

Geometry I

Coursework 7

To hand in on 5th March 2014

A solution to the Feedback Question, stapled if on more than one sheet, and with your full name (last name underlined) and student number, should be handed in to your tutor first thing in your Week 9 Exercise Class.

By the beginning of your Week 9 Exercise Class (on 5th March 2014), you should already have tried the Practice Questions, on which you can ask for help in the Exercise Class. You will not receive any help on your Feedback Question.

Show clearly all the steps in your calculations.

Practice Question 1.

Let $A = \begin{pmatrix} -1 & -3 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & -3 \\ 1 & -3 \end{pmatrix}$. Evaluate the matrix expressions:

(a) AB ; (b) BA ; (c) $5A - 3BA$; (d) $(B + 3I_2)^2$; (e) $3A + I_2^{17} - 5B$.

Practice Question 2.

(a) Find a 2×2 matrix A such that $A \neq 0_{2 \times 2}$ but $A^2 = 0_{2 \times 2}$.

(b) Find 2×2 matrices A and B such that $AB = 0_{2 \times 2}$ but $BA \neq 0_{2 \times 2}$.

Practice Question 3.

Let $A = (a_{ij})_{m \times n}$, let $B = (b_{ij})_{n \times p}$, and let λ and μ be scalars. Prove each of the matrix equalities stated below, by determining that the sizes of the left-hand and right-hand sides are the same, and that, for all possible i and j , the (i, j) -entry of the left-hand side equals the (i, j) -entry of the right-hand side.

[Hint: Review the proofs of matrix equalities in your Week 8 lecture notes.]

(a) $A + (-A) = 0_{m \times n}$;

(b) $(\lambda + \mu)A = \lambda A + \mu A$;

(c) $\lambda(AB) = A(\lambda B)$.

Feedback Question.

Let $A = \begin{pmatrix} -7 & -1 & 3 \\ 1 & 0 & -3 \\ 1 & 2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 \\ -3 & 1 \\ 2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} -2 & -2 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & -2 \end{pmatrix}$.

In each of the following cases, either evaluate the given matrix expression, or explain why the expression has no meaning.

- (a) $A - 3B$; (b) $-A + 5C$; (c) $(1 \ 5 \ -2)B$; (d) BA ; (e) CB ; (f) CA .

Dr John N. Bray, 24th February 2014