Solutions

(1). $f(x) = e^{\sin x}$ , $f(0) = e^0 = 1$

$f'(x) = \cos x\, e^{\sin x}$ ; $f'(0) = e^0 \cos(0) = 1$

$f''(x) = \cos x(\cos x\, e^{\sin x}) - \sin x\, e^{\sin x}$ , $f''(0) = 1$

Maclaurin Series $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$

$\therefore f(x) = e^{\sin x} = 1 + x + \frac{x^2}{2!} + \dots$

(2). (a) $\lim\limits_{n\to\infty} u_n = \lim\limits_{n\to\infty} \frac{n-3}{2n+4} = \lim\limits_{n\to\infty} \frac{1-3/n}{2+4/n} = \frac{1}{2} \neq$ series divergent

(note Ratio test is inconclusive).

(b). Ratio test $k = \lim\limits_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim\limits_{n\to\infty} \left|\frac{3^{n+1}}{(n+2)!} \frac{(n+1)!}{3^n}\right|$

$= \lim\limits_{n\to\infty} \frac{3}{n+2} = 0 < 1$ convergent.

(3). Ratio test $k = \lim\limits_{n\to\infty} \left|\frac{u_{n+1}}{u_n}\right| = \lim\limits_{n\to\infty} \left|\frac{5^{n+1}}{(n+1)^2} \frac{n^2}{5^n} \frac{x^{2(n+1)}}{x^{2n}}\right|$

$= 5 \lim\limits_{n\to\infty} |x^2| \left|\left(\frac{n}{n+1}\right)^2\right| <$ for convergence

$\therefore 5 \lim\limits_{n\to\infty} |x^2| \left|\left(\frac{1}{1+1/n}\right)^2\right| < 1$

$\therefore 5|x^2| < 1 \Rightarrow |x^2| < 1/5$ or $-\frac{1}{\sqrt{5}} < x < \frac{1}{\sqrt{5}}$ range of convergence.

(4). $\int x \sin x\, dx = -x\cos x + \int \cos x\, dx$ $\left\{ \begin{array}{l} u = x,\ du = dx \\ dv = \sin x\, dx \\ v = -\cos x \end{array} \right.$

$= -x\cos x + \sin x + C$

(5) $I = \int \frac{1}{x^2+2x-8}\, dx$ . $\frac{1}{x^2+2x-8} = \frac{1}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$

$\therefore 1 = A(x-2) + B(x+4)$

put $x = -4$ $1 = -6A$ or $A = -1/6$, put $x = 2$ $1 = 6B$ or $B = 1/6$

$\therefore I = -\frac{1}{6}\int \frac{dx}{x+4} + \frac{1}{6}\int \frac{dx}{x-2} = -\frac{1}{6}\ln(x+4) + \frac{1}{6}\ln(x-2) + C$

(6). $I = \int_0^{\pi/2}\int_0^{\pi/2} \cos(x+y)\, dx\, dy$

$= \int_0^{\pi/2} \Big[\sin(x+y)\Big]_0^{\pi/2} dy = \int_0^{\pi/2} \Big[\sin(\pi/2+y) - \sin y\Big] dy$ $[\sin(\pi/2+y) = \cos y]$

$= \int_0^{\pi/2} (\cos y - \sin y)\, dy = \Big[\sin y + \cos y\Big]_0^{\pi/2}$

$= \left[\sin\pi/2 + \cancel{\cos\pi/2}\right] - \left[\sin(0) + \cos(0)\right]$

$= 1 - 1 = 0.$