Queen Mary,
UNIVERSITY OF LONDON

BEng, MEng Examination by Course Units

MAE 111 ENGINEERING MATHEMATICS II

Date: May 4, 2005 Time: 14:30 – 16:30 (2 hours)

Duration: 2 hours

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination.

Vectors are in boldface, thus: ***A****.*
A table of standard integrals is provided at the end of this examination paper.

Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question. For Section A you may answer more than one question on a page in the answer book, but make it clear where each answer starts.

SECTION A

Each question carries 6 marks. You should attempt ALL questions.

1. Using the appropriate rules for differentiation, calculate $\frac{dy}{dx}$, when $y(x)$ is given by

$$y = x^4 ln(1 + 2x).$$

2. Calculate $\frac{dy}{dx}$, where

$$y(t) = \sinh^2 t, x(t) = \cosh^3 t.$$

3. Differentiate implicitly in order to determine $\frac{dy}{dx}$ when $y(x)$ is given by

$$x^3 + y^3 + x^2 = 4xy.$$

4. Evaluate the integral

$$I = \int x^2 e^x \, dx.$$

5. By using partial fractions, evaluate the following integral

$$I = \int \frac{dx}{x^2 + x - 12}.$$

6. Given that

$$f = 5x^3 + 6x^2y^2 + 2xy + 7y.$$

obtain,

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x \partial y}.$$

7. Using the equation

$$s = \int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, dx.$$

find the length s of the curve given by $9y^2 = 4x^3$ between $x = 3$ and $x = 8$.

8. Evaluate the double integral

$$I = \int_0^2 \int_{x^2}^{2x} (x^3 + 4y) dy\, dx.$$

9. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = 12\widehat{\boldsymbol{i}} + 3\widehat{\boldsymbol{j}} - 4\widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 5\widehat{\boldsymbol{i}} - 8\widehat{\boldsymbol{j}} + 2\widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) $3\boldsymbol{b} - \boldsymbol{a}$, (b) the magnitude of $\boldsymbol{a}$, and (c) the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

10. Use L'Hopital's Rule to evaluate the limit

$$\lim_{x \to 0} \left\{ \frac{x - sinx}{x^3} \right\}.$$

11. Two complex numbers z and Z are given by

$$z = 2 - j5, Z = 3 + j4.$$

Calculate (i) $4Z - 2z$, (ii)zZ, (iii)z/Z, expressing your answer in each case in the form $a + jb$.

12. Given that z and $\bar{z}$ are conjugate complex numbers, find the values of all the complex number that satisfy the equation

$$3z\bar{z} + 2(z - \bar{z}) = 39 + j12.$$

SECTION B

Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

1. (i) Use the MACLAURIN series method to obtain the first four terms in the series for e^x.

(ii) Find the nth term of the series. Use the RATIO test to find the range of x for which this series is convergent.

(iii) Write down the equivalent series for e^{-x} and hence deduce the first three non-zero terms in the series for $coshx$

2. (a) Using the basic definition of the tangent function, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x.$$

(b) Given that $I_n = \int \tan^n dx$ show that, for $n \geq 2$,

$$(n-1)I_n = \tan^{n-1} x - (n-1)I_{n-2}.$$

(c) Hence calculate I_4.

3. In a Cartesian coordinate system, point A is given by $(-3,\ 0, -5)$ and point B by $(3,\ 4,\ 3)$. Find the Direction Cosines of the line AB and write down the vector equation for this line.

A second line passes through point C with coordinates $(2,\ 5,\ 5)$, and is in the direction $(2\widehat{\boldsymbol{i}} + 3\widehat{\boldsymbol{j}} + 6\widehat{\boldsymbol{k}})$. Show that these two lines intersect at a point D and write down the coordinates of D.

Find a vector that is perpendicular to both $\boldsymbol{AB}$ and $\boldsymbol{CD}$.

4. i) State de Moivre's theorem. Use this to show that

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

By using the above result or otherwise, find all solution to the equation

$$\cos 3\theta = \cos^3\theta.$$

 [This question continues overleaf ...]

(ii) Find the four roots of the equation $z^4 = j3$, expressing your answer in the form $re^{j\theta}$, where r and θ are real. Sketch their positions in an Argand diagram.

[Next section overleaf.]

Table of Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln \lvert x \rvert$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln \lvert\cos x\rvert = \ln \lvert\sec x\rvert$
$\cot x$	$\ln \lvert\sin x\rvert$
$\sec x$	$\ln \lvert\sec x + \tan x\rvert = \ln \tan \lvert(\frac{\pi}{4} + \frac{1}{2}x)\rvert$
$\operatorname{cosec} x$	$-\ln \lvert\operatorname{cosec} x + \cot x\rvert = \ln \lvert\tan(\frac{1}{2}x)\rvert$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln \cosh x$
$\coth x$	$\ln \lvert\sinh x\rvert$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

[End of examination paper.]