

## B.Sc. EXAMINATION BY COURSE UNITS

## MAS115 Calculus I (first sit paper)

Thursday 1 May 2008, 14:30 – 16:30

*The duration of this examination is 2 hours.*

*You should attempt all questions. Marks awarded are shown next to the questions.  
Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.*

Candidates must not remove the question paper from the examination room.

**YOU ARE NOT PERMITTED TO START READING THIS QUESTION  
PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR**

1. (a) [5 marks] Find the limits

$$\lim_{x \rightarrow -3^-} (x+4) \frac{|x+3|}{x+3} \quad \text{and} \quad \lim_{x \rightarrow -3^+} (x+4) \frac{|x+3|}{x+3}.$$

- (b) [5 marks] Find equations of all lines having slope  $-2$  that are tangent to the curve

$$y = \frac{18}{x+9}.$$

- (c) [5 marks] If

$$x^3 + y^3 = 56,$$

find the values of  $dy/dx$  and  $d^2y/dx^2$  at the point  $(-2, 4)$ .

- (d) [5 marks] Find the absolute maximum and minimum values of the function

$$f(x) = \frac{5}{6}x - 9$$

on the interval  $[-4, 7]$ .

- (e) [5 marks] Find the limit

$$\lim_{x \rightarrow 0} \frac{1 - \cos(6x)}{36x^2}.$$

- (f) [5 marks] Find

$$\frac{d}{dx} \int_{\sqrt[3]{x}}^{\pi/6} \cos(t^3) dt.$$

- (g) [5 marks] Find the area of the region enclosed by the curves

$$y^2 - x = 1 \quad \text{and} \quad x - y = 1.$$

- (h) [5 marks] Find the derivative  $y'(t)$  of

$$y = 7^{\sin(6t)}.$$

- (i) [5 marks] Evaluate the integral

$$\int \frac{16 dx}{1 - \cos(8x)}.$$

- (j) [5 marks] Evaluate the integral

$$\int_{1/3}^7 8x \ln(3x) dx.$$

[Next question overleaf]

- 2.** [20 marks] Consider the curve  $y = f(x)$  for the function

$$f(x) = \frac{x}{1+x^2} .$$

- (a) Identify the domain of  $f$  and any symmetries the curve may have.
- (b) Find  $f'(x)$  and  $f''(x)$ .
- (c) Find the critical points of  $f$ , and identify the function's behaviour at each one.
- (d) Find where the curve is increasing and where it is decreasing.
- (e) Find the points of inflection, if any occur, and determine the concavity of the curve.
- (f) Identify any asymptotes.
- (g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve.
- (h) Is the area enclosed by the curve and the positive  $x$ -axis finite? If so, what is its value?

- 3.** [10 marks] Consider the family of curves given by

$$f_a(x) = 2x^3 + ax^2 + 1 , \quad a \in \mathbb{R} .$$

- (a) For fixed  $a$ , compute the critical point(s) of each curve.
  - (b) When varying  $a$ , the set of all critical points lie on a new curve. Compute the equation of that curve.
- 4.** [10 marks]

- (a) What are the hypotheses and conclusions of the Mean Value Theorem For Definite Integrals?
- (b) Using this Theorem, deduce the following statement:

If

$$\int_a^b f(x) dx = 0$$

for a function  $f$  which is continuous on the interval  $[a, b]$  with  $a < b$ , then  $f(x) = 0$  at least once in  $[a, b]$ .

- 5.** [10 marks]

- (a) State the definition of the derivative of the function  $f(x)$  with respect to the variable  $x$ .
- (b) Given that

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 ,$$

differentiate from first principles  $f(x) = e^x$ .