VECTORS

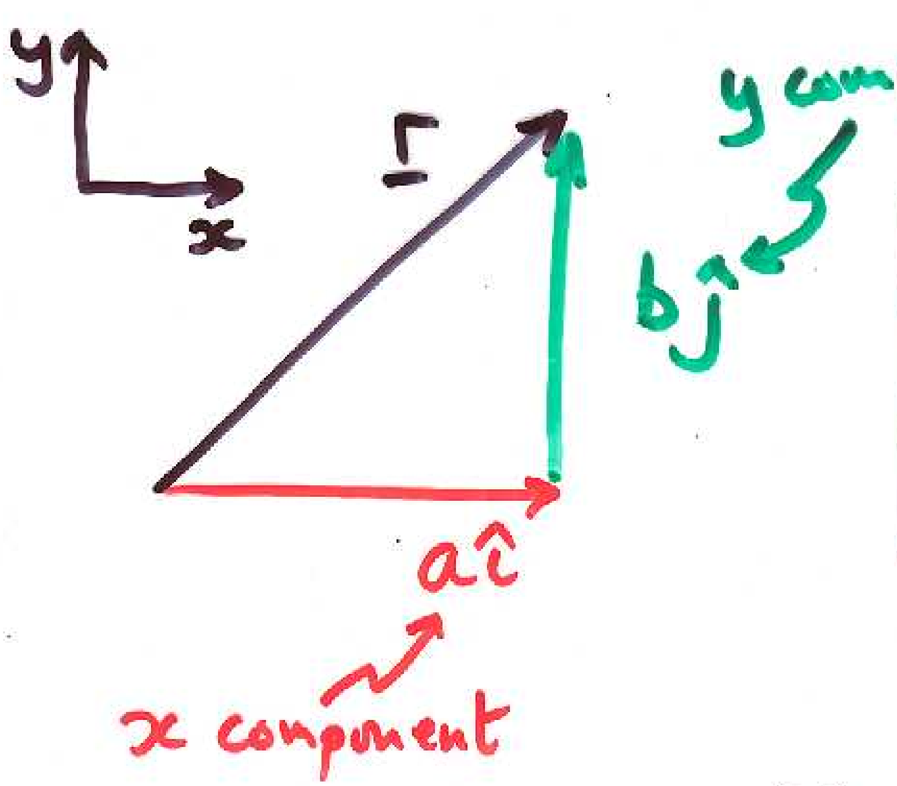
- MATHEMATICAL OBJECT WITH
  - DIRECTION
  - MAGNITUDE

- VECTOR ALGEBRA
  - RULES FOR COMBINING VECTORS AND MANIPULATING THEM.

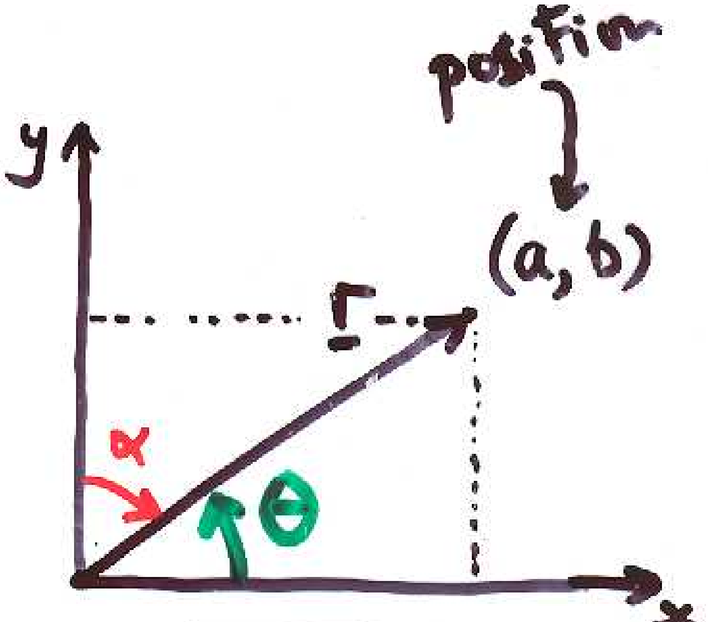
Examples of Vectors	SCALAR
- FORCE	TEMPERATURE
- VELOCITY	MASS
- POSITION	VOLTAGE
	SPEED

