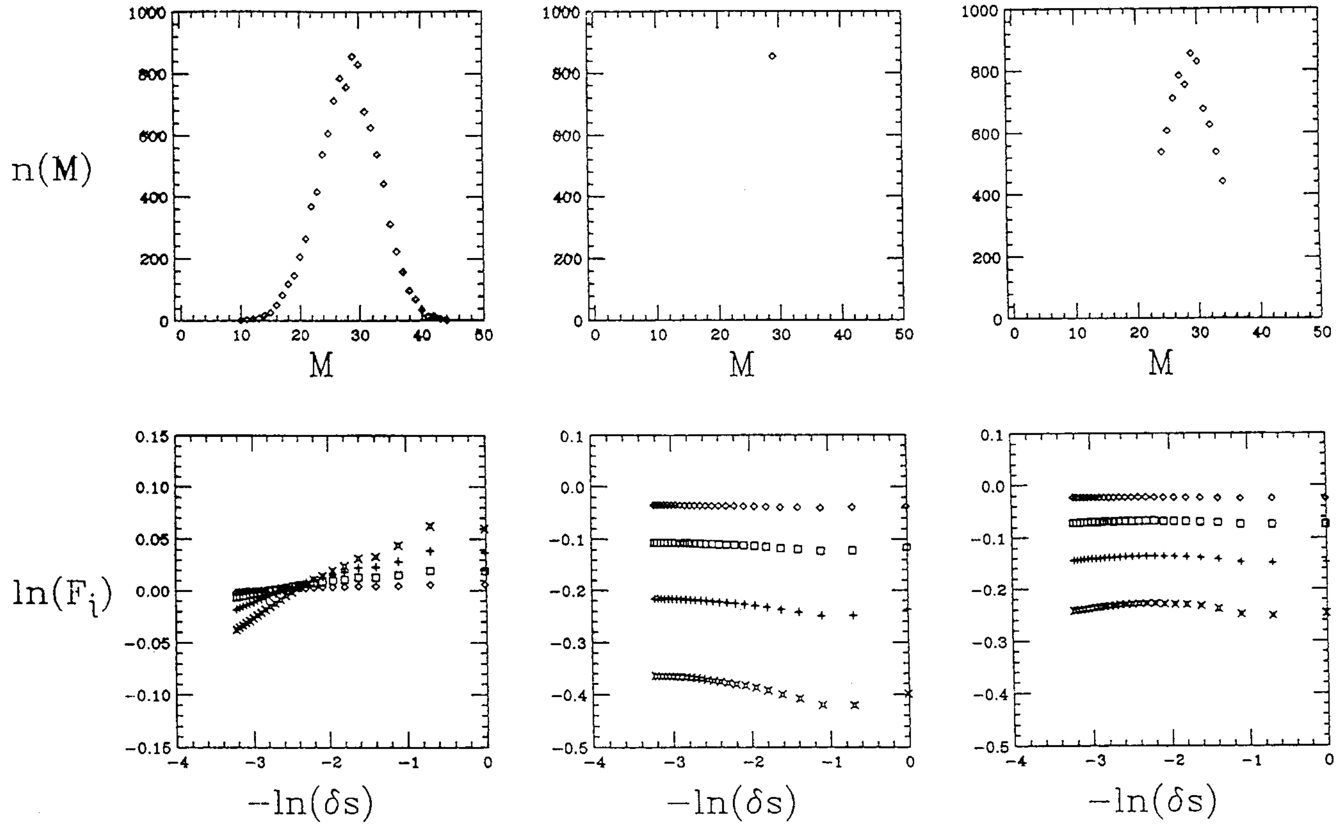

FIG. 7. In the upper panels we show the multiplicity distributions for the expansion of the system $A=100$ when different ranges of multiplicities are chosen and in the lower panels the corresponding intermittency signal at $T=5$ MeV. Left part: the multiplicity and intermittency when the complete set of 10 000 events is considered. Central part: in the upper panel we show the effect of retaining just one multiplicity (the most probable one) that involves 748 events; in the lower panel we see that the intermittency signal is gone. In the right panels we show the effect on the IS of using only the central events of the multiplicity distribution.

take $A_{\mathrm{tot}}=200$ shown in the lower part of the figure for the case where the multiplicity is fixed.

