Probability III – 2009/10

Exercise Sheet 10 JRJ

You are strongly encouraged to attempt all the questions on this sheet. Questions 3 and 4 are probably the most important.

If you want feedback on your answers to any of the questions then see me in a class or office hour.

1. Describe the evolution of the (rather simple) $D(d_1)/D(d_2)/1$ queue. Can you generalise to $D(d_1)/D(d_2)/s$?

2. Customers arrive at a single server according to a Poisson process of rate 1 per minute. The service times are exponentially distributed, with mean 40 seconds. The queue is run for a long time.

a) What is the expected queue length?

b) How would halving the expectation of the service time (with the distribution still being exponential) change the expected queue length?

c) The manager of a $M(\lambda)/M(\mu)/1$ queue believes that halving the expectation of the service times will halve the expected queue length in the long-run. Are they correct? Are they ever approximately correct?

3. Let $Q(t)$ be the number of customers in a $M(\lambda)/M(\mu)/3$ queue at time t.

a) Find the equilibrium distribution for $Q(t)$ under suitable conditions (which you should determine) on λ and μ. (You may use the expression for the equilibrium distribution of a birth-death process from your notes.)

b) Under the asumption that the equilibrium distribution does exist and the queue has been run for a long-time (or equivalently is at equilibrium) find the following:

i) The probability that all three servers are idle.

ii) The probability that all three servers are busy.

iii) The probability that there are exactly three customers waiting for a server to become available.

4. Consider an $M(\lambda)/M(\mu)/1$ queue with $\lambda < \mu$. Find the expected time that an arriving customer spends in the system (that is the total waiting time and time being served) conditioned on the fact that there are n customers in the system when he arrives.

Find the expected time a customer spends in the system if the queue is in equilibrium. (You may use the expression for the equilibrium distribution of an $M(\lambda)/M(\mu)/1$ queue from your notes without deriving it.)

5. Suppose that the $M(\lambda)/M(\mu)/1$ model is modified so that if there are n customers in the system when a new customer arrives, they join the queue as usual with probability p_n and otherwise leave in disgust.

a) Say why this does not satisfy the assumptions we made in lectures.

b) Nevertheless, show that this queue is described by a birth-death process. What are the parameters?

c) How could you model a queue with a waiting room which can only hold k people (with anyone finding the room full leaving in disgust) using this approach?