V3.1

Vector Component Form



$$\boxed{\underline{r} = a\hat{i} + b\hat{j}}$$

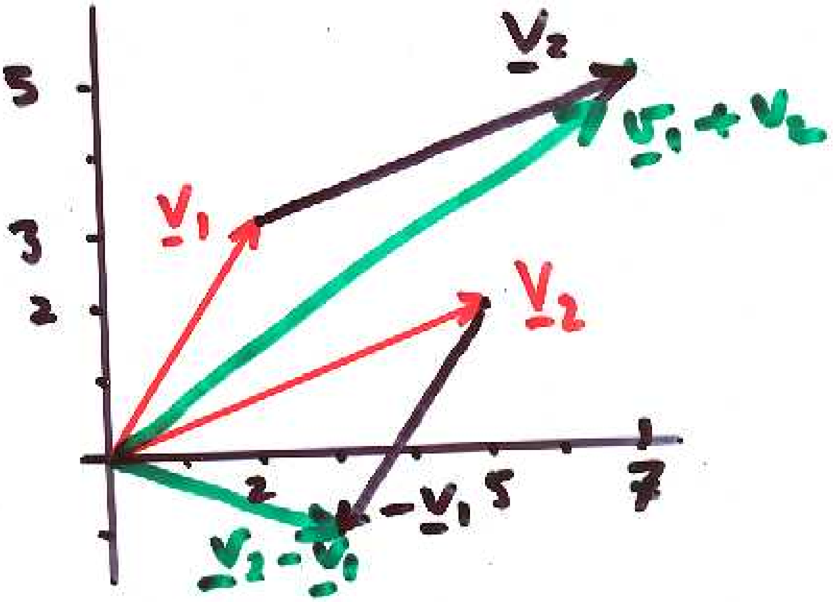


Magnitude $|\underline{r}| = r = \sqrt{a^2+b^2}$

Angle to x axis $\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+b^2}}$

Note: $\cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$ ← Angle to y axis

VECTOR ADDITION : EXAMPLE



$\underline{v}_1 = 2\hat{i} + 3\hat{j}$ $\quad\quad$ $\underline{v}_2 = 5\hat{i} + 2\hat{j}$

$$\underline{v}_1 + \underline{v}_2 = (2+5)\hat{i} + (3+2)\hat{j} = 7\hat{i} + 5\hat{j}$$

$$\underline{v}_2 - \underline{v}_1 = (5-2)\hat{i} + (2-3)\hat{j}$$

$$= 3\hat{i} - \hat{j}$$

USING VECTORS:

CENTRE OF MASS OF SET OF POINT MASSES

POINT MASS: A given mass acting at a point

CENTRE OF MASS: (C.M.)

- POINT ABOUT WHICH TURNING MOMENT IS ZERO

ALTERNATIVE

- TURNING MOMENT ABOUT SOME POINT SAME AS IF ALL MASS IS AT POSITION OF CENTRE OF MASS

EXAMPLE

Masses 3g, 5g and 1g at

$\underline{a} = 2\hat{\imath} + 4\hat{\jmath}$ $\quad \underline{b} = -3\hat{\imath} + \hat{\jmath}$

