SUMMARY SHEET: SERIES
Engineering Maths II (MAE111), 2011

Sequence

$$u_1, u_2, u_3, \ldots u_n, \ldots$$

Terms may be numbers or any mathematical expression; there may ne a finite or infinite number of terms.

Series May be finite:

$$S_n = u_1 + u_2 + u + 3 + \ldots + u_n = \sum_{r=1}^{n} u_r$$

Or infinite

$$S_\infty = u_1 + \ldots + u_n + \ldots = \sum_{r=1}^{\infty} u_r$$

Sum of integers from 1 to n

$$\sum_{r=1}^{n} r = \frac{1}{2} n(n+1)$$

Sum of squares of integers from 1 to n

$$\sum_{r=1}^{n} r^2 = \frac{1}{6} n(n+1)(2n+1)$$

Arithmetic Series

$$S_n = a + (a+d) + (a+2d) + \ldots + (a+(n-1)d) = na + \frac{1}{2} n(n-1)d$$

Geometric Series

$$S_n = a + ar + ar^2 + \ldots + an^{n-1} = \frac{a(1-r^n)}{1-r} = \frac{a(r^n - 1)}{r-1}$$

Limiting behaviour For $p > 0$, and $x > 0$

$$\lim_{x\to 0} x^p = 0, \qquad \lim_{x\to 0} (1/x^p) = \infty$$

And ...

$$\lim_{x\to\infty} x^p = \infty, \qquad \lim_{x\to\infty} (1/x^p) = 0$$

Terms in a Sequence:

$$u_1, u_2, u_3, \ldots u_r, \ldots$$

$\lim_{r\to\infty} u_r$ can behave in different ways: either get smaller and smaller ($\to 0$), get bigger and bigger ($\to \infty$), get closer and closer to some definite value ($\to K$), oscillate, even behave randomly.

Infinite Series Partial sum of first n terms of an infinite series S_∞ is

$$S_n = \sum_{r=1}^{n} u_r$$

If S_n tends to some definite (finite) value as more and more terms are added (ie as $n \to \infty$) then the series is **convergent.** If such a value does not exist then the series is **divergent**.

Convergence Tests: Size of terms If the size of terms does not tend to zero, ie if

$$\lim_{n\to\infty} u_n \neq 0$$

then the series is **divergent**. (Note: the converse is not true, ie, even if the size of the terms goes to zero, then the series might still be divergent.)

Convergence Tests: Comparison with Known Series Compare term by term with another series, whose convergence properties are known. If each term is larger, and the known series diverges, then the test series also diverges. If each term is smaller, and the known series converges, then the test series also converges.

Convergence Tests: Ratio Test Consider the quantity which is the limit of the (absolute) ratio of the last two terms in the series as $n \to \infty$:

$$k = \lim_{n\to\infty} \left| \frac{u_{n+1}}{u_n} \right|$$

	$k < 1$	series converges
If ...	$k > 1$	series diverges
	$k = 1$	inconclusive (ie: no information)

Power Series

$$S = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots \qquad S_\infty = \sum_{n=0}^{\infty} a_n x^n$$

Radius of convergence for Power Series

$$|x| < \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right|$$

Maclaurin Series It is possible to write down a power series representation of a function, together with a remainder term, in the form

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots + \frac{x^n}{n!} f^{(n)}(0) + R_n(x)$$

The remainder term becomes smaller as the number of terms is increased. $f(0)$ means the function evaluated at $x = 0$, and similarly for the derivatives such as $f'(0)$.

Taylor Series

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \ldots$$

Binomial Series

$$\begin{aligned} (1+x)^p &= 1 + px + \frac{p(p-1)}{2!} x^2 + \ldots \\ &= \sum_{n=0} \frac{p! x^n}{(p-n)! n!} \end{aligned}$$

L'Hopital's Rule If the numerator and the denominator of a quotient separately tend to zero, then the limit of the quotient is the same as the limit of the quotient of the functions' derivatives.

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

If, even after differentiation, numerator and denominator are still both zero, then the process is simply repeated.

Text Book References

	Mathematics for Engineers Croft & Davison	Engineering Mathematics Stroud (4th Edition)	Engineering Mathematics Stroud (5th Edition)
Series	Ch 15 (not convergence tests)	Programmes 13.22 – 13.48, 14.1 – 14.45	Programmes 13, 14