Strong Quantum Coupling and Relativity, Part 1: Sections 1-3 and References only. Physical Interpretations of Relativity Theory. Conference Proceedings. British Society for the Philosophy of Science, Imperial College London, September 2002

James G. Gilson j.g.gilson@qmul.ac.uk *

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Abstract

Part 1 of this article begins with a preamble, preceding the introduction and headed section 1, which has the objective of showing how important the *technical* aspects of fine structure theory are to furthering a more complete scientific and philosophical understanding of life and the universe in general. This line of work arose out of the application of an alternative version of Schrödinger quantum mechanics, based on the idea of vacuum polarization, to develop a theory of α, the fine structure constant. It turned out that the theory for α could equally well be based on orthodox Schrödinger theory and that, further, the new theory for α was itself the key to a theory of quantum coupling constants in general. The introduction section 2 is still about α but also takes the work forward into the general quantum coupling constant context of the geometry of cyclical group representation symmetries. The work then proceeds by discussing the relation of the quantum coupling constants with π and the significance of this connection which was suggested some forty years ago by R. P. Feynman[23]. Section 3 is used to explains the connection between the *wave capture diagram* figure 1 and the numerical values of quantum coupling constants in general, using the quantum strong coupling constant's diagrammatic structure as a working model. The reference section then concludes this first part of the article. The missing sections that will constitute part 2 will hopefully have been completed in time for the PIRT conference in September 2002.

*School of Mathematical Sciences, Queen Mary College University of London, Mile End Road, London E1 4NS, United Kingdom.

1 Preamble

The work to be discussed in this article is a theory elaboration and refinement related to a recent discovery of a theoretical formula for the fine structure constant α and a prediction for its value of great accuracy. This prediction and the formula that made it possible represents a substantial step forward in the solution of the α *mysteries* which are fundamental and pervade the whole of theoretical physics from the smallest systems, elementary particles, to the very largest one, the universe. This is no mere speculative fantasy or science fiction extrapolation of imagination about the importance of α. The fine structure constant impacts on our every day life here and now through the physics of the semi-conductor devices via the *Hall effect* that is for example exploited in electronic switches. The fine structure constant's ubiquitous influence in such every day objects of the electronic arena is now an all pervasive adjunct to the human environment. This influence in our society extends our philosophical view of the world by the possibility of making explorational tools such as computer controlled telescopes, microscopes, digital photo devices, fibre optical light communication channels all of which involve, at some stage of their operational systems, the coupling of the electric field with electrons and thus α. Our primary personal interface with the world, our eyes are such photonic-electronic coupled biological signal conversion devices. The fine structure constant, has been for many years a source of scientific and philosophical questions regarding its value and significance and as a *fundamental object of theory*, it has an unavoidable place in our attempted explanations of those extraordinary images of distant galaxies returned by the Hubble telescope. Thinking persons cannot ignore those new spectacular views of our world such as that of the *Eagle Nebula* which have recently been returned by Hubble. They jolt us back to the age old question of *what is it all about?*. The mass of such an object relative to the mass of a human observer may be greater than $\approx 10^{40}$ an incredible and overwhelming comparison. When we think about the relative scale of those distant monolithic and magnificent assembles of raw particles, fields and atomic systems, how can we have any doubt that molecular, biological and social structures are out there in abundance. However, it largely depends on α and its contribution to the strength of the Coulomb potential. If α where to suddenly be switched off, here on earth out there in those massive astronomical objects and indeed everywhere, atomic systems would shed all their orbiting electrons as the Coulomb attraction reduced to zero. All those bound electrons would slingshot tangentially out of orbit into a universal orgy of randomness. All atomic, molecular, biological systems would be destroyed in the process. All life including our own, all human aspirations, society and institutions would be consumed in the instant catastrophic fireball and there would be no record left that they had ever existed. It might not be *The End of The World* but it would certainly be the end of us.

Technical astronomical, spectroscopic information about the properties of α at great distances from earth is being obtained by examining light that started on its journey to earth some six billion years ago to arrive currently at the world's largest telescope, the 10-metre Keck facility in Hawaii. The team of scientists involved in that project, Chris Churchill et al, are questioning the constancy of α over cosmological time and what bearing this might have on light velocity in distant places or in the distant past. Answers to such questions are essential if we are to make real progress with our fundamental scientific view of the universe. A full understanding of the significance of α will transform present day science at the technical and practical device level and greatly help with finding the answers to those many fascinating philosophical questions in physics and cosmology of existence, meaning, significance and beginnings generally. Some of these philosophical issues will be discussed in this article. My objective in writing this short preamble has been to bring out how central and important the study if the fine structure constant is to our *Scientific and Philosophical View of the World*.

