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**Tab. 1.1** Prof. Radu Balescu († June 1st, 2006); photo by Jacques Misguich, July 2005, Provence

## 1

### **In Memoriam: Radu Balescu**

*Boris Weyssow, Angelo Vulpiani, Francesco Mainardi, Raul Sánchez and Diego del-Castillo-Negrete*

The first chapter of this book is dedicated to Professor Radu Balescu, who very unexpectedly passed away on June 1st, 2006, during a trip to Romania at the age of 73 years. Radu Balescu was invited as one of the keynote speakers to the conference on *Anomalous transport: experimental results and theoretical*

*challenges* in Bad Honnef, Germany, which took place about one month later in July 2006. This conference initiated the writing of the present book. Being a world class leader in the theory of nonequilibrium transport in plasmas, before his death Radu Balescu developed a very strong interest in anomalous dynamical processes. We start this chapter by including the abstract of the talk that Radu Balescu was intending to give in Bad Honnef.

Radu Balescu was planning to meet several colleagues at this conference for the very first time, with whom before he had intensive e-mail correspondences on topics of anomalous transport. These colleagues, as well as other conference participants who earlier had the privilege to meet Radu Balescu, have been invited to contribute to this book with their own obituaries. Their memories are added after a short summary of the scientific career of Radu Balescu, written by one of his closest collaborators.

### 1.1

#### **Radu Balescu's abstract for the conference on anomalous transport in Bad Honnef**

##### **Statistical mechanics and strange transport**

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In recent years Continuous Time Random Walks (CTRW) and Fractional Differential Equations (FDE) have proved to be extremely successful modelling techniques for describing a wide range of applications for which standard diffusive transport is found experimentally to be inadequate. Yet there does not exist a complete justification of these concepts based on first principles of mechanics. A tractable starting point is provided by a “semi-dynamical” approach, based on a V-Langevin equation: an equation of motion of Newtonian (or Hamiltonian) type for a tracer particle moving in presence of a random potential. Associated with it there is a “hybrid kinetic equation” (HKE) for the (stochastic) distribution function  $f(x, t)$  of the positions. By standard methods of statistical mechanics an equation of evolution of the ensemble-average of this function, called the “density profile”  $n(x, t) = \langle f(x, t) \rangle$  is derived. The latter is, however, not closed because of its non-local character: in its right hand side appears, under an ensemble-average, the density profile evaluated at the fluctuating position of the particle, together with other fluctuating quantities. The usual “local approxi-

mation" provides a good description of normal diffusive processes, but is inadequate for strange transport (sub- or supra-diffusive).

Recently, Sánchez et al.<sup>1</sup> used an elegant method for overcoming the non-locality difficulty, based on functional integration techniques applied to the fluctuating particle trajectories (supposed to be self-similar) in order to derive a closed non-local equation for the density profile. Under special assumptions this equation can be reduced to a FDE.

In the present work we use a quite different approach, based on an analysis of the various types of propagators appearing in the treatment of the HKE. We introduce a non-local extension of an approximation similar to the Corrsin factorization assumption of turbulence theory. The result is a non-Markovian and non-local, formally linear equation, in which the rate of change of the density profile  $n(x, t)$  at point  $x$  and time  $t$  is related to the values of this function at neighbouring points  $x + r$  and at past times  $t - T$ . Its structure is similar, but not identical (because of different approximations) with the equation of Sánchez et al. On the other hand, it can be shown that there exists an "equivalent" CTRW. The transition probability in the Montroll-Shlesinger equation describing the latter is related to the Eulerian velocity autocorrelation and to the ensemble-averaged propagator of the HKE  $\langle G(x, t|x', t') \rangle$ . Under certain special assumptions on the form of the latter two quantities (such as self-similar power-law forms), the equation for the density profile can be reduced to a fractional differential equation.

When viewed from a more general point of view, the equation of evolution for  $n(x, t)$  is readily transformed into an equation for the average propagator  $\langle G(x, t|x', t') \rangle$ . The latter provides, as usual, the solution of the Dirichlet problem of the former equation for arbitrary initial condition  $n(x, 0)$ . Besides its non-Markovian and non-local character, this equation appears to be explicitly nonlinear. Thus, – not surprisingly – even in this simplest "non-local Corrsinlike" approach, one is faced with the complexity of a nonlinear process. A selfconsistent theory of strange transport should therefore involve adequate approximation methods (such as renormalization techniques) for treating this equation.

