Queen Mary
University of London

B. Eng. Examination by course unit 2011

MAE 111 ENGINEERING MATHEMATICS II

Duration: 2 hours

Date and time: 20th May 2011, 10.00h–12.00h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

The paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

A table of standard integrals is provided at the end of this examination paper.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J.R. Donnison, W.J. Sutherland

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Section A: Each question carries 6 marks with a section total of 72 marks . You should attempt ALL questions.

Vectors are in Boldface, thus $\boldsymbol{A}$.

Question 1 Calculate $\frac{\mathrm{d}y}{\mathrm{d}x}$, when $y(x)$ is given by

$$y = (x^3 + \cos(3x))\ln(1 + x^2).$$

Question 2 By differentiating implicitly, determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ when the function $y(x)$ is given by

$$y + 4xy^2 = 3x^3 + x^2 y.$$

Question 3 Evaluate, using partial fractions, the following integral

$$I = \int \frac{1}{x^2 - x - 6}\,dx.$$

Question 4 The volume generated by rotating a curve about the x-axis between $x = a$ and $x = b$ is given by $V = \int_a^b \pi y^2\,dx$. Calculate this volume of rotation for the curve $y = x^2 + 4$ between $x = 1$ and $x = 2$.

Question 5 Use L'Hopital's Rule to evaluate the limit

$$\lim_{x\to 0}\left\{\frac{1-\cos 4x}{2x^2}\right\}.$$

Question 6 Evaluate the double integral

$$I = \int_1^3 \int_0^y (x^2 + xy)dx\,dy.$$

Question 7 The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are given in component form by

$$\boldsymbol{a} = 2\widehat{\boldsymbol{i}} + \widehat{\boldsymbol{j}} - 2\widehat{\boldsymbol{k}}, \qquad \boldsymbol{b} = \widehat{\boldsymbol{i}} - 2\widehat{\boldsymbol{j}} + 3\widehat{\boldsymbol{k}}.$$

For these vectors, calculate

(a) their sum $\boldsymbol{a} + \boldsymbol{b}$, (b) the scalar product $\boldsymbol{a}\cdot\boldsymbol{b}$, (c) the vector cross product $\boldsymbol{a}\wedge\boldsymbol{b}$.

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Question 8 Point masses 3g, 1g, 4g and 2g are situated at the points A, B, C and D with position vectors $\widehat{\boldsymbol{i}}+2\widehat{\boldsymbol{j}}$, $3\widehat{\boldsymbol{i}}-\widehat{\boldsymbol{j}}$, $-2\widehat{\boldsymbol{i}}+\widehat{\boldsymbol{j}}$, and $4\widehat{\boldsymbol{i}}+2\widehat{\boldsymbol{j}}$, respectively. Find the position vector of the centre of mass for these four particles.

Question 9 Three complex numbers are given as $a=3+j$, $b=1+j2$ and $c=1-j2$. Calculate

(a) $2a-b$,
(b) ab,
(c) b/c.

Question 10 A complex number z is given by $z=\sqrt{3}+j$.

(a) Express this number in polar form $re^{j\Theta}$,
(b) compute z^2 in polar form
(c) express z^2 in algebraic form $z^2=x+jy$.

Question 11 A cylinder with radius r and height h has a volume $V=\pi r^2 h$. Given that initially $r=50$ mm and $h=30$ mm, find the approximate change in V when r decreases by 3.0 mm and h increases by 4.0 mm.

(You may assume that these changes can be considered small.)

Question 12 Given that

$$f=3x^4+4xy^2+2xy+5y,$$

obtain the expressions of

$$\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial^2 f}{\partial x\partial y}.$$

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Section B: Each question carries 14 marks. You may attempt all questions but only marks for the best TWO questions will be counted.

Question 13 (a) Use the Maclaurin series method to obtain the first four non-zero terms in the series for $\ln(1+2x)$.

[6].

(b) Check that the first four non-zero terms are given by

$$U_n = \frac{(-1)^{n+1}(2x)^n}{n}, \ n = 1, 2, 3, 4.$$

[4]

(c) Assuming that this formula holds for all $n \geq 1$, use the Ratio Test to find the range of values of x for which this series is convergent. [4]

Question 14 (a) Using the basic definition of the tangent function, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x.$$

[2]

(b) Defining $I_n = \int \sec^n x \, dx$ show that, for $n \geq 2$,

$$(n-1)I_n = \sec^{n-2} x \ \tan x + (n-2)I_{n-2}.$$

[8]

(c) Hence calculate I_4. [4]

Question 15 a) Given the function

$$u(s,t) = f(x,y),$$

and that $x = s^2 - t^2$, and $y = 2st$, prove that

$$s\frac{\partial u}{\partial s} - t\frac{\partial u}{\partial t} = 2(s^2+t^2)\frac{\partial f}{\partial x}.$$

[6]

(b) Determine the position and nature (i.e.,whether minimum, maximum or saddle point) of the stationary points of the function

$$f(x,y) = x^3 + y^3 - 3xy.$$

(Only real solutions should be considered)

[8]

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Question 16 (a) State de Moivre's theorem. Use this to show that

$$\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1.$$

[6]

(b) Write down in the form $a + jb$
(a) $e^{j\pi}$ (b) $e^{j\frac{\pi}{2}}$.
Hence find the three roots of the equation $z^3 = j27$, expressing your answer in the form $a + jb$, where a and b are real. Sketch their positions in an Argand diagram. [8]

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$f(x)$	$\int f(x)dx$
$x^n, n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}, x > 0$	$\ln\|x\|$
e^{ax}	$\frac{1}{a}e^{ax}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln\|\cos x\| = \ln\|\sec x\|$
$\cot x$	$\ln\|\sin x\|$
$\sec x$	$\ln\|\sec x + \tan x\| = \ln\tan\|(\frac{\pi}{4} + \frac{1}{2}x)\|$
$\operatorname{cosec} x$	$-\ln\|\operatorname{cosec} x + \cot x\| = \ln\|\tan(\frac{1}{2}x)\|$
$\sec^2 x$	$\tan x$
$\operatorname{cosec}^2 x$	$-\cot x$
$\sec x \tan x$	$\sec x$
$\operatorname{cosec} x \cot x$	$-\operatorname{cosec} x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\operatorname{sech}^2 x$	$\tanh x$
$\operatorname{cosech}^2 x$	$-\coth x$
$\tanh x$	$\ln\cosh x$
$\coth x$	$\ln\|\sinh x\|$
$\operatorname{sech} x \tanh x$	$-\operatorname{sech} x$
$\operatorname{cosech} x \coth x$	$-\operatorname{cosech} x$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x}, (x < a)$
$\frac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2+a^2)}}{a}\right]$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[\frac{x+\sqrt{(x^2-a^2)}}{a}\right]$
$\frac{1}{x\sqrt{(x^2-a^2)}}$	$\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)$

End of Paper

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