As an illustrative example we show the results of NVM when the simple power law distribution used in the random partition procedure for generating the mass spectra is replaced by the Fisher's law, which reads

$$Y(A)=Y_0A^{-\tau}X^{A^{2/3}}Y^A, \qquad (4)$$

where Y_0, τ, X, and Y are parameters which have been obtained by fitting Eq. (4) to the asymptotic spectra of molecular dynamics simulations at various initial temperatures (see I for details). The values of these parameters are summarized in Table I for different initial temperatures. It is worth emphasizing that the value obtained for τ is 2.23, in agreement with Fisher droplet model prediction for a liquid-gas phase transition at the critical point. In Fig. 3 we show the results of the NVM analysis for the randomly generated spectra. Once again no signal is obtained. It must be mentioned that for the case $T=3$ MeV the resulting parameters only fit appropriately the region of mass $1\leqslant A\leqslant 60$, so we introduced an artificial cut at mass $A=60$ to get spectra close to the results of the CMD calculation [14].

(2) We test our NVM in a system that displays true critical behavior. For this, we choose again, due to its simplicity, the bond percolation model [19]. In this case we take a simple cubic grid of different sizes, $5\times5\times5$, $10\times10\times10$, and $20\times20\times20$. Although true scaling can only be verified in the asymptotic limit, the well-known theory of renormalization group shows that even in small lattices the essential features of the phase transition can be verified [20].

The results of this analysis are displayed in Fig. 4. The upper part of this figure shows the resulting NVM for the three lattices as a function of the bond activation probability Q for unrestricted total multiplicity M, while the lower part shows the same quantity but for fixed multiplicity (once again taken as the most probable one for each value of Q) and for lattice sizes $5\times5\times5$ and $10\times10\times10$. A maximum can be readily observed in all the cases. This maximum has a location which depends on the size of the lattice, which is usually attributed to a manifestation of finite size effects.

From these results it is immediate to see that NVM is a good candidate for a criticality signal that is not triggered by unphysical fluctuations superimposed on arbitrary distributions.

We now calculate the NVM for the results of CMD simulations of the evolution of systems with 100 particles ($Z=50$, $N=50$) and no Coulomb interaction. We have chosen initial temperatures of 2, 3, 4, 4.5, 5, 5.5, 7, 10, and 15 MeV. In I it was found via intermittency analysis that the critical