## 1.2

### The scientific career of Radu Balescu by Boris Weyssow

Professor Emeritus Radu Balescu unexpectedly passed away first June 2006 during a trip in Romania at the age of 73 years. Professor at the Université Libre de Bruxelles, Member of the Royal Academy of Science, Humanities and Fine Arts of Belgium, Honorary Member of the Romanian Academy, Radu

1) see Ref. [1], the editors

Balescu was at a very young age recognized internationally as a leader in the development of the Statistical Physics of charged particles (the well-known Balescu-Lenard collision operator) and of the theory of transport in magnetically confined plasmas.

Professor Radu Balescu was born in 1932 in Bucharest, Romania, of a Belgian mother, and acquired Belgian nationality in 1959. He studied at the Université Libre de Bruxelles and from 1957 until 1961 was the assistant of Professor Illya Prigogine (Nobel laureate in 1977). Professor Radu Balescu made original contributions in the kinetic theory of plasmas and published his first book *Statistical Mechanics of Charged Particles* (Interscience, 465 pp) in 1963 followed in 1975 by a comprehensive treatise of statistical mechanics *Equilibrium and Nonequilibrium Statistical Mechanics* (Wiley, 742 pp). The main purpose of this book was to achieve a presentation that would be as unified as possible with a very wide coverage of non-equilibrium theory, not treated in standard textbooks of that time. It also introduced for the first time in a textbook some new concepts, such as the renormalization group theory of critical phenomena, which turned out to be very successful in forthcoming years. This book is still very widely cited in the literature. Both this book and the previous one have been translated into Russian by leading Russian theorists.

Until becoming Emeritus in 1997, he had been Ordinary Professor at the University of Brussels. He has lectured or has been visiting Professor all over Europe, in Kyoto, Austin and Mexico. Professor Radu Balescu was involved in the European fusion programme for more than 30 years as a scientist and as the head of research unit of the ULB group in the Euratom-Belgian state Association.

In 1988, Professor Balescu published a magnum opus *Transport Processes in Plasmas* (North-Holland, 803 pp) *Volume 1: Classical transport; Volume 2: Neoclassical transport*. This set of two volumes presents a comprehensive review of the classical and the neoclassical theories of transport in plasmas, especially in the regime relevant to thermonuclear fusion. This is one of the first fully consistent presentations of the theory. It starts from first principles (Hamiltonian mechanics), going through kinetic theory, and ending in the explicit calculation of the transport coefficients and the discussion of the thermodynamic aspects of the transport. The second volume describes the non-trivial influence of the toroidal geometry (existing in magnetically confined plasmas) on the transport. Again, the main aim here was a unified presentation of the theory. It should be emphasized that these volumes encompass not only the work of many authors but that they are original in their conception and include many hundreds of pages of new consistent calculations and results. They are considered by many theorists as the current bible for plasma transport processes and has already been translated into Chinese.

A third volume on transport was about to be finished at that time. However Professor Balescu found that some elements of the mathematical theory of stochastic processes and of turbulence were not advanced enough to be published as chapters of a book. It is in 2005 that *Aspects of anomalous transport in plasmas* was finally published by IOP press. The final version contains several chapters on the kinetic theory of turbulence (quasilinear theory, renormalization, DIA) but also includes a thorough discussion on random processes with applications to fusion experiments. This book also extends *Statistical dynamics: matter out of equilibrium* published in 1997 by Imperial College Press which is a survey of nonequilibrium statistical mechanics, from Hamiltonian systems to complex systems and summarizes the first considerations of the author on CTRW and percolation processes.

This survey of Professor Balescu's work amply demonstrates the breadth and the depth of his contribution, not only to theoretical basic plasma and fusion physics, but also to statistical physics in general. It encompasses the conceptual roots of the disciplines and addresses the most challenging present-day problems. The excellence of his contributions has early been recognized by the most prestigious Belgian Scientific prize : "Le Prix Francqui" in 1970. He was a member or foreign member of several academies. He was awarded the first von Engel Prize of the International Conference on Phenomena in Ionised Gases in 1999 and the first Hannes Alfvén Prize of the Plasma Physics Division of the European Physical Society in 2000.

Radu Balescu was not only a recognized scientist but also a very appreciated teacher. All along his career, he spread his love of physics amongst his students with brilliance and enthusiasm. Radu Balescu will also be remembered as a true and respected humanist. He was constantly guided by exceptional honesty and integrity in his scientific career, in the management of his group and in his relationship with colleagues.