$\underline{c} = -8\hat{\jmath}$

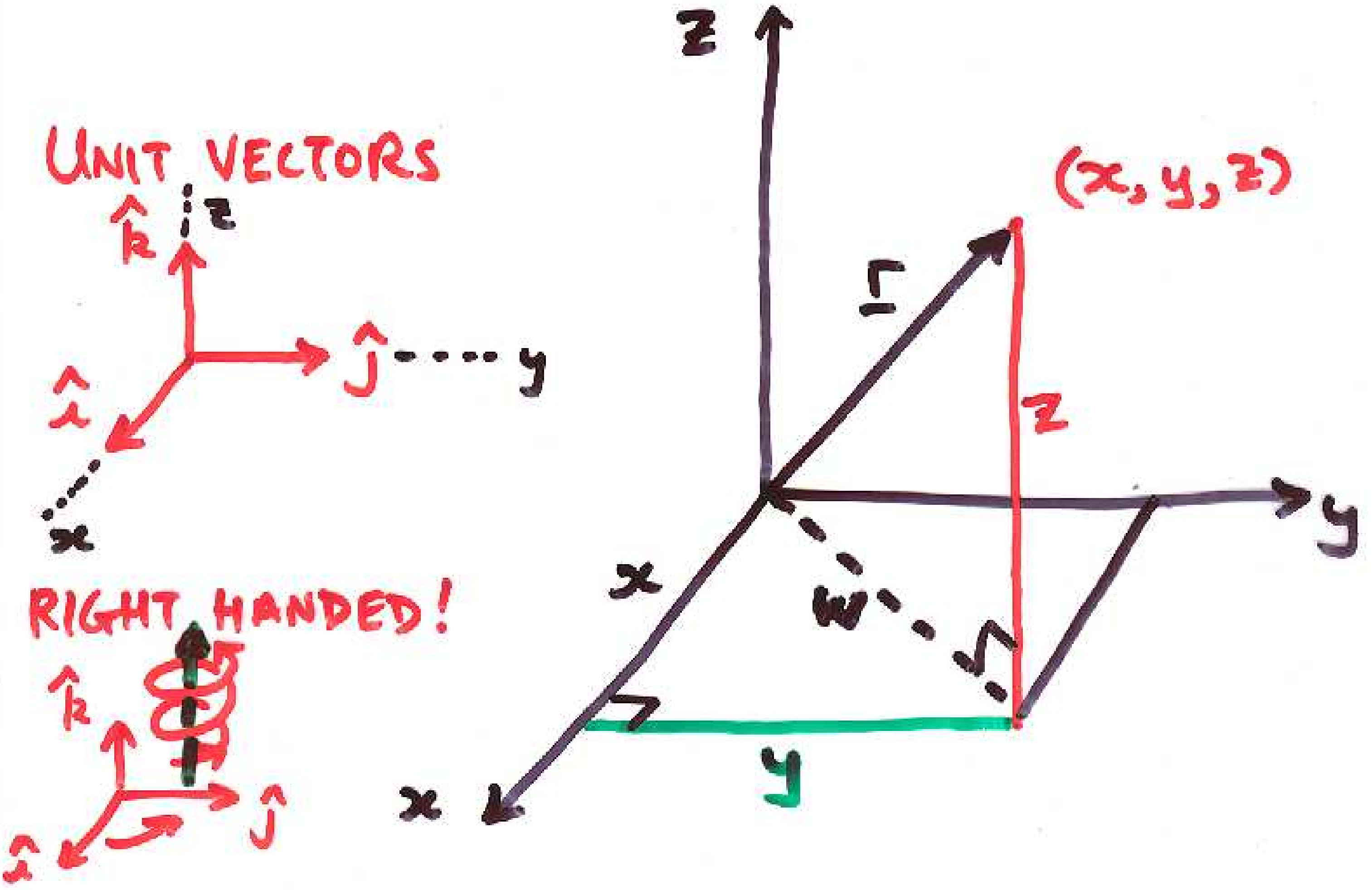
$$M = 3 + 5 + 1 = 9\,g$$

$$\underline{r}_{cm} = \frac{1}{9}\Big[(3 \cdot 2 - 5 \cdot 3)\hat{\imath} + (3 \cdot 4 + 5 \cdot 1 - 1 \cdot 8)\hat{\jmath}\Big]$$

$$= \frac{1}{9}\Big[-9\hat{\imath} + 9\hat{\jmath}\Big]$$

$$\underline{r}_{cm} = -\hat{\imath} + \hat{\jmath}$$

THREE DIMENSIONAL VECTOR COMPONENTS



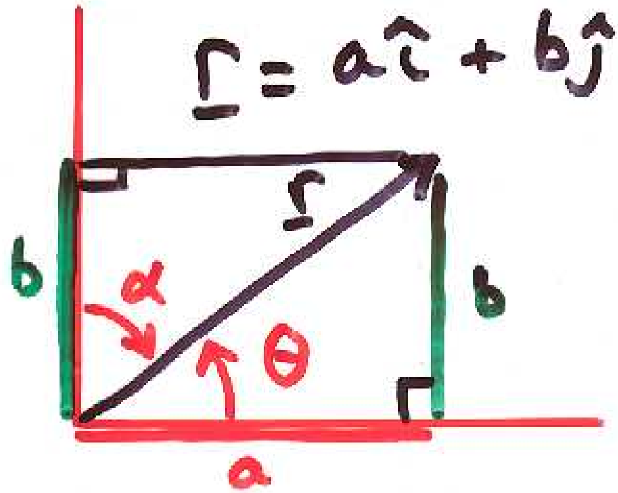
$$\underline{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Same rules for EQUALITY & ADDITION:

APPLY COMPONENT BY COMPONENT (SEPARATELY)

DIRECTION COSINES

Remember 2D:

