QUANTIZATION OF BOLTZMANN'S GAS CONSTANT

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October 27, 2005

Abstract

The Planck length and the Planck mass are the quantities that are usually used to describe the connection between the microscopic world of the vacuum and the cosmological world of the expanding universe. In this article, it is shown that another mass-length combination introduced by Stoney in *1881* is more effective in that context in that it enables the quantization of Boltzmann's gas constant k in terms of cosmological scale quantities. The quantization formula $k = N_B k_q$ is derived and the quantum number N_B is found to have the specific numerical value $N_B = 10^{13}$ associated with the quantum, k_q of k. The relations between the various quantities is discussed in relation to the part played by the gravitation constant, G, and connections of the structure with vacuum thermodynamics.

Keywords: Boltzmann, Stoney, Cosmology
PACS Nos.: 04.06-m, 04.20-q, 05.30-d, 05.70.Ce

1 New Quantum Cosmology Link

The objective of this short paper is to report the discovery of an unexpected link between classical gas dynamics theory and quantum cosmology theory[2][3]. This takes the form of a quantization of the classical gas dynamics Boltzmann's constant, k, in terms of the classical electromagnetic length, r_S, associated with the Stoney[1] mass, m_S, and which is also closely related to both the Planck length[9], l_P, and the gravitation constant[5][7][8][9], G. We first define the quantities involved and some relations between them,

$$r_S = e^2/(4\pi\epsilon_0 m_S c^2) = \alpha l_S = \alpha\hbar/(m_S c) = G m_S/c^2. \tag{1.1}$$

$$m_S^2 = \xi m_p m_e = \alpha m_P^2, \tag{1.2}$$

where ξ is Dirac's first large number and α is the fine structure constant[4]. The relation between the classical electromagnetic Stoney length r_S and the smaller Compton Planck length l_P is

$$r_S = \alpha^{-1/2} l_P. \tag{1.3}$$

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Assume that Boltzmann's gas constant, k, can be expressed in the form,

$$k = N_B k_q, \tag{1.4}$$

where N_B is an integer and k_q is a *cosmological* quantum of the Boltzmann constant, k, such that

$$k_q = \zeta_0 r_S, \quad r_S = \alpha\hbar/(m_S c) \tag{1.5}$$

where ζ_0 is an unknown dimensioned multiplier quantity to be determined. It follows that

$$k = N_B \zeta_0 r_S = N_B \zeta_0 \alpha\hbar/(m_S c), \tag{1.6}$$

or

$$k/(hc) = N_B \zeta_0 \alpha/(2\pi m_S c^2) = \ 69.50356(12), \tag{1.7}$$

from CODATA's value[6] for the ratio $k/(hc)$. It follows that

$$N_B \zeta_0 = 1.0000771036063 \times 10^{13} = 10000771036063. \tag{1.8}$$

Therefore the quantum pure number integer, N_B, ζ_0 and the Boltzmann quantum k_q are respectively given by

$$N_B = 10^{13} \tag{1.9}$$

$$\zeta_0 = 1.0000771036063 \quad JK^{-1}m^{-1} \tag{1.10}$$

$$k_q = \zeta_0 r_S = 1.3806502980108 \times 10^{-36} \quad JK^{-1} \tag{1.11}$$

with ζ_0 having a value notably close to unity and expressed in terms of known physical constants it is

$$\zeta_0 = (k^2 c^3/(\alpha\hbar G))^{1/2} 10^{-13} \tag{1.12}$$

and r_S in terms of known physical constants is

$$r_S = (\alpha\hbar G/c^2)^{1/2}. \tag{1.13}$$

It follows that the product $k_q = \zeta_0 r_S$ is

$$k_q = \zeta_0 r_S = 10^{-13} k \quad JK^{-1} \tag{1.14}$$

which is the original equation (1.5) re-expressed and which shows that k_q does not depend on the value of G.

The numerical index, 13, in the value of N_B is the power of 10 that enables the expression of $N_B\zeta_0$ as a fourteen digit *integer* at equation (1.8). Such a number would have the same value over a very large variation of numerical values for ζ_0. Thus the quantum number N_B is very stable and does not depend on the value of G even under new measurement or possible physical dependence on universe epoch. Further, it is also of importance to notice that N_B is a pure *dimensionless* number with the form,

$$N_B = 10^{13} = \frac{k}{k_q} = \frac{k}{\zeta_0 r_S}. \tag{1.15}$$

That is to say, N_B is the ratio of the two *equal* dimensional quantities k and k_q or k and $\zeta_0 r_S$. It follows conclusively from this that it is dimensionless and therefore would not change value under any consistent change of units.

The value for k is given by CODATA as

$$k = 1.3806503(24) \times 10^{-23} \quad JK^{-1} \tag{1.16}$$

2 Conclusions

It has been shown that Boltzmann's constant k can be thought of as additively composed of a large number, 10^{13}, of the smaller parts or quanta k_q. The formula is definite and specific and will hold even if CODATA revises the numerical measurements involved. One of our interests in this connection is the question of how an epoch varying G cosmology should be described in terms of classical or statistical thermodynamics where quantities analogous to temperature, pressure and entropy in the usual macroscopic context are involved. In the quantum cosmology context, things are not *usual* and we anticipate that this quantization discovery will help in that arena. We expect that the quanta k_q and a related entropy will help describe the thermodynamics of, for example, dark energy. The k quantum expansion formula can also be taken to be a possible addition to Dirac's collection of large number relations between micro and cosmological quantities with the further distinction that it is exactly derivable and the large number involved is a strong invariant quantity.

The blame for the quantization formula (1.4) for Boltzmann's constant, k, should be apportioned between McPherson for its discovery and Gilson for its formulation.

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