### 1.3

#### **My memory of Radu Balescu by Angelo Vulpiani**

I had the opportunity, and privilege, to know Radu Balescu and to collaborate with him. I guess it is not necessary to spend time to remind of his long career, his seminal contributions to Statistical Physics and Plasma Physics (one can just mention the celebrated Lenard-Balescu equation and its basic role in the statistical kinetic theory of charged particles), and the important awards, as the Hannes Alfvén Prize. For a discussion of his late contributions to anomalous transport see [2].

In autumn 1984 I was in the CEA Plasma Physics Center of Fontenay-aux-Roses (then the Center moved to Cadarache) working on the diffusion of

particle-guiding centers across a strong magnetic field, caused by a turbulent electrostatic field. The research was a project with Marco Pettini and Jacques Misguich and was mainly based on some ideas by Radu Balescu. As consequence of a funny interactions with the CEA bureaucracy, Marco Pettini and I went to Bruxelles where we had the opportunity to use a free good computer, and overall to get to know Radu.

My knowledge of plasma physics was (and still is) rather elementary, fortunately our research was basically a statistical mechanics problem. So Marco and I had not particular difficulties in the computational and theoretical aspects, but we had very vague ideas of the real physical relevance of our results. We thought that the agreement between the numerical results and the theoretical prediction for the diffusion coefficient was very bad (their ratio was between 1.5 and 4), so we informed Radu that the computation based on his theory was not so good. Immediately Radu realized how naive we were. With tolerance and irony he explained to us that plasma physics is a very intricate field, where it is not enough to solve equations, but overall it is necessary to be aware of the role of the assumptions and approximations. Then he concluded by saying that, since in that issue the ratio between theory and experiment was typically between 10 and 100, the result was really nice [3,4]. At first glance I thought he was joking, but Radu invited us for dinner to celebrate.

At that time when I was a young assistant professor, Radu was already a well known scientist, although he was still associate professor (sic!). In spite of the clear difference of our positions in the international scientific ranking, Radu treated me as a young colleague. I remember him in his small office, in a forest of partial differential equations, special functions and asymptotic expansions, spending the major part of his time writing a magnum opus on transport processes in plasma physics [5]. However everybody could enter and discuss science with him. Radu Balescu was a master of statistical mechanics with a deep physical insight. He will certainly be missed, but his scientific legacy will continue to be influential.

#### 1.4

##### **My memory of Radu Balescu by *Francesco Mainardi***

My first acquaintance with Radu Balescu was in the late 1990's through his book *Statistical Mechanics - Matter out of Equilibrium* (Imperial College Press, 1997). I found his treatment of statistical mechanics very innovative compared with those in the classical books and I much appreciated his chapter on non-Gaussian stable probability distributions and generalized master equations. A few years earlier, these topics had reawaken in me the interest towards the

potential of fractional calculus in statistical physics, in particular to explore the phenomena of anomalous diffusion and transport. So, during a visit at the Université Libre de Bruxelles (ULB), I had the opportunity to meet Radu Balescu in his office and to express to him my appreciation and interest for his approach to statistical mechanics. I also pointed out where in that book possible connections with fractional calculus could be envisaged. Unfortunately, this brief colloquium did not have any immediate consequence, since it was forgotten by himself, as it was recognized later.

It was a (nice) surprise for me when on 1st November 2005 I received the following kind letter from him:

*Dear Professor Mainardi,*

*Please let me introduce myself. I am Radu Balescu, professor emeritus at the Université Libre de Bruxelles. I have worked throughout my career on non-equilibrium statistical mechanics, and in particular on its application to plasma physics. I wrote (among others) two books which may be of interest to you: "Statistical Mechanics - Matter out of Equilibrium" (Imperial College Press, 1997) and "Aspects of anomalous transport in plasmas" (Taylor and Francis, 2005). They contain sections on strange (= anomalous) transport, a subject that fascinated me for a long time.*

*Recently I discovered your works on fractional differential equations - in particular the one in Fractional Calc. Appl. Anal. 2001 - (see [6]) which I found wonderful, by their content and the clarity of the exposition. I regretted strongly that I did not know them before the completion of my latest book. I should greatly appreciate your sending me your latest works in this area.*

*I am attaching a text containing my latest work in this domain. It is a "long version", not intended for publication as such. It contains many details (it was the basis of some seminars I delivered here). I should sincerely appreciate your having a look at it and letting me know your comments and suggestions.*

*With best regards, Radu Balescu*

From then on, a kind exchange of e-mails was carried out with information about the conferences appointed in the near past and future, related in some way to the applications of fractional calculus in statistical physics, that is the Workshop *In Search of a Theory of Complexity* organized by A. Allegrini, P. Grigolini and B.J. West at Denton (Texas), August 2005 and the Workshop *Anomalous Transport: Experimental Results and Theoretical Challenges* organized by R. Klages, G. Radons and I.M. Sokolov at Bad Honnef (Germany), July 2006.