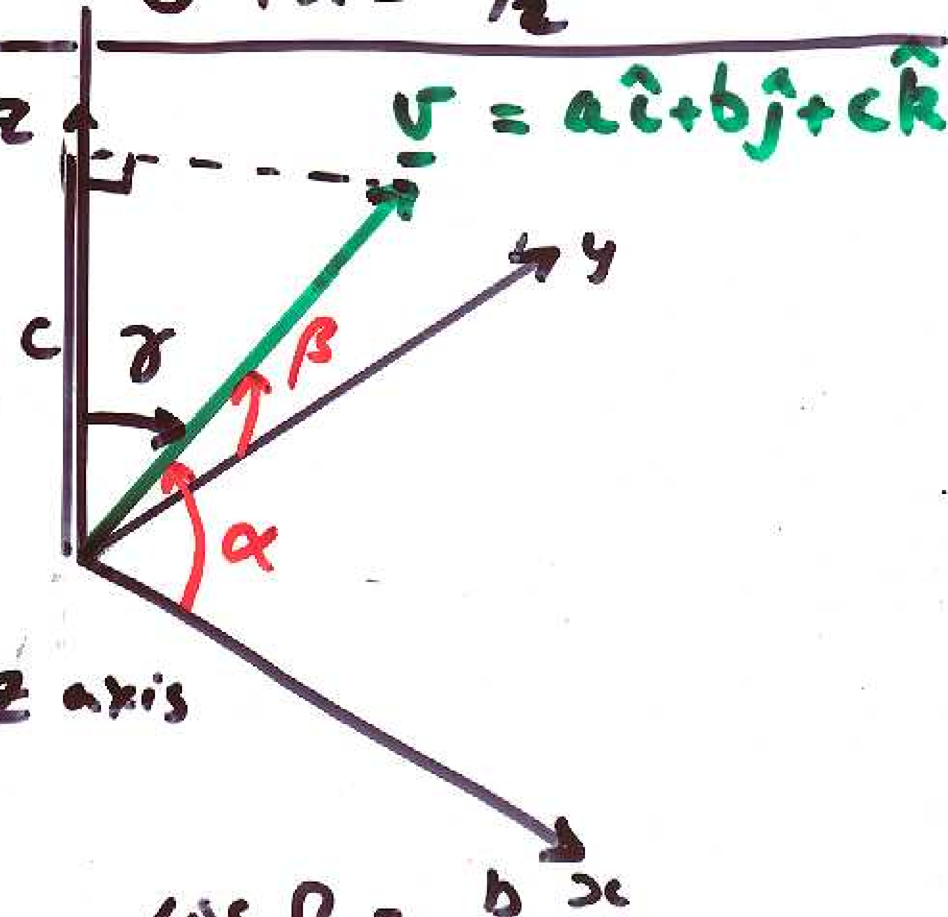


$$\cos\theta = \frac{a}{|\underline{r}|}$$

$$\cos\alpha = \frac{b}{|\underline{r}|}$$

$$\theta + \alpha = \pi/2$$

3D....



$$\cos\gamma = \frac{c}{|\underline{v}|}$$

γ angle $\underline{v}$ to z axis

$$\cos\alpha = \frac{a}{|\underline{v}|} \quad , \quad \cos\beta = \frac{b}{|\underline{v}|}$$

