**Queen Mary,**
UNIVERSITY OF LONDON

## BEng, MEng Examination by Course Units

# MAE 111 ENGINEERING MATHEMATICS II

Date: May 2, 2006    Time: 10:00 – 12:00 (2 hours)

*Duration: 2 hours*

*This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.*

*Calculators are NOT permitted in this examination.*

*Vectors are in boldface, thus:* $\boldsymbol{A}$.
*A table of standard integrals is provided at the end of this examination paper.*

*Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question. For Section A you may answer more than one question on a page in the answer book, but make it clear where each answer starts.*

 

## SECTION A

*Each question carries 6 marks. You should attempt ALL questions.*

**1.** Using the appropriate rules for differentiation, calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $y(x)$ is given by

$$(i)\ y = x^3 \sinh 2x.$$

**2.** Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, where

$$y(t) = \sin(t+7), \quad x(t) = t \ln|t|.$$

**3.** Differentiate implicitly in order to determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $y(x)$ is given by

$$3x^2 - 4x^2y^2 = y^2.$$

**4.** By completing the square and using a standard integral, evaluate the integral

$$I = \int \frac{dx}{\sqrt{3 + 2x - x^2}}.$$

**5.** Given that

$$f = 3x^2 + 5xy^2 + 6xy + 8y.$$

obtain,

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x \partial y}.$$

**6.** Find the range of values for $x$ for which

$$S = \sum_{n=1}^{\infty} \frac{9^n x^{2n}}{n^3}$$

converges. [You need not consider the convergence properties at the endpoints of the range.]

**7.** Evaluate the double integral

$$I = \int_1^2 \int_0^y (x^3 + xy) dx\ dy.$$

**8.** The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = 3\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 3\widehat{\boldsymbol{\imath}} - \widehat{\boldsymbol{\jmath}} + 3\widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) the scalar product (or dot product), and (b) the angle between the vectors .

**9.** Point masses 1g, 2g, 4g and 3g are situated at the points $A$, $B$, $C$ and $D$ with position vectors $\widehat{\boldsymbol{\imath}} + \widehat{\boldsymbol{\jmath}}$, $-9\widehat{\boldsymbol{\imath}} + 2\widehat{\boldsymbol{\jmath}}$, $2\widehat{\boldsymbol{\imath}} + 3\widehat{\boldsymbol{\jmath}}$, and $5\widehat{\boldsymbol{\imath}} - 3\widehat{\boldsymbol{\jmath}}$, respectively. Find the position vector of the centre of mass for these four particles.

**10.** Use L'Hopital's Rule to evaluate the limit

$$\lim_{x \to 0} \left\{ \frac{x \cos x}{\sin 2x} \right\}.$$

**11.** Express the following complex expressions in the form $a + jb$.

$$(a) \qquad (1 - j2)(3 + j),$$

$$(b) \qquad \frac{1 - j3}{5 + j}.$$

**12.** Express the complex numbers $(3 + j3)$ in the form $re^{j\theta}$ .

## SECTION B

*Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.*

**1.** (a) Use the MACLAURIN series method to obtain the first three non-zero terms in the series for

$$f(x) = \cos^2 x.$$

(b) Find the length of the curve

$$y = ln(1 - x^2).$$

between $x = 0$ and $x = 1/2$.

**2.** (a) Using the basic definition of the cotangent function, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x.$$

(b) Given that $I_n = \int \cos^n x dx$ show that, for $n \geq 2$,

$$nI_n = \sin x \ \cos^{n-1} x + (n-1)I_{n-2}.$$

(c) Hence calculate $I_4$.

**3.** (a) Determine the position and nature (i.e.,whether minimum, maximum or saddle point) of the stationary points of the function

$$f(x, y) = x^2 - 4xy + y^3 + 4y.$$

(b) The radius, $r$, of a cylinder decreases at the rate of 2.0 mm/s and the height, $h$, increases at a rate of 3.0 mm/s, find the rate of change of the volume, $V$, of the cylinder($V = \pi r^2 h$)at the instant when $r = 50mm$ and $h = 30mm$.

**4.** (a) State de Moivre's theorem. Use this to show that

$$\sin 3\theta = 3\sin\theta - 4\cos^3\theta.$$

(b) Find the four roots of the equation $z^4 = -16$, expressing your answer in the form $a + jb$, where $a$ and $b$ are real. Sketch their positions in an Argand diagram.

**Table of Standard Integrals**

| $f(x)$ | $\int f(x)dx$ |
|---|---|
| $x^n, n \neq -1$ | $\frac{x^{n+1}}{n+1}$ |
| $\frac{1}{x}, x > 0$ | $\ln \lvert x \rvert$ |
| $e^{ax}$ | $\frac{1}{a}e^{ax}$ |
| $\sin x$ | $-\cos x$ |
| $\cos x$ | $\sin x$ |
| $\tan x$ | $-\ln \lvert \cos x \rvert = \ln \lvert \sec x \rvert$ |
| $\cot x$ | $\ln \lvert \sin x \rvert$ |
| $\sec x$ | $\ln \lvert \sec x + \tan x \rvert = \ln \tan \lvert (\frac{\pi}{4} + \frac{1}{2}x) \rvert$ |
| $\operatorname{cosec} x$ | $-\ln \lvert \operatorname{cosec} x + \cot x \rvert = \ln \lvert \tan(\frac{1}{2}x) \rvert$ |
| $\sec^2 x$ | $\tan x$ |
| $\operatorname{cosec}^2 x$ | $-\cot x$ |
| $\sec x \tan x$ | $\sec x$ |
| $\operatorname{cosec} x \cot x$ | $-\operatorname{cosec} x$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |
| $\operatorname{sech}^2 x$ | $\tanh x$ |
| $\operatorname{cosech}^2 x$ | $-\coth x$ |
| $\tanh x$ | $\ln \cosh x$ |
| $\coth x$ | $\ln \lvert \sinh x \rvert$ |
| $\operatorname{sech} x \tanh x$ | $-\operatorname{sech} x$ |
| $\operatorname{cosech} x \coth x$ | $-\operatorname{cosech} x$ |
| $\frac{1}{a^2+x^2}$ | $\frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right)$ |
| $\frac{1}{a^2-x^2}$ | $\frac{1}{2a} \ln \frac{a+x}{a-x}, (x < a)$ |
| $\frac{1}{\sqrt{(a^2-x^2)}}$ | $\sin^{-1} \left(\frac{x}{a}\right)$ |
| $\frac{1}{\sqrt{(x^2+a^2)}}$ | $\sinh^{-1} \left(\frac{x}{a}\right) = \ln \left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$ |
| $\frac{1}{\sqrt{x^2-a^2}}$ | $\cosh^{-1} \left(\frac{x}{a}\right) = \ln \left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$ |
| $\frac{1}{x\sqrt{(x^2-a^2)}}$ | $\frac{1}{a} \sec^{-1} \left(\frac{x}{a}\right)$ |



*[End of examination paper.]*