Overview

Rather than describing in detail particular projects and its technicalities, this document gives a brief description of research fields from where projects can be created. Topics are not necessarily limited to this list. I am, of course, open for suggestions and if you have an idea for a project then speak to me.

I am happy to supervise 3rd year, MSc/MSci, or PhD projects. A 3rd year as well as an MSc/MSci project involves a review of the literature, including recent related results. Original research is normally not required. A PhD project requires, of course, original research and the topics indicated below may be the starting point for a PhD thesis.

The list covers topics in Nonlinear Dynamics and Statistical Physics. The description is rather brief and uses sometimes specialised terms. A full description of a project is not given, as projects have to be discussed with the applicant and have to be tailored according to his/her prerequisites and needs. For further details please contact me:

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A: Intermittency

Intermittency is used as a term for dynamics where different types of motion, say a calm "laminar" state and a chaotic "turbulent" state alternate in a non-periodic way. Such type of motion can be observed quite frequently, for instance, in fluids at the onset of turbulent behaviour, in Internet traffic, close to phase transitions, e.g., boiling water etc.. According to the nature of the "laminar" and the "turbulent" state, different types of Intermittency can be identified (cf. e.g. *Order within chaos : towards a deterministic approach to turbulence*, by Pierre Bergé, Yves Pomeau, and Christian Vidal, Wiley 1984 - QA614.8 BER).

Various tools from dynamical systems' theory can be used to study intermittent motion, for instance, correlation functions, power spectra, Markov maps, bifurcation theory, and others. Intermittency is a universal phenomenon in the sense that the dynamics does not depend on the details of the particular model. Thus, it provides the simplest example where renormalisation group approaches can be applied in a simple and rigorous way (cf. e.g. *Deterministic chaos: an introduction*, by Heinz Georg Schuster, VCH 1995 - QA614.8 SCH). Recently, new types of Intermittency have been discovered which are related with the synchronisation of chaotic systems. Such types of so called on-off Intermittency illustrate that aspects of synchronisation, for instance, phase synchronisation, generalised synchronisation, or strange non-chaotic attractors, become important concepts to understand intermittent motion in a broader context.

Possible topics for projects could cover, for instance, the investigation of simple models for intermittent motion either by analytical or by numerical means.

B: Dynamics with time delay

Time evolution changes considerably when a finite reaction time becomes relevant. That is, in fact, the scientific reason why in most countries driving a car is prohibited when the driver is drunken. In mathematical terms effects of a finite reaction time can be taken into account by an equation of motion which depends on the state of the system, say $x(t)$, and its value at some time in the past, say $x(t-\tau)$, where $\tau > 0$ denotes the time delay. The corresponding equations of motion, say a differential difference equation like $\dot{x}(t) = f(x(t), x(t-\tau))$ for a time-continuous set up, or delayed maps like $x_{n+1} = T(x_n, x_{n-\tau})$ in the case of time-discrete dynamical systems, are then analysed with respect to the impact of the delay time τ.

For linear systems, as usual, there exists a fairly complete and analytical method of solution (cf. e.g. *Differential-difference equations*, by Richard Bellman and Kenneth Lloyd Cooke, Academic Press 1963 - QA373 BEL). With regards to applications time delay dynamics is, for instance, relevant for stabilisation and control problems (cf. e.g. *Handbook of chaos control*, by Heinz G. Schuster (ed.), Wiley-VCH 1999 - QA614.8 HAN). The associated nonlinear eigenvalue problems can be handled with mathematical tools like complex analysis or number theory. Other aspects of time delay dynamics concern the change of chaotic motion when time delay becomes important. Last but not least dynamics with time delay can be related to the motion of spatially extended systems and pattern formation.

Possible topics for projects could cover, for instance, the investigation of simple models to estimate the impact of time delay by analytical or by numerical means.

C: Complex coupled systems

If the quote “the whole is more than the sum of its parts” (I guess you may recognise the author) applies to a research field, then, it is valid for the study of coupled dynamical units. How do swarms manage to coordinate their motion, why do certain fireflies blink in complete synchrony, how is it possible that sometimes traffic jams appear out of the blue? Even if a single unit, for instance, a single neural cell shows completely boring dynamics, a whole lot of coupled units, i.e. the brain, self-organises it's dynamics and displays surely an amazing complex behaviour.

Various types of mathematical models have been developed to study fundamental aspects of such complex coupled systems. For instance coupled maps can be used to understand how the interaction of random chaotic units can lead to coherent pattern formation (cf. e.g. *Theory and applications in coupled map lattices*, by Kunihiko Kaneko, Wiley 1993). Furthermore, many aspects of oscillatory systems are captured by the famous Kuramoto model (cf. e.g. *The Kuramoto model: A simple paradigm for synchronization phenomena*, Rev. Mod. Phys. 77, 137-185 (2005)). Various tools can be used to study such models, for instance, bifurcation theory to identify the mechanisms for coherent pattern formation, Lyapunov exponents to quantify the information transfer in coupled chaotic systems, or master stability functions to investigate synchronisation of a large number of units. Furthermore, symbolic dynamics and piecewise linear Markov maps can be used to link the theory of coupled map lattices with probabilistic cellular automata, and to establish, e.g., a link between space time intermittency and percolation problems in nonequilibrium Statistical Physics.

Possible topics for projects could cover, for instance, the investigation of simple models for coherent pattern formation and synchronisation by analytical or by numerical means.

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