Queen Mary
University of London

BEng, MEng Examination by Course Units

MAE 111 ENGINEERING MATHEMATICS II Paper

Date: May 7, 2008   Time: 10:00 – 12:00 (2 hours)

Duration: 2 hours

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination.

Vectors are in boldface, thus: $\boldsymbol{A}$.
A table of standard integrals is provided at the end of this examination paper.

Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question. For Section A you may answer more than one question on a page in the answer book, but make it clear where each answer starts.

SECTION A

Each question carries 6 marks. You should attempt ALL questions.

1. Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $y(x)$ is given by

$$y = x^3(\ln x)\sin 2x.$$

.

2. By differentiating implicitly, determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ when y is given by

$$3x^2 + 4y^3 = xy + y.$$

3. Evaluate the integral

$$I = \int \frac{\mathrm{d}x}{x \ln x}.$$

4. Determine the distance above the x-axis of the centroid of the laminar defined by the curves $y = x^2$ and $y = 1$.

5. Use L'Hopital's Rule to evaluate the limit

$$\lim_{x \to 0} \left\{ \frac{x - \sin x}{x^2} \right\}.$$

6. Evaluate the double integral

$$I = \int_0^1 \int_{x^2}^{x} (3y + 2x) dy\, dx.$$

7. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = \widehat{\boldsymbol{i}} + 2\widehat{\boldsymbol{j}} - \widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 2\widehat{\boldsymbol{i}} - 3\widehat{\boldsymbol{j}} + \widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) their sum $\boldsymbol{a} + \boldsymbol{b}$, (b) the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$, and (c) the vector cross product $\boldsymbol{a} \wedge \boldsymbol{b}$.

8. Point masses 4g, 1g, 5g and 2g are situated at the points A, B, C and D with position vectors $2\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}}$, $-\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}}$, $3\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}}$, and $\widehat{\boldsymbol{i}} - \widehat{\boldsymbol{j}}$, respectively. Find the position vector of the centre of mass for these four particles.

[Next question overleaf.]

9. Three complex numbers are given as $a = 3 + j$, $b = 2 - j$ and $c = 5j$. Calculate

(a) a/b,
(b) c/b,
(c) ab.

10. A complex number z is given by $z = \sqrt{3} + j$.

(a) Express this number in polar form $re^{j\Theta}$,
(b) compute z^2 in polar form
(c) express z^2 in algebraic form $a + jb$.

11. The density ρ of a metal is measured by weighing a cylindrical sample of this metal with diameter D and height h. The mass m, height h and diameter D are measured with errors Δm, Δh and ΔD, respectively. Using a Taylor series the error $\Delta\rho$ can be estimated as

$\Delta\rho \approx a\Delta D + b\Delta h + c\Delta m.$

Express a, b and c in terms of D, h and m.

12. Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions and simplify:

(a) $f = \frac{1}{\sin(xy)}$

(b) $f = ax^n y^m$, where a is a constant.

[Next section overleaf.]

SECTION B

Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

1. (a) Use the MACLAURIN series method to obtain the first three non-zero terms in the series for

$$f(x) = \sin(x).$$

[7 marks]

(b)Assuming that the n-th term of the series is given by

$$u_n = \frac{(-1)^{2n-1}(x)^{2n-1}}{(2n-1)!}.$$

show, by using the Ratio Test, that the series is convergent for all x. [5 marks]
(c) By differentiating the series find the first three terms of the cosine series. [2 marks]

2. (a)

(i) Prove, by using the substitution $u^2 = a^2 + x^2$, that

$$\int \frac{x}{\sqrt{a^2 + x^2}} dx = \sqrt{a^2 + x^2} + constant.$$

[3 marks]

(ii) Given that

$$I_n = \int \frac{x^n}{\sqrt{a^2 + x^2}} dx,$$

show that, for $n \geq 2$,

$$nI_n = x^{n-1}\sqrt{a^2 + x^2} - a^2(n-1)I_{n-2}.$$

[5 marks]

(b) By first changing the order of integration, show that

$$\int_0^a dy \int_y^a \frac{x^2}{\sqrt{x^2 + y^2}} dx = \frac{a^3}{3} \ln(1 + \sqrt{2}).$$

[6 marks]

(Hint: Use table of standard integrals))

3. (a) Show that the function

$$u(t,x) = \frac{1}{\sqrt{4\pi t}}\exp\left(\frac{-x^2}{4t}\right).$$

satisfies the following equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

[6 marks]

(b) Determine the position and nature (i.e.,whether minimum, maximum or saddle point) of the stationary points of the function

$$z = x^3 + y^3 - 3xy.$$

[8 marks]

4. (a) State de Moivre's theorem. Use this to show that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta.$$

[6 marks]

(b) Find the four roots of the equation $z^4 = -1$, expressing your answer in the form $a + jb$, where a and b are real. Sketch their positions in an Argand diagram. [8 marks]

[Next section overleaf.]

Table of Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln \lvert x \rvert$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln \lvert \cos x \rvert = \ln \lvert \sec x \rvert$
$\cot x$	$\ln \lvert \sin x \rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert = \ln \tan \lvert (\frac{\pi}{4} + \frac{1}{2}x) \rvert$
$\operatorname{cosec} x$	$-\ln \lvert \operatorname{cosec} x + \cot x \rvert = \ln \lvert \tan(\frac{1}{2}x) \rvert$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln \cosh x$
$\coth x$	$\ln \lvert \sinh x \rvert$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1} \left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1} \left(\frac{x}{a}\right) = \ln \left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1} \left(\frac{x}{a}\right) = \ln \left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a} \sec^{-1} \left(\frac{x}{a}\right)$

[End of examination paper.]