For the former Workshop, which was attended by myself but not by him, the organizers invited him to submit his contribution to the special issue of *Chaos Solitons and Fractals*. For the latter Workshop, both of us agreed to participate: in our intentions this event would have provided the occasion for our meeting

and exchange of ideas on topics of common interest. Unfortunately, just a few weeks before the Workshop in Bad Honnef, exactly on June 2nd, I received from Prof. Jacques Misguich a circular mail (being entered in the mailing list of Radu) with the bad news of Balescu's death on June 1st 2006 in Romania.

Consequently, his interesting contribution for the Denton special issue appeared posthumous [7]: the Guest Editors dedicated that issue to his memory, see [8]. The abstract of his invited lecture at Bad Honnef has been included by the Editors in the present book. The last version of his lecture notes on anomalous transport, based on the applications of fractional calculus (the object of our exchange of e-mails), was submitted by his colleagues to the Physics section of the *Cornell University Archives* in April 2007, see [9]. Both posthumous contributions appeared with the same title "V-Langevin Equations, Continuous Time Random Walks and Fractional Diffusion" but they were edited with different purposes: the former as a research paper, the latter as a didactic review paper. They provide the last messages to the scientific community from a great man, who had spent his life for research and teaching with enthusiasm. Personally, I have lost with him a great pen pal but overall a possible mentor and collaborator.

### 1.5

#### **In Memoriam: Radu Balescu by Raul Sánchez**

The unexpected passing away of Radu Balescu saddened deeply his numerous friends and colleagues. Our group had the privilege of interacting with him frequently during the last two years of his life. In what follows, I will recall some of my personal memories about him, rather than regarding discussing his great scientific achievements, which are well known.

I first heard Radu Balescu's name when I was in my fourth year in college, pursuing my Physics degree. That year, I had to take a rather hard course on Statistical Mechanics, and I remember being forever grateful to this Radu Balescu guy for the great book he had written on this topic [10], which helped me get through the course successfully and even learn some Physics along the way. At the time, I knew nothing of who he was and of his many important contributions to the field of Plasma Physics. Little did I suspect that I was to end up, a few years later, working in that very same beautiful but extremely hard part of Physics myself. As I was becoming acquainted with this new phase of my career, his books paid again another great service to me: I pretty much learnt neoclassical transport in toroidally confined plasmas by using, side by side, Hirshman & Sigmar's famous review article [11] together with Radu Balescu's two-volume book on the subject [5].

Therefore, having used extensively his books for many years, I held a deep respect for Radu Balescu's scientific achievements and pedagogical abilities

as I advanced in my scientific career. I had never met him personally and he almost surely ignored my existence. But precisely for that reason, I became extremely excited when, one spring morning of 2004 (I was visiting Ben Carreras at Oak Ridge National Laboratory at the time), Boudewijn van Milligen wrote us from CIEMAT (Madrid, SPAIN) to tell Ben and me that Radu Balescu himself had e-mailed him to congratulate us for a very recent paper of ours [12]. The paper dealt with some initial exploration of the use of ideas based on the Continuous-time Random Walk (CTRW) concept to advance the understanding of radial turbulent transport in magnetically confined plasmas. We knew that Radu Balescu was himself a CTRW aficionado, since he had published an earlier paper on CTRWs and magnetic turbulent transport [13], but his great interest in our findings came to us as a very pleasant surprise.

This confluence of common interests triggered a very intense collaboration between Radu Balescu and us during the last two years of his life. We started to exchange numerous e-mails with notes, drafts of papers, calculations, comments, questions, doubts, criticisms and what not. In them, we discussed details of what, to the best of my knowledge, was the last physics problem Radu Balescu attacked: the derivation of superdiffusive fractional differential equations (FDEs) from what he called the  $V$ -Langevin kinetic equation [14]. Fractional equations are differential equations that contain the rather esoteric fractional derivatives and which have become quite fashionable lately. They appear as the natural fluid limit (at very long times and long distances) of CTRWs. Thus, if CTRWs or FDEs are to provide a reasonable description of turbulent transport, they should be derivable, at least in some limit, from more standard continuous equations. The  $V$ -Langevin equation, which is simply the continuity equation for the density of particles which are advected by a flow ( $V$ ) with prescribed statistical and correlation properties, provides one such starting point for the derivation. Both Radu and us attempted to complete it frantically. Radu did not work together with our team (composed of Ben, Boudewijn, David Newman from the University of Alaska, Vickie Lynch from Oak Ridge and myself), but in parallel. The path we chose to attack the derivation was quite different from the one Radu chose to pursue. But Radu kept our progress under close scrutiny as much as we kept his. In the end we completed the derivation, not without Radu pointing out several shortcomings and improving it through his comments [1]. Radu also completed his own derivation, but we never got to see it. He died in Romania a few weeks before we were to meet in person at Bad Honef, for the first time, to discuss the details of his approach and compare it with ours. But even when we never got to see each other in person, we got to meet his real persona during this time of intense collaboration. And he turned out to be much more than the very bright and technically gifted scientist we expected him to be. We met a man with the greatest scientific honesty and as critical with our ideas as he was