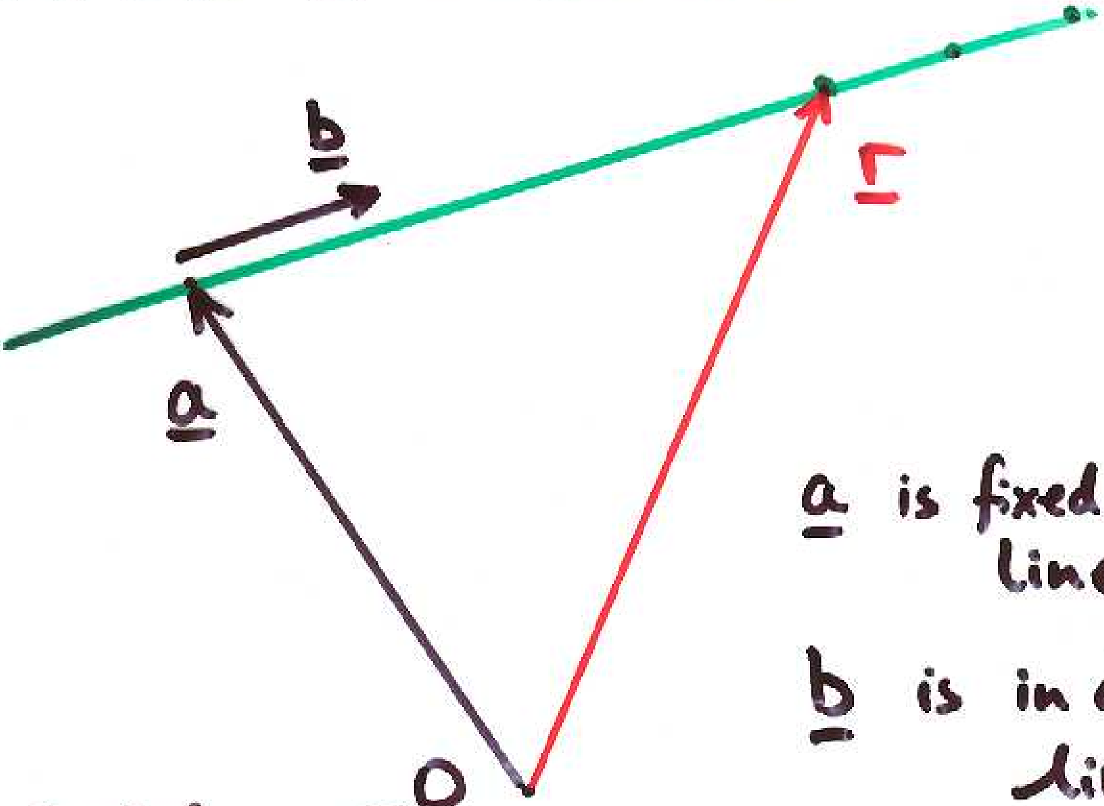
α angle to x axis

β angle to y axis

V3.6

EQUATION OF LINE

... IN 3 DIMENSIONS



Origin → O

[$\underline{a}$, $\underline{r}$ are position vectors]

$\underline{a}$ is fixed point on line

$\underline{b}$ is in direction of line

$\underline{r}$ is vector to <u>any</u> point on line

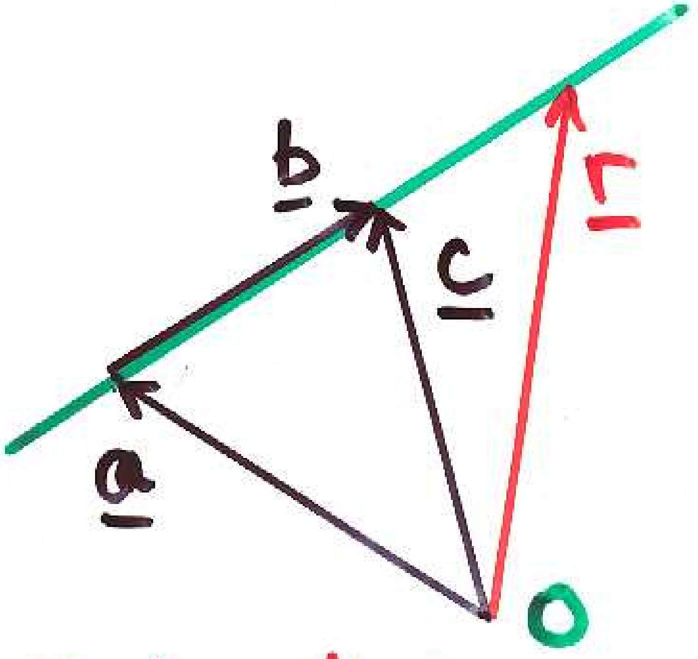
Remember: $\underline{a}$ is fixed (constant)

$\underline{r}$ is different for different points on line

$$\underline{r} = \underline{a} + s\,\underline{b}$$

↑ PARAMETER

EQUATION OF LINE GIVEN TWO POINTS ON LINE



Vector $\underline{b}$ in direction of line

$$\underline{b} = \underline{c} - \underline{a}$$

Point on line:

