Queen Mary,
UNIVERSITY OF LONDON

BEng, MEng Examination by Course Units

MAE 111 ENGINEERING MATHEMATICS II

Date: May 7, 2003   Time: 10:00 – 12:00 (2 hours)

Duration: 2 hours

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination.

Vectors are in boldface, thus: $\boldsymbol{A}$.
A table of standard integrals is provided at the end of this examination paper.

Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question.

SECTION A

Each question carries 6 marks. You should attempt ALL questions.

1. Using the product rule for differentiation, calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $y(x)$ is given by

$$y = (x^3 + 2x^2)(\sin x + 2\cos x).$$

2. Calculate $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} / \frac{\mathrm{d}x}{\mathrm{d}t}$ in terms of the parameter t where

$$\begin{aligned} y(t) &= 2at\sin t \\ x(t) &= at^2. \end{aligned}$$

3. By differentiating implicitly, determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $y(x)$ is given by

$$y + 4xy^2 - 2x^2 - x = 0.$$

4. Evaluate, using partial fractions, the following integral

$$I = \int \frac{x-2}{2x^2 + x - 1}\, dx.$$

5. Evaluate the integral

$$I = \int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx.$$

6. Evaluate the following integral by parts

$$I = \int x^2 \ln x \, dx.$$

7. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = 3\widehat{\boldsymbol{i}} + 4\widehat{\boldsymbol{j}} + 7\widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 2\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}} - 2\widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$, (b) the vector cross product $\boldsymbol{a} \wedge \boldsymbol{b}$, and (c) $|(\boldsymbol{a} + \boldsymbol{b})|$.

8. Point masses 1g, 2g and 5g are situated at the points A, B, and C, with position vectors $2\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}}$, $\widehat{\boldsymbol{i}} + 2\widehat{\boldsymbol{j}}$ and $\widehat{\boldsymbol{i}} - 3\widehat{\boldsymbol{j}}$, respectively. Find the position vector of the centre of mass for these three particles. If the 1g and 2g masses have their positions swapped, what is the consequent change in the x position of the centre of mass?

9. The function $f(x, y)$ is given by

$$f = x^2 + \sin(xy) - 2y^3.$$

Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial x \partial y}$.

10. The complex numbers z_1 and z_2 are given by $z_1 = 2 - j3$ and $z_2 = 1 + j$. Calculate:
(a) $z_2 - 3z_1$,
(b) $z_1 z_2$
(c) z_1/z_2.

11. Express in the form $r\, e^{j\theta}$ the following complex number:

$$z = \sqrt{2}(-1 + j).$$

Hence, or otherwise, find the value of z^4 in the form $a + jb$, where a and b are real.

12. Find the values of all complex numbers that satisfy the equation

$$z^2 + 4z + 8 = 0.$$

[Next section overleaf.]

SECTION B

Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

1. Use the Maclaurin's series method to obtain the first four non-zero terms in the series for

$$f(x) = \ln(1 + 2x).$$

Check that the first four non-zero terms are given by

$$u_n = \frac{(-1)^{n+1}(2x)^n}{n}.$$

Assuming that this formula holds for all $n \geq 1$, use the Ratio Test to find the range of values of x for which this series is convergent.

2. The line $y = 2\sqrt{x}$ when rotated about the x-axis between $x = 1$ and $x = 4$ generates a solid of revolution. Determine:
(a) the volume of the solid,
(b) the position of the centroid of the solid from the y axis,
(c) the total surface area of the solid.

3. (a) The buckling load of a thin walled circular column is given by

$$P = k\frac{r^3 t}{L^2}$$

where r and t are the radius and thickness of the column, L is its length, and k is some constant.

The lengths r and t can be measured to an accuracy of $\pm 2\%$ and L can be measured to an accuracy of $\pm 0.1\%$. Consequently, what is the maximum possible percentage error in the calculation of P?

(b) If $u = \ln r$ and $r^2 = x^2 + y^2$, prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

4. (a) State de Moivre's Theorem.

(b) Use this to show that

$$\sin(4\theta) = 4cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta.$$

(c) Find the four roots of the equation $z^4 = -1$. Express your answer in the form $a + jb$, where a and b are real. Sketch the positions of the roots in an Argand diagram.

[Next section overleaf.]

Table of Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln\|x\|$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln\|\cos x\| = \ln\|\sec x\|$
$\cot x$	$\ln\|\sin x\|$
$\sec x$	$\ln\|\sec x + \tan x\| = \ln\tan\|(\frac{\pi}{4} + \frac{1}{2}x)\|$
$\operatorname{cosec} x$	$-\ln\|\operatorname{cosec} x + \cot x\| = \ln\|\tan(\frac{1}{2}x)\|$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln\cosh x$
$\coth x$	$\ln\|\sinh x\|$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

[End of examination paper.]