Queen Mary,
UNIVERSITY OF LONDON

BEng, MEng Examination by Course Units

MAE 111 ENGINEERING MATHEMATICS II Paper

Date: 3 May, 2007 Time: 10:00 – 12:00 (2 hours)

Duration: 2 hours

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination.

Vectors are in boldface, thus: $\boldsymbol{A}$.
A table of standard integrals is provided at the end of this examination paper.

Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question. For Section A you may answer more than one question on a page in the answer book, but make it clear where each answer starts.

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SECTION A

Each question carries 6 marks. You should attempt ALL questions.

1. Calculate $\frac{dy}{dx}$, when $y(x)$ is given by

$$y = \frac{2}{x}\tan\frac{x}{2} + \sin\frac{x}{12}.$$

2. Calculate $\frac{dy}{dx}$ in terms of x and y if

$$y = 1 + xe^y.$$

3. Use L'Hopital's Rule to evaluate the limit

$$\lim_{x\to 0}\left\{\frac{x\cos x}{\sin 2x}\right\}.$$

4. By using partial fractions, evaluate the following integral

$$I = \int \frac{x+3}{x^2-3x+2}dx.$$

5. Evaluate the double integral

$$I = \int_0^2 \int_{x^2}^{2x} (3x+2)dy\ dx.$$

6. Derive the first four non-zero terms of the Maclaurin power series for

$$\ln(1+x).$$

7. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = \widehat{\boldsymbol{i}} + 3\widehat{\boldsymbol{j}} - \widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 3\widehat{\boldsymbol{i}} - \widehat{\boldsymbol{j}} + 2\widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) their sum $\boldsymbol{a}+\boldsymbol{b}$, (b) the scalar product $\boldsymbol{a}\cdot\boldsymbol{b}$, and (c) the vector cross product $\boldsymbol{a}\wedge\boldsymbol{b}$.

8. Point masses 2g, 4g, 5g and 1g are situated at the points A, B, C and D with position vectors $\widehat{\boldsymbol{i}}+\widehat{\boldsymbol{j}}$, $2\widehat{\boldsymbol{i}}+\widehat{\boldsymbol{j}}$, $-\widehat{\boldsymbol{i}}+\widehat{\boldsymbol{j}}$, and $3\widehat{\boldsymbol{i}}-\widehat{\boldsymbol{j}}$, respectively. Find the position vector of the centre of mass for these four particles.

[Next question overleaf.]

9. Three complex numbers are given as $a = 3 + 2j$, $b = -3 + 2j$ and $c = 2 - 3j$. Calculate

(a) $2a - b$,
(b) ab,
(c) b/c.

10. A complex number z is given by $z = 2 - 2j$.

(a) Express this number in polar form $re^{j\Theta}$,
(b) compute $d = \sqrt{z}$ and
(c) express d in algebraic form $a + jb$.

11. Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the following functions and simplify:

(a) $f = ax^2 + bxy + cy^2 + d$
(b) $f = \frac{y}{x^2} - \frac{x}{y^2}$.

12. Verify that $f_{xx} + f_{yy} = 0$ for

$f = y \ln(x^2 + y^2) - 2x \tan^{-1} \frac{x}{y}$.

(Note that $\frac{d}{du} \tan^{-1} u = \frac{1}{1+u^2}$.)

SECTION B

Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

1. (a) Prove:

$$\cosh^2 x - \sinh^2 x = 1.$$

(b) Evaluate

$$\int \operatorname{sech} u du.$$

[You may find the substitution $\cosh u = \sec\theta$ of use.]

2. A half circular disk of radius a lies above the $x - axis$ and with its diameter on the $x - axis$.

(a) Prove that the centroid is positioned a distance $4a/3\pi$ above the $x - axis$ [Hint: use the substitution $x = a\cos\theta, y = a\sin\theta$]
(b) Determine the volume generated when the half circular disk is rotated about the $x - axis$.

(c) Determine the moment of inertia of the half circular disk about

(i) the $x - axis$.
(ii) the centroid.

[You may assume $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$ and $\sin^4\theta = \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3)$].

3. The average velocity in a circular pipe with laminar flow is given by

$$u = \frac{1}{32}\frac{d^2}{\mu}\frac{\Delta p}{l}.$$

In an experiment this relationship is verified. The pipe diameter d and the length l are measured to 0.5% accuracy and the pressure difference Δp to 1%. The fluid is a mixture and its viscosity μ is precise to 3%.

1. Find the maximum percentage error.
2. There is a budget to improve measurement accuracy of either the diameter measurement to 0.25%, of the length to 0.20% or the pressure difference to 0.75%. Which option results in the largest improvement?
3. Using the original setup of part(1), how much would the accuracy of the viscosity μ have to be improved to have an error of at most 3% of the velocity measurement.

4. (a) State de Moivre's theorem. Use this to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

(b) Find the three roots of the equation $z^3 = -8j$, expressing your answer in the form $a + jb$, where a and b are real. Sketch their positions in an Argand diagram.

[Next section overleaf.]

Table of Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln \lvert x \rvert$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln \lvert \cos x \rvert = \ln \lvert \sec x \rvert$
$\cot x$	$\ln \lvert \sin x \rvert$
$\sec x$	$\ln \lvert \sec x + \tan x \rvert = \ln \tan \lvert (\frac{\pi}{4} + \frac{1}{2}x) \rvert$
$\operatorname{cosec} x$	$-\ln \lvert \operatorname{cosec} x + \cot x \rvert = \ln \lvert \tan(\frac{1}{2}x) \rvert$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln \cosh x$
$\coth x$	$\ln \lvert \sinh x \rvert$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

[End of examination paper.]