$$\underline{r} = \underline{a} + s(\underline{c} - \underline{a})$$

Where $\underline{a}$ and $\underline{c}$ are given

SPECIAL CASES

$$\underline{r} = \underline{a} + s\,\underline{b}$$

OR: $$s = \frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$$

* If $\underline{b}$ is <u>unit vector</u> then

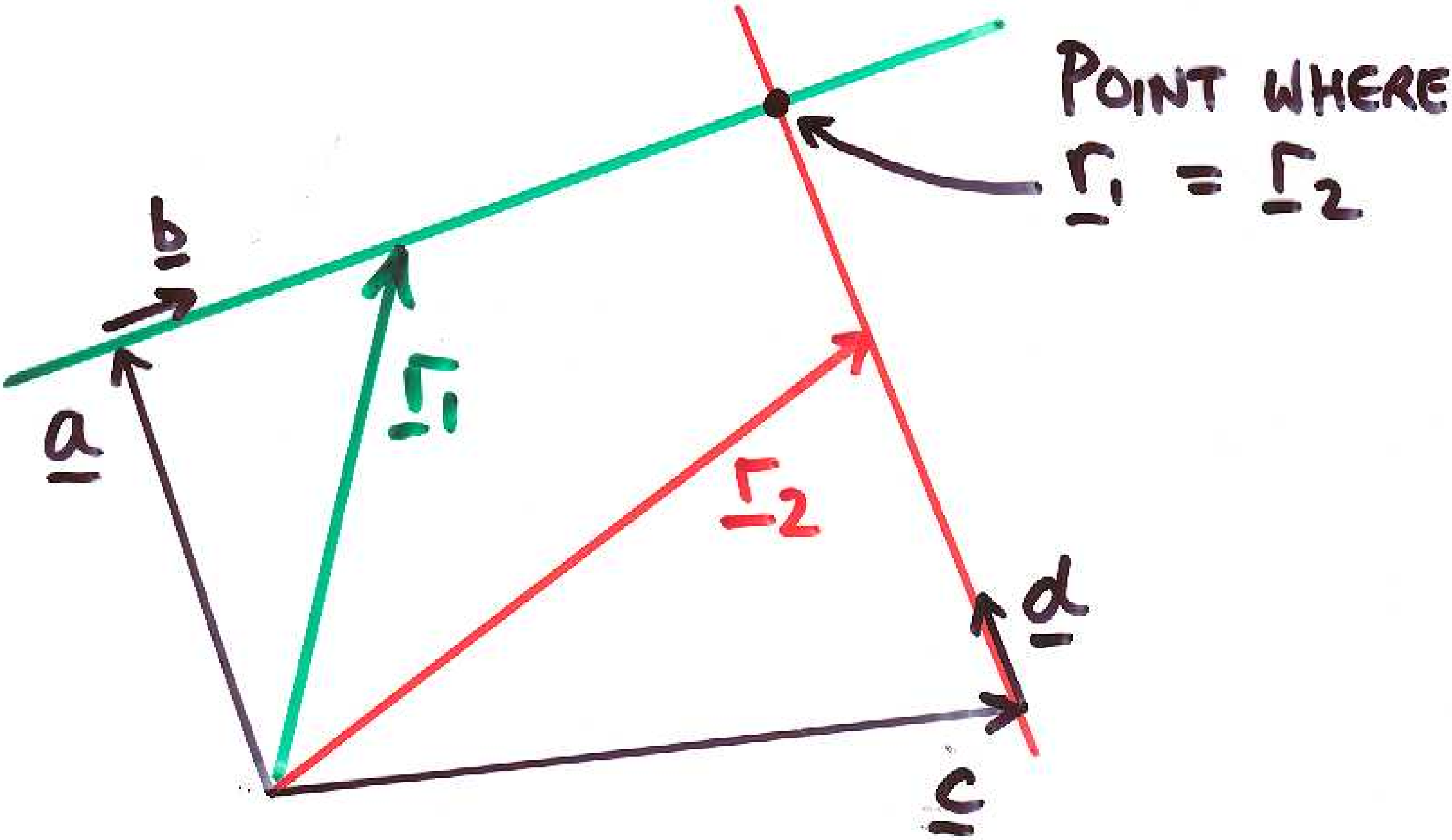
b_1, b_2, b_3 are <u>Direction Cosines</u>

* If line passes through origin

<u>then</u> $\underline{a} = \underline{0}$

OR: $a_1 = a_2 = a_3 = 0$

INTERSECTION OF TWO LINES



$$\underline{r}_1 = \underline{a} + s\,\underline{b}$$

$$\underline{r}_2 = \underline{c} + t\,\underline{d}$$

INTERSECTION...

$$\underline{a} + s\,\underline{b} = \underline{c} + t\,\underline{d}$$

VECTOR MULTIPLICATION

Remember:

(scalar)(vector) → vector

There are TWO ways to multiply vector by vector.

(vector) • (vector) → scalar

"DOT" product (scalar product)

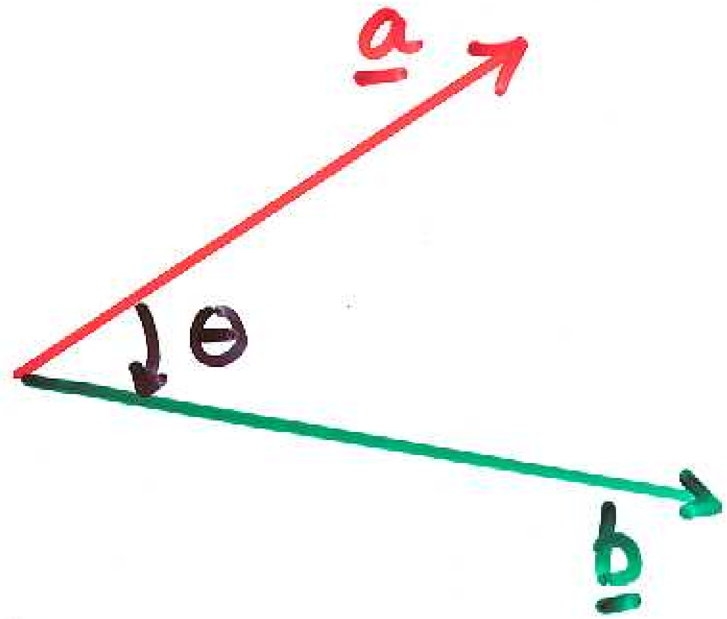
(vector) ∧ (vector) → vector

"CROSS" product

(or: vector product)

