

Possible MSci project: Sylow's Theorems

Here are some topics around Sylow's Theorems, which could possibly make a suitable MSci project. A project could be built out of one or more of these topics.

1. Alternative proofs There are several different proofs of Sylow's Theorems scattered in the literature. The original proof (I believe) used double cosets; Wielandt's proof used groups acting on sets; and there are inductive proofs as well. The project would involve presenting several such proofs with commentary.

2. Extensions For soluble groups, Philip Hall proved that analogues of Sylow's theorems hold where a single prime p is replaced by a set π of primes (so that the analogue of a Sylow p -subgroup is a subgroup whose order is the product of the p -parts of the order of the group for all primes $p \in \pi$). The project would involve a presentation of these theorems and their proofs.

3. Applications There are many applications of Sylow's theorems in group theory – they are absolutely fundamental – but one particular application is in proving that groups of certain orders cannot be simple. The project could involve, say, taking as many as possible of the composite integers below 1000 as possible, and showing that groups of those orders cannot be simple, or that a simple group of such an order is of known type.

4. Infinite versions For some families of infinite groups which satisfy some finiteness condition (for example, locally finite groups or profinite groups), analogues of Sylow's theorems hold. The project would involve a summary of such results, with proofs where possible.