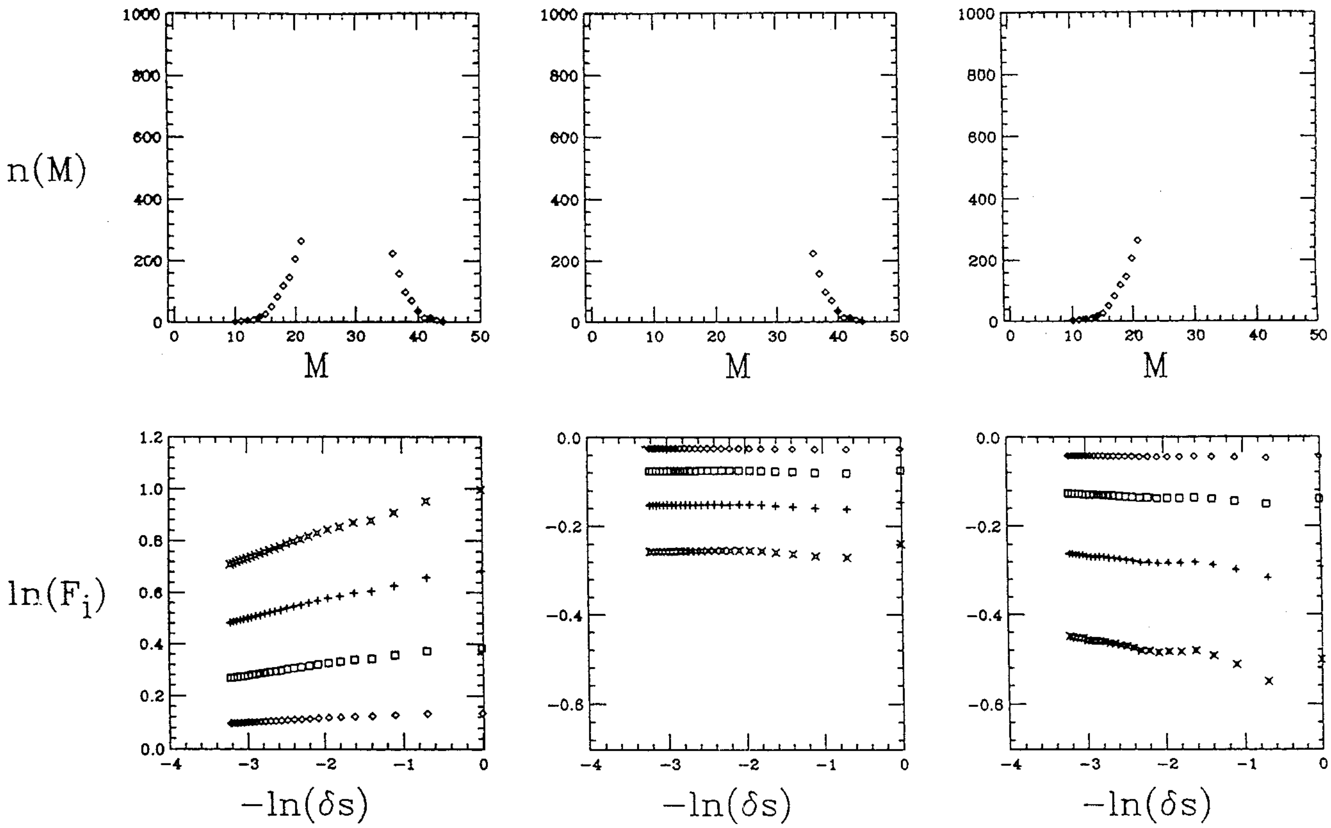

FIG. 8. Same as Fig. 7 but we show the effect of retaining only the liquidlike tail (right), the vaporlike tail (center), and when the two above mentioned are retained at the same time (left). It is clearly seen that for this last case not only intermittency is again present, but also it gets stronger.

region should be around the range of initial temperatures 4.5–5 MeV, the same region where the final spectra are of pure power law type. Within the same model it was also found from an analysis of the Lyapunov exponents that the maximum value for these exponents is obtained at an initial temperature of about 4.5–5 MeV [15].

In Fig. 5, we show the probability distribution of the mass of the maximum fragment for different initial temperatures (see caption for details). Figure 6 displays the average mass of the maximum fragment (upper part) and the normalized variance NVM (lower part) versus the initial temperature T. It is seen that the NVM signal shows a maximum located at $T=4$ MeV which is quite close to the value obtained from the IS and from the Lyapunov exponents of 4.5–5 MeV. This is completely consistent with the findings from the percolation analysis: the NVM shows a peak but the value for the critical point is somewhat shifted.

IV. INTERMITTENCY SIGNAL IN CMD SIMULATIONS

In order to look for critical behavior, one of the most widespread approaches is to make an intermittency analysis. Intermittency is a statistical concept used to analyze the fluctuations and correlations of a distribution. This concept has been widely used in various fields of physics such as turbulent flow [21], astrophysics, and magnetohydrodynamics, among others [22]. Bialas and Peschanski introduced this idea to study the dynamical fluctuations in rapidity distributions of particles from high-multiplicity events produced in ultrarelativistic reactions [10]. More recently Ploszajczak and Tucholski suggested looking for intermittency in the fragment distributions in nuclear multifragmentation at intermediate energies. They were able to see evidence for intermittent pattern of fluctuations in the fragment charge distributions both in data and in models [11]. Furthermore, lots of efforts have been devoted to find evidence for the occurrence of a phase transition of nuclear matter in the intermittent behavior of the multiplicity distributions [12,14,23–25].

Generally, the occurrence of intermittency is associated with the existence of large nonstatistical fluctuations which have self-similarity over a broad range of scales. This signal can be deduced from the scaled factorial moments which, for flat distributions, measure the properties of dynamical fluctuations without the bias of statistical fluctuations [10,11]:

$$F_i(\delta s)=\frac{\sum_{k=1}^{X_{\max}/\delta s}\langle n_k\times(n_k-1)\times\cdots\times(n_k-i+1)\rangle}{\sum_{k=1}^{X_{\max}/\delta s}\langle n_k\rangle^i}. \quad (5)$$