v4.6

SCALAR (DOT) PRODUCT



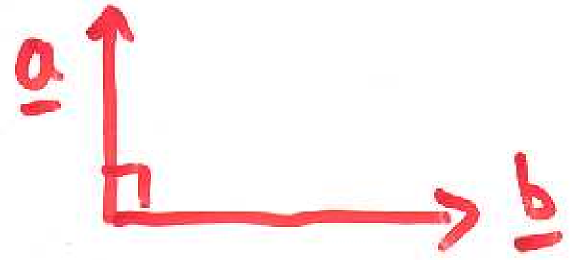
Definition

$$\underline{a} \cdot \underline{b} = |\underline{a}|\,|\underline{b}| \cos\theta$$

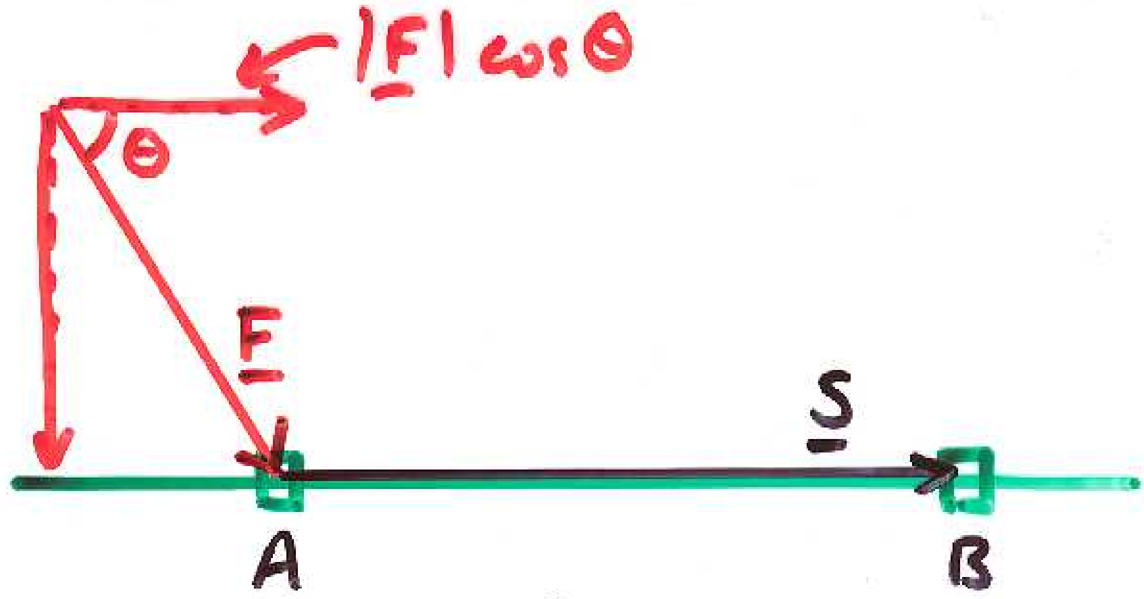
NOTE: * $\underline{a} \cdot \underline{b} = \underline{b} \cdot \underline{a}$ The order doesn't matter

* $\underline{a} \cdot \underline{a} = |\underline{a}|^2$

* If $\underline{a} \perp \underline{b} \Rightarrow \underline{a} \cdot \underline{b} = 0$



EXAMPLE: WORK BY FORCE



$$\vec{AB} = \underline{s}$$

Force $\underline{F}$ causes displacement $\underline{s}$

Work done is ~~Force in direction of~~

(Force in direction $\vec{AB}$) $\times$ (Distance AB)

$$\boxed{W = \underline{F} \cdot \underline{s}}$$

SCALAR PRODUCT IN COMPONENTS

Unit vectors $\hat{i}, \hat{j}, \hat{k}$

$\hat{i} \cdot \hat{i} = 1$ since magnitude unity

$\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = 0$ since $\hat{i} \perp$ to $\hat{j}$ <u>and</u> $\hat{k}$

$$\underline{a} \cdot \underline{b} = (a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \cdot (b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k})$$