with his own. Regretfully, this kind of "symmetry" is not commonly found in the competitive arena that scientific research has become. It is just one of the reasons for which, without a doubt, I can say that he will be missed.

## 1.6

### Remembering Radu Balescu by *Diego del-Castillo-Negrete*

Radu Balescu once wrote: "I have been 'in love' with the statistical mechanics of plasma physics over my whole scientific career" [5]. Out of this love, important contributions to the study of anomalous transport in plasmas in general, and magnetically confined fusion plasmas in particular, were born. Of special value are Balescu's seminal contributions on the application of non-Gaussian stochastic processes to the description of anomalous transport. These works set the path of a very promising and elegant research direction that was sadly interrupted by Radu's unexpected death. Understanding anomalous transport is one of the main roadblocks to achieve controlled nuclear fusion in magnetically confined plasmas. Radu Balescu's original and unique perspective on this subject will prove to be very valuable in the years to come.

It is a great honor to share in this brief note my personal acquaintance with Radu during the last years of his life. Although at a long distance, this interaction was extremely valuable to me. As many scientists of my generation, I first learned about Radu Balescu from his books on statistical mechanics and plasma transport. However, my first contact with Radu at a more personal level came at the end of 2004 when I sent him a brief e-mail along with a paper that I had published in August of that year. The work focused on anomalous, superdiffusive transport in plasma turbulence. In particular, it discussed how the CTRW (continuous time random walk) formalism was a natural framework to describe this problem, and showed that the probability distribution function of particle displacements can be obtained from the propagator of a space-time fractional diffusion equation. Radu's response was prompt and enthusiastic, he was "...happy to see that the CTRW philosophy continues its development!"

Radu's interest on the application to the CTRW model to study transport in plasmas goes back to 1995 when he proposed for the first time the application of this formalism to study anomalous transport in magnetized plasmas [13]. However, despite the close connection between the CTRW and fractional diffusion, Radu was at the beginning skeptical about the value of fractional diffusion. This explains why fractional diffusion did not play a prominent role in his book on anomalous transport [14].

However, this situation changed rapidly during 2005. In an e-mail exchange on February of that year he wrote "...I became convinced (for the first time)

of the usefulness of fractional calculus. All the papers I read previously about the subject left me with an impression of a heavy and untransparent formalism, which did not give me an impetus for further study. After reading your paper I clearly understand that fractional calculus is, indeed, an indispensable tool for a fine analysis of the CTRW." Over the next few months Radu became increasingly interested in the fractional diffusion approach to plasma turbulence, and we had the opportunity to discuss the topic in considerable detail. During this time, I was thankful to Radu's kindness, enthusiasm, and support.

In August 2005 Radu circulated a set of notes entitled "V-Langevin Equations leading to superdiffusion". Although with his characteristic modesty Radu referred to them as preliminary, the notes contained not only a valuable summary of fractional calculus, but also a very interesting discussion on the connection between particle dynamics with random velocity (V-Langevine equation), the CTRW model and fractional diffusion. Elaborating on these notes, Radu prepared an extended manuscript including review material, and a short research paper, both entitled "V-Langevine Equations, Continuous Time Random Walks and Fractional Diffusion." At this point Radu was fully engaged in the study of fractional diffusion. The year was 2006 and I had received an invitation to participate in the summer workshop on "Anomalous Transport: Experimental Results and Theoretical Challenges" in Bad Honnef, Germany. I was delighted to accept the invitation that would give me the opportunity to meet Radu in person, and to learn more about his most recent ideas. Sadly, he unexpectedly passed away before the meeting on June 1st during a trip to Romania. His last two papers were published early this year [7,9]. Radu left us with great memories, and long lasting contributions to statistical mechanics and plasma physics that will inspire many generations to come.

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