Here $X_{\max}$ is an upper characteristic value of the system (i.e., total mass or charge, maximum transverse energy or momentum, etc.) and i is the order of the moment. The total interval $0-X_{\max}$ ($1-A_{\max}$, $Z_{\max}$ in the case of mass or charge dis-

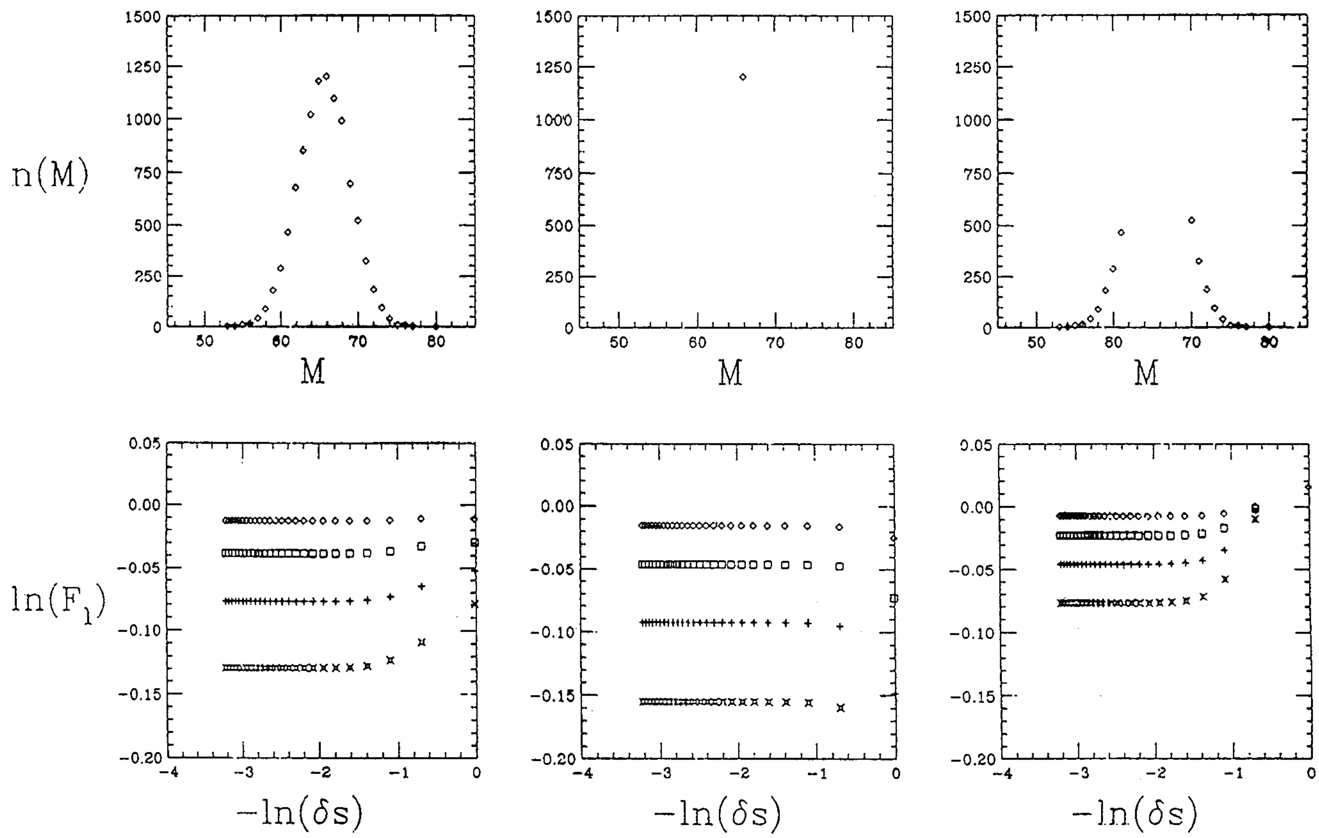

FIG. 9. Same as Fig. 7 but for the $T=15$ MeV case.

tributions) is divided in $M=X_{\max}/\delta s$ bins of size δs, n_k is the number of particles in the kth bin for an event, and the brackets $\langle\cdot\rangle$ denote the average over many events. If self-similar fluctuations exist at all scales δs, the scaled factorial moments (SFM's) follow the power law $F_i(\delta s)\propto(\delta s)^{-\lambda_i}$ where λ_i are called intermittency exponents. So the intermittent behavior is defined as a linear rise in a plot of $\ln(F_i)$ versus $-\ln(\delta s)$.