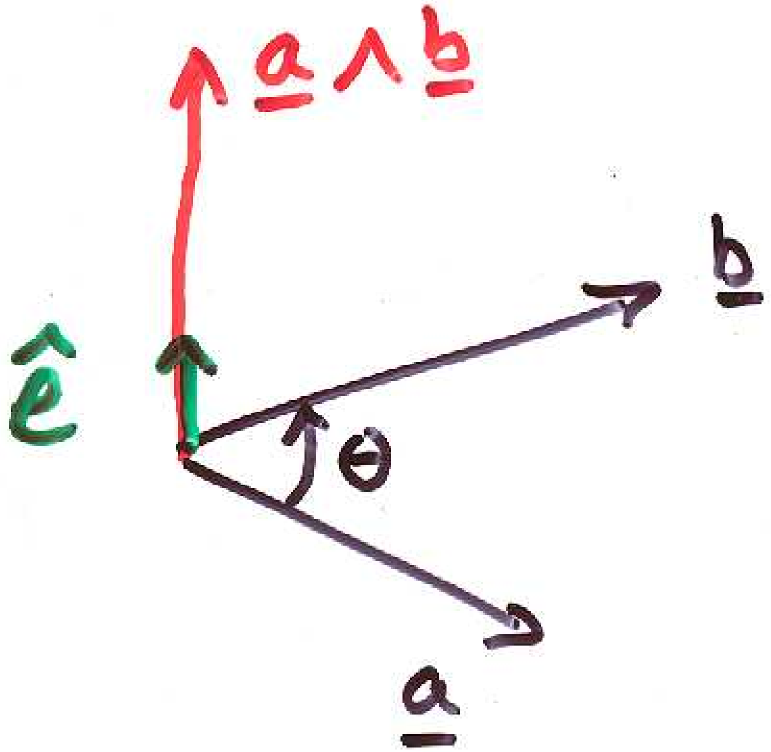
$$\boxed{\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3}$$

Multiply components

ADD UP

$\underline{a} \cdot \underline{b}$

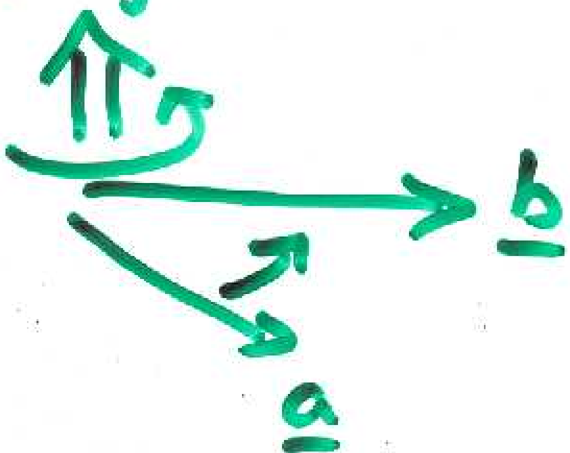
THE VECTOR (CROSS) PRODUCT



$\hat{\underline{e}}$ unit vector perp. to $\underline{a}$ and $\underline{b}$

$$\boxed{\underline{a} \wedge \underline{b} = |\underline{a}|\,|\underline{b}| \sin\theta \; \hat{\underline{e}}}$$

$\hat{\underline{e}}$ is Right handed !



VECTOR PRODUCT IN COMPONENT FORM

$$\hat{i} \wedge \hat{i} = \hat{j} \wedge \hat{j} = \hat{k} \wedge \hat{k} = \underline{0}$$

$$\hat{i} \wedge \hat{j} = \hat{k} \quad \Rightarrow \quad \hat{j} \wedge \hat{i} = -\hat{k}$$

$$\hat{j} \wedge \hat{k} = \hat{i} \quad \Rightarrow \quad \hat{k} \wedge \hat{j} = -\hat{i}$$

$$\hat{k} \wedge \hat{i} = \hat{j} \quad \Rightarrow \quad \hat{i} \wedge \hat{k} = -\hat{j}$$

$$\underline{a} \wedge \underline{b} =$$

$$(a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}) \wedge (b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k})$$

$$= a_1 b_2 \hat{k} - a_1 b_3 \hat{j} +$$

$$a_2 b_3 \hat{i} - a_2 b_1 \hat{k} +$$

$$a_3 b_1 \hat{j} - a_3 b_2 \hat{i}$$

$$= (a_2 b_3 - a_3 b_2)\hat{i} + (a_3 b_1 - a_1 b_3)\hat{j}$$

$$+ (a_1 b_2 - a_2 b_1)\hat{k}$$