Preface

The term *small systems* denotes objects composed of a limited, small number of particles, as is typical for matter on meso- and nanoscales. The interest of the scientific community in small systems has been boosted by the recent advent of micromanipulation techniques and nanotechnologies. These provide scientific instruments capable of measuring tiny energies in physical systems under *nonequilibrium* conditions, that is, when these systems are exposed to external forces generated by gradients or fields. Prominent examples of small systems exhibiting nonequilibrium dynamics are biopolymers stretched by optical tweezers (as shown in the lower picture on the book cover), colloidal particles dragged through a fluid by optical traps, and single molecules diffusing through meso- and nanopores.

Understanding the *statistical physics* of such systems is particularly challenging, because their small size does not allow one to apply standard methods of statistical mechanics and thermodynamics, which presuppose large numbers of particles. Small systems often display an intricate interplay between microscopic nonlinear dynamical properties and macroscopic statistical behavior leading to highly non-trivial fluctuations of physical observables (cf. the upper picture on the book cover). They can thus serve as a laboratory for understanding the emergence of complexity and irreversibility, in the sense that for a system consisting of many entities the dynamics of the whole is more than the sum of its single parts.

Studying the behavior of small systems on different spatio-temporal scales becomes particularly interesting in view of nonequilibrium transport phenomena such as diffusion, heat conduction, and electronic transport. To understand these phenomena in small systems requires novel theoretical concepts that blend ideas and techniques from nonequilibrium statistical physics, thermodynamics, stochastic theory and dynamical systems theory. More recently it has become clear that a central role in this field is played by *fluctuation relations*, which generalize fundamental thermodynamic relations to small systems in nonequilibrium situations.

The aim of this book is to provide an introduction for both theorists and experimentalists to small systems physics, fluctuation relations, and the associated research topics listed in the word cloud diagram shown below. The book should also be useful for graduate-level students who want to explore this new field of research. The single chapters have been written by internationally recognized experts in small systems physics and provide in-depth introductions to the directions of their research. This approach of a multi-author reference book appeared to be particularly useful in view of the vast amount of literature available on different forms of fluctuation relations. While there exist excellent reviews highlighting single facets of fluctuation relations, we feel that the field lacks a reference that brings together the most important contributions to this topic in a comprehensive manner. This book is an attempt to fill the gap. In a way, it may act itself as a complex system, in the sense that the book as a whole ideally yields a new picture on small systems physics and fluctuation...
relations emerging from a synergy of the individual chapters. Along these lines, our intention was to embed research on fluctuation relations into a wider context of small systems research by pointing out cross-links to other theories and experiments. We thus hope that this book may serve as a catalyst both to fuse existing theories on fluctuation relations and to open up new directions of inquiry in the rapidly growing area of small systems research.

Accordingly, the book is organized into two parts: *Part I* introduces both the theoretical and experimental foundations of *Fluctuation Relations*. It starts with a three-fold opening on basic theoretical ideas: The first chapter features a pedagogical introduction to fluctuation relations based on an approach that was coined ‘stochastic thermodynamics’. The second chapter outlines a fully deterministic theory of fluctuation relations by working it out both analytically and numerically for a particle in an optical trap. Chapter three generalizes these deterministic ideas by also establishing cross-links to the Gallavotti-Cohen fluctuation theorem, which historically was the first to be established, with mathematical rigor, for nonequilibrium steady states. After this theoretical opening, the following two chapters summarize groundbreaking experimental work on two fundamental types of fluctuation relations: Along the lines of Gallavotti and Cohen, the first subset of them is often referred to as ‘fluctuation theorems’ generalizing the second law of thermodynamics to small systems (see the first formula on the book cover). This type of fluctuation formulas is tested experimentally in systems where particles are confined by optical traps under nonequilibrium conditions. ‘Work relations’, on the other hand, generalize an equilibrium relation between work and free energy to nonequilibrium (see the second formula on the book cover). The result is tested in experiments where single DNA and RNA chains are unzipped by optical tweezers. The remaining
three chapters of Part I elaborate on aspects of fluctuation relations that moved into the focus of small systems research more recently: The first one introduces the nonequilibrium thermodynamics of information processing by using feedback control. The second one reviews quantum mechanical generalizations of fluctuation relations applied to electron transport in mesoscopic circuits. The third one discusses generalizations of fluctuation relations for stochastic anomalous dynamics with cross-links to experiments on biological cell migration.

Part II goes Beyond Fluctuation Relations by reviewing topics that, while centered around nonequilibrium fluctuations in small systems, do not elaborate in particular on fluctuation relations. It starts with a discussion of fluctuation-dissipation relations, which are intimately related to, but may not be confused with, fluctuation relations. A cross-link to the foregoing chapter is provided in terms of partially studying anomalous dynamics, a topic that becomes particularly important for heat conduction in nanostructures, as is demonstrated from both an experimental and a theoretical point of view in the subsequent chapter. Fluctuation relations bear an important relation to large deviation theory, as is outlined in the next chapter, with applications to interacting particle systems. The book concludes with a summary about Lyapunov modes, which provide important information about the phase space dynamics in deterministically chaotic interacting many-particle systems, and experiments about diffusion in meso- and nanopores by performing single molecule spectroscopy.

We finally remark that the various points of view expressed in the single chapters may not always be in full agreement with each other. This became clear in lively discussions between different groups of authors when the book was in preparation. As editors, we do not necessarily aim to achieve a complete consensus among all authors, as differences in opinions are typical for a very active field of research such as the one presented in this book.

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