In order to explore the properties of the IS we calculate it for the asymptotic mass spectra of the already described fragmentation experiments in the CMD model. In Fig. 7 we plot the total multiplicity distributions (top panel) and the SFM's (bottom panel) for this $T=5$ MeV. In the left-upper panel the multiplicity distribution of *all* the particles is given. The corresponding SFM's are given below and show a clear IS. It has been shown in [8] that intermittency is a property of the distribution function describing the process. Experimentally we can calculate the event frequencies. Both quantities can only be related if we fix the multiplicity, in such a case

$$p\equiv m_0\langle n\rangle.$$

In accordance with to this relation we fix the multiplicity to, say, the most probable value $M=29$ in order to have more events, the corresponding IS completely disappears, and actually the slopes become negative (central panels). We also found that if we perform the analysis by considering the first bin only, the SFM's remain essentially the same in agreement with what was found in [8]. Thus the SFM's are essentially determined by the fluctuations of the lightest particles, and if we fix the total multiplicity, we impose a strong self-correlation that destroys the IS.

We now further explore the properties of the IS when the total multiplicity is not fixed. In order to see the physical origin of the slope in the SFM's, see Fig. 8, we made cuts in the multiplicity distributions by selecting all the events with multiplicity $M<22$, i.e., events where a big size fragment is present (mostly liquid than gas) and events with $M>35$, i.e., with many small particles (mostly gas than liquid) (right panels). The signal becomes stronger. Thus the physical origin of the slope is the mixing of events where sometimes there is mostly liquid and sometimes there is mostly vapor. In other words the IS is stronger when the initial temperature of the finite fragmenting system allows the presence of big fluctuations in the multiplicity of the asymptotic mass distribution. Note that when selecting only one of the tails of the distribution (the liquid part or the gas part) or when selecting the central part of the multiplicity distribution (i.e., $22\leqslant M\leqslant 35$), the IS does not show up. To confirm this feature, we have performed calculations at a much higher temperature $T=15$ MeV. In this case the mass distribution is exponentially decreasing, i.e., vaporization events (see Fig. 1). In Fig. 9 we plot the total multiplicity distributions (top panels) and the SFM's (bottom panels). In the first column one sees a Gaussian-type multiplicity distribution, similar to the $T=5$ MeV case, but centered at multiplicity $M=66$. Thus many small fragments are formed as a result of the large

