Queen Mary,
UNIVERSITY OF LONDON

BEng, MEng Examination by Course Units

MAE 111 ENGINEERING MATHEMATICS II

Date: May 4, 2004 Time: 14:30 – 16:30 (2 hours)

Duration: 2 hours

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination.

Vectors are in boldface, thus: $\boldsymbol{A}$.
A table of standard integrals is provided at the end of this examination paper.

Answers to questions should be given in order in the answer books. Leave space to return to any uncompleted question. For Section A you may answer more than one question on a page in the answer book, but make it clear where each answer starts.

SECTION A

Each question carries 6 marks. You should attempt ALL questions.

1. Using the appropriate rules for differentiation, calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $y(x)$ is given by

$$(a) \qquad y = (x^7 + 2x)\sin x,$$

$$(b) \qquad y = \frac{2x^2}{x + \cos x}.$$

2. Differentiate implicitly in order to determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $y(x)$ is given by

$$3x^3 + 9x^2y = y^3.$$

3. Use L'Hopital's Rule to evaluate the limit

$$\lim_{x \to 0} \left\{ \frac{\cos 5x - 1}{x^2} \right\}.$$

4. Using integration by parts, evaluate the integral

$$I = \int x^2 \sin x \; dx.$$

5. By completing the square and using a standard integral, evaluate the integral

$$I = \int \frac{dx}{\sqrt{3 + 2x - x^2}}.$$

6. Evaluate the double integral

$$I = \int_1^2 \int_0^\pi (2 + \cos\theta) d\theta \; dr.$$

7. Decide whether the series $S = \sum u_n$ is convergent or divergent when:

$$(a) \qquad u_n = \frac{n-2}{2n+1},$$

$$(b) \qquad u_n = \frac{2^n}{n!}.$$

8. The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = 2\widehat{\boldsymbol{\imath}} + \widehat{\boldsymbol{\jmath}} - \widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = 3\widehat{\boldsymbol{\imath}} - 2\widehat{\boldsymbol{\jmath}} + 2\widehat{\boldsymbol{k}}.$$

For these vectors, calculate (a) their sum $\boldsymbol{a} + \boldsymbol{b}$, (b) the scalar product $\boldsymbol{a} \cdot \boldsymbol{b}$, and (c) the vector cross product $\boldsymbol{a} \wedge \boldsymbol{b}$.

9. Point masses 2g, 4g, 5g and 1g are situated at the points A, B, C and D with position vectors $3\widehat{\boldsymbol{\imath}} - 4\widehat{\boldsymbol{\jmath}}$, $-\widehat{\boldsymbol{\imath}} + 5\widehat{\boldsymbol{\jmath}}$, $\widehat{\boldsymbol{\imath}} + 7\widehat{\boldsymbol{\jmath}}$, and $5\widehat{\boldsymbol{\imath}} + \widehat{\boldsymbol{\jmath}}$, respectively. Find the position vector of the centre of mass for these four particles.

10. Express the following complex expressions in the form $a + jb$.

$$(a) \qquad (2 - j3)(3 + j),$$

$$(b) \qquad \frac{1 - j3}{5 + j2}.$$

11. Express in the form $r\,e^{j\theta}$ the following complex number:

$$z = 2\sqrt{3} + j2.$$

[Next question overleaf.]

12. Given the function $f(x, y)$

$$f = \frac{x}{y}\ \cos(xy),$$

show that

$$x\frac{\partial f}{\partial x} - y\frac{\partial f}{\partial y} = 2f.$$

SECTION B

Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

1. Use the Maclaurin's series method to obtain the first three non-zero terms in the series for

$$f(x) = \sin^{-1}(x).$$

(You may use the Table of Standard Integrals.)

Assuming that the n-th term of the series is given by

$$u_n = \frac{2(2n)!}{(n!)^2(2n+1)}\left(\frac{x}{2}\right)^{2n+1},$$

show, by using the Ratio Test, that the series is convergent for $|x| < 1$.

2. Find the mean value of the $y = 2\sin t + 3\cos t$ between $t = 0$ and $t = \pi$.

Find also the Root Mean Square (RMS) value of the same function between the same limits. [You may find the substitution $\cos^2 t = (1/2)(1 + \cos 2t)$ useful.]

3. (a) Using the basic definition of the tangent function, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x.$$

(b) Given that $I_n = \int \sec^n x dx$ show that, for $n \geq 2$,

$$(n-1)I_n = \sec^{n-2} x\ \tan x + (n-2)I_{n-2}.$$

(c) Hence calculate I_4.

[Next question overleaf.]

4. (a) State de Moivre's Theorem.

(b) Find the three cube roots of the complex number $-j8$. Express your answer in the form $a + jb$, where a and b are real. Sketch the positions of the roots in an Argand diagram.

 [Next section overleaf.]

Table of Standard Integrals

$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln\lvert x\rvert$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln\lvert\cos x\rvert = \ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\tan\lvert(\frac{\pi}{4} + \frac{1}{2}x)\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln\cosh x$
$\coth x$	$\ln\lvert\sinh x\rvert$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

[End of examination paper.]