ENG MATHS SAMPLE TEST SOLUTIONS

#1

$$y(\theta) = \sin(\theta + \pi/4) \Rightarrow \frac{dy}{d\theta} = \cos(\theta + \pi/4)$$

$$x(\theta) = \cos(\theta + \pi/4) \Rightarrow \frac{dx}{d\theta} = -\sin\left(\theta + \frac{\pi}{4}\right)$$

$$\frac{dy}{dx} = {dy/d\theta} \Big/ \frac{dx}{d\theta} = -\frac{\cos(\theta + \pi/4)}{\sin(\theta + \pi/4)} = -\cot\left(\theta + \frac{\pi}{4}\right)$$

#2 –

$$x^3 + 2x^2 y = 3xy$$

$$3x^2 + 2x^2\frac{dy}{dx} + y \cdot 4x = 3x\frac{dy}{dx} + 3y$$

$$\frac{dy}{dx}\left(2x^2 - 3x\right) = 3y - 4xy - 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3y - 4xy - 3x^2}{2x^2 - 3x}$$

#3:

$$y = \sinh^{-1}(e^x) \Rightarrow \sinh y = e^x$$

diff implicitly: $\cosh y \frac{dy}{dx} = e^x \Rightarrow \frac{dy}{dx} = \frac{e^x}{\cosh y}$

But $\cosh y = \sqrt{1 + \sinh^2 y} = \sqrt{1 + e^{2x}}$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x}{\sqrt{1 + e^{2x}}}$$

#4

$$\underline{a} = (2, 2, -1) \qquad \underline{b} = (5, -3, 2)$$

(a) $\underline{a} \cdot \underline{b} = (2)(5) + (2)(-3) + (-1)(2) = 10 - 6 - 2 = 2$

(b) $\underline{a} \wedge \underline{b} = \left((2)(2) - (-1)(-3)\right)\hat{i} + \left((-1)(5) - (2)(2)\right)\hat{j} + \left((2)(-3) - (2)(5)\right)\hat{k}$

$= \hat{i} - 9\hat{j} - 16\hat{k}$

OR: $\underline{a} \wedge \underline{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 5 & -3 & 2 \end{vmatrix} = \hat{i} - 9\hat{j} - 16\hat{k}$

#5

$$\underline{r}_{cm} = \frac{1}{M}\sum m_i \underline{r}_i \qquad M = 2 + 5 + 1 = 8\text{ kg}$$

$$8\underline{r}_{cm} = 2(3\hat{i} - 4\hat{j}) + 5(-\hat{i} + 5\hat{j}) + (5\hat{i} - \hat{j})$$

$$= \hat{i}(6 - 5 + 5) + \hat{j}(-8 + 25 - 1)$$

$$= 6\hat{i} + 16\hat{j}$$

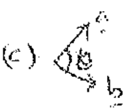
$$\underline{r}_{cm} = \frac{3}{4}\hat{i} + 2\hat{j}$$

#6 (a) $2\underline{a} = -4\hat{i} + 4\hat{j} - 2\hat{k}$

$\Rightarrow 2\underline{a} + \underline{b} = -\hat{i} + 6\hat{j} - 4\hat{k}$

(b) $|\underline{a}| = \sqrt{(-2)^2 + (2)^2 + (-1)^2} = \sqrt{9} = 3$

(c)



$\cos\theta = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|\,|\underline{b}|}$

$\underline{a} \cdot \underline{b} = (-2)(3) + (2)(2) + (-1)(-2) = 0$

$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \pi/2$