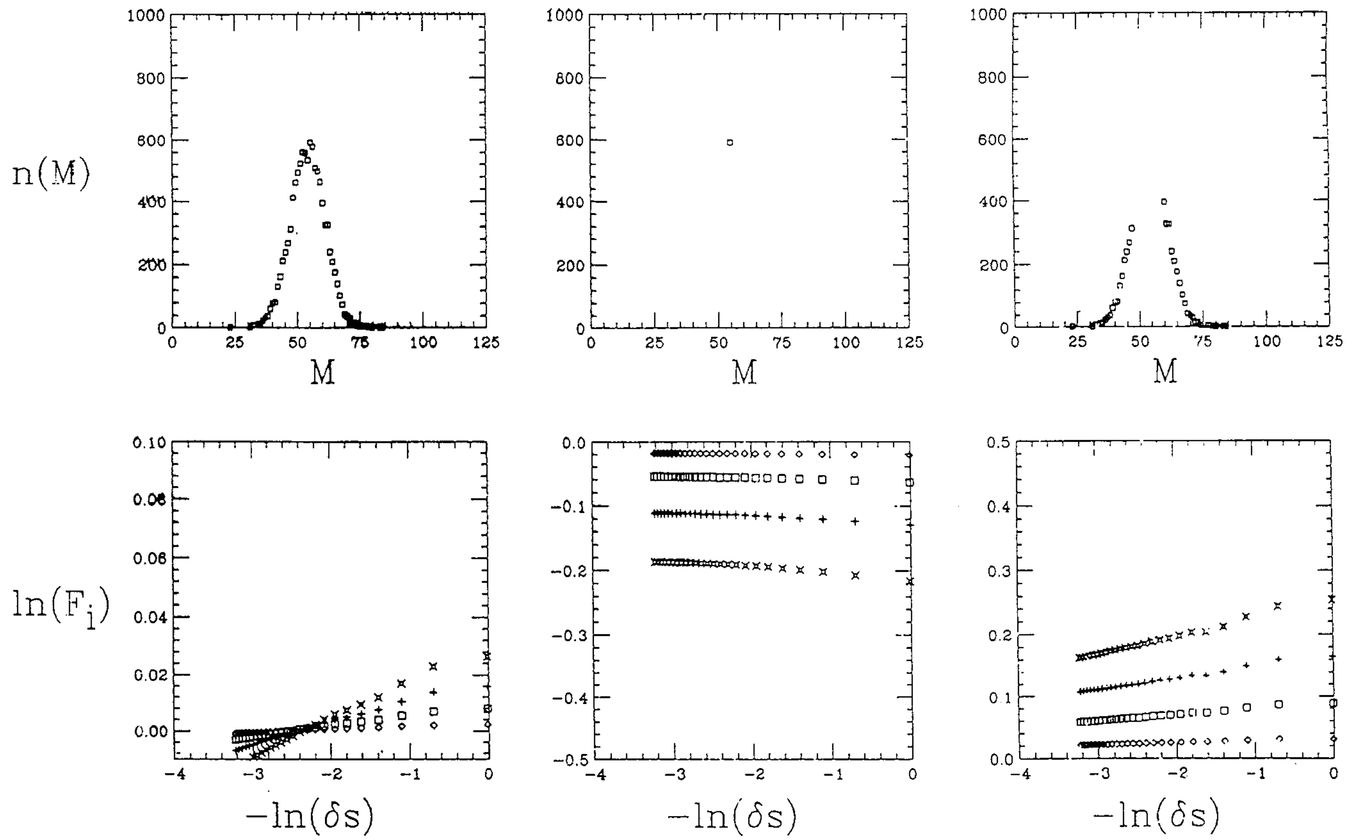

FIG. 10. Same as Fig. 7 but for percolation on a $5\times5\times5$ lattice for a value of the bond parameter $Q=0.24$.

initial excitation energy, and the corresponding fluctuations of the lightest fragments are small as is revealed by the SFM's (bottom panel). Also in this case if we fix the multiplicity of the events to the average value of 66, we get flatter and more negative SFM's, since the self-correlation is destroying the small fluctuations present in these events. If we repeat the game of choosing the events in the tails of the multiplicity distribution, we still get no signal in the SFM's. This is so because all the events have high multiplicity; i.e., we are always in the gas side.

In order to confirm our findings, we have performed similar calculations in a simple cubic bond percolation model of size $5\times5\times5$ [19]. In Fig. 10 we plot our results for values of the bond parameter Q: $Q=0.24$ close to the critical point; we have also calculated for $Q=0.44$ ''evaporation'' events and $Q=0.08$ ''vaporization'' events. We have found exactly the same behavior as in CMD even for the evaporation events. Note, however, some differences in the average values (for instance, at the respective critical points) of the multiplicity and the slopes of the IS which cannot be recovered even after a simple scaling to take into account the slight different masses (100 in CMD and 125 in percolation). We would like to stress that the value for which the IS is obtained is very close to the critical value for an infinite system $Q_c=0.23$, thus suggesting that IS can give information about the true critical point.

To summarize the findings of this section, we have shown that the the maximum of the IS is found for the events in which stronger fluctuations are expected, i.e., those for which the spectra is power like and for which stronger mixing of ''liquid''-like events and ''vapor'' like ones is expected. In this way, the IS could signal the presence of such a mixing. Nevertheless, its serious drawbacks (for example, being triggered by trivial fluctuations as in the random partition model) disqualify it as a useful signature of criticality.

V. CONCLUSIONS

In conclusion, we have studied the intermittency signal and the normalized variance of the size of the largest fragment in the framework of the bond percolation model and molecular dynamics model. According to the obtained results the following aspects clearly emerge.

The NVM displays a clear maximum in the critical region for all the truly critical models analyzed, while it is not triggered by the unphysical fluctuations emerging in the random partition model

On the other hand, the IS, which is ill defined theoretically, displays a maximum when liquidlike events are mixed with vaporlike events; moreover, its occurrence in some data, as for example, in the random partition model described above, is not a sufficient condition for criticality.

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