The resolution of the *big* philosophical problems will likely only come about when the more technical aspects of α are fully clarified. With that thought in mind, let us return to the technical question of the correctness of the prediction for the value of α. The predicted value published by the present author was confirmed by the value given in CODATA's latest report[11], the one that appeared in 1999, on the experimental and recommended values of the fundamental constants. Effectively, the prediction added four additional decimal places to the eight decimal places that were previously accepted as correct. The theoretical formula for α is an inevitable consequence of the kinematics and geometry of the cyclic group of order 29×137. However, the analysis to be carried out in the following article refers to quantum coupling constants in general where the value of α is a special case obtained from a general formula $\alpha(n_1, n_2)$ which yields the values for various quantum coupling constants depending on the integer pair (n_1, n_2) input. The geometry resides in a simple relation between $\alpha(n_1, n_2)$ and an integer dependent generalization $\pi(n)$ of π, the $n = n_1 \times n_2$ order cyclic group and an unusual application of the idea of projection quantization, a concept from orthodox quantum mechanics. Coupling here will mean the tendency for a coupled object to remain in some specific locality by *cyclically* visiting some defined region over some period of time. This is not necessarily the same as an object being attracted to some position by a force such as the Coulomb field generated by a proton. However, these two aspects of coupling are related. The cyclicity implies that π plays a central role and this will be discussed in the introduction to follow. Results obtained earlier from this theory are further discussed in this paper. One such result is that the fine structure constant α has the same value that *characterises* a relation, denoted by $\alpha_{137}(29 \times 137)$, between a representation of the cyclic group of order 29×137 and the induced representation for the cyclic subgroup of order 137. The value of this characteristic is $\alpha_{137}(29 \times 137) = 0.007297352532....$ Another such result further discussed is that the complementary characteristic $\alpha_{29}(29 \times 137) = 0.034280626357...$ for the cyclic subgroup of order 29 represents the gauge theory electro-weak coupling quantity $g^2/4\pi$. A third such result obtained earlier from this *theoretical* structure and further discussed here is a generalized version of the Weinberg electro-weak mixing angle θ_W. These are the more technically orientated results that come from the formula for alpha. However, as remarked earlier, the mysteries of α are very much also in the domain of the philosophy of science and while the technical progress that has occurred is a first step there is still a long way to go with those more tantalising philosophical aspects of what the fine structure really represents and how understanding it can help with the big questions of space-time and such issues as *Mach's Principle*. Such possible further steps beyond the technical developments will be considered in this article.

2 Introduction

Two sets of numbers $\Pi_{\alpha,i}$ and $\Pi_{\alpha,o}$ composed of elements each having analogous properties to π can be defined for the n sided polygon similarly to the way that π is defined in relation to the circle. π is defined in relation to the circle by dividing the circumference C_r of a circle of radius r by twice its radius, $\pi = C_r/(2r)$. The generalizations of π which will be denoted by $\pi_{\alpha,i}$ and $\pi_{\alpha,o}$ will be defined by dividing the perimeter length $P_n = 2n\tan(\pi/n)r_{n,b}$ of an n sided polygon by twice the radius $r_{n,b}$ of its inscribed circle in the case of the i subscript or twice the radius $r_{n,a}$ of its circumscribed circle in the case of the o subscript. Here the subscript b just means *inscribed* and the subscript a means *circumscribed*. Thus the elements of the sets $\Pi_{\alpha,i}$ and $\Pi_{\alpha,o}$ are

$$\pi_{\alpha,i}(n) = n\tan(\pi/n) = \pi\beta_i(n), \tag{2.1}$$

or

$$\pi_{\alpha,o}(n) = n\sin(\pi/n) = \pi\beta_o(n), \tag{2.2}$$

where

$$\beta_i(n) = n\tan(\pi/n)/\pi \tag{2.3}$$

or

$$\beta_o(n) = n\sin(\pi/n)/\pi \tag{2.4}$$

and n is an integer. We shall call the two sets $\Pi_{\alpha,i}$ and $\Pi_{\alpha,o}$ $\pi's$ sibling sets, the i subscript meaning inner and the o subscript meaning outer. From now on we concentrate on the subscript i set which is the more important set for the analysis to be carried through in this paper. We note that $\beta_i(2) = \infty$. The reason for the subscript α notation for these generalization of π arises from the fact that the i subscripted set of elements is related to the set of quantum coupling constants, a class of constants typified by the fine structure constant α. This aspect will be discussed next. The elements, $\pi_{\alpha,i}$, of the set $\Pi_{\alpha,i}$ of π siblings have the following property in relation to the elements, $\alpha(n_1,n_2)$, of the set of quantum coupling constants $C_Q = \{\alpha(n_1,n_2)\}$,

$$\frac{\pi_{\alpha,i}(n_1n_2)}{\pi_{\alpha,i}(\infty)} = \frac{\alpha(n_1,n_2)}{\alpha(n_1,\infty)}, \qquad \pi_{\alpha,i}(\infty) = \pi \tag{2.5}$$

and

$$\alpha(n_1,\infty) = \cos(\pi/n_1)/n_1. \tag{2.6}$$

When $n_1 = 2$

$$\alpha(2,n_2) = n_2\cos(\pi/2)\tan(\pi/(2n_2))/\pi = \tag{2.7}$$

$$\cos(\pi/2)\beta_i(2n_2)/2 = 0 \tag{2.8}$$

except when $n_2 = 1$. In which case the apparently indeterminate product of zero cosine and infinite β factor is clearly trigonometrically interpreted as a numerical multiple of a finite sine,

$$\alpha(2,1) = \sin(\pi/2)/\pi = 1/\pi. \tag{2.9}$$

Thus only the member $\alpha(2,1)$ of elements of the form $\alpha(2,n_2)$ is nonzero. This one none zero element in the $n_1 = 2$ run is particularly important physically. $\alpha(2,1)$ is the numerically largest valued member of the set C_Q of quantum coupling constants. Its value can be identified with the value of the strong coupling constant α_s at the energy of the τ-meson. The relation (2.5) can alternatively be expressed as

$$\alpha(n_1,n_2) = \alpha(n_1,\infty)\beta_i(n_1n_2). \tag{2.10}$$

Thus we can write

$$\alpha(n_1,n_2)\pi = \alpha(n_1,\infty)\pi_{\alpha,i}(n_1n_2). \tag{2.11}$$

The subscript α will be dropped from now on leaving just the i subscript for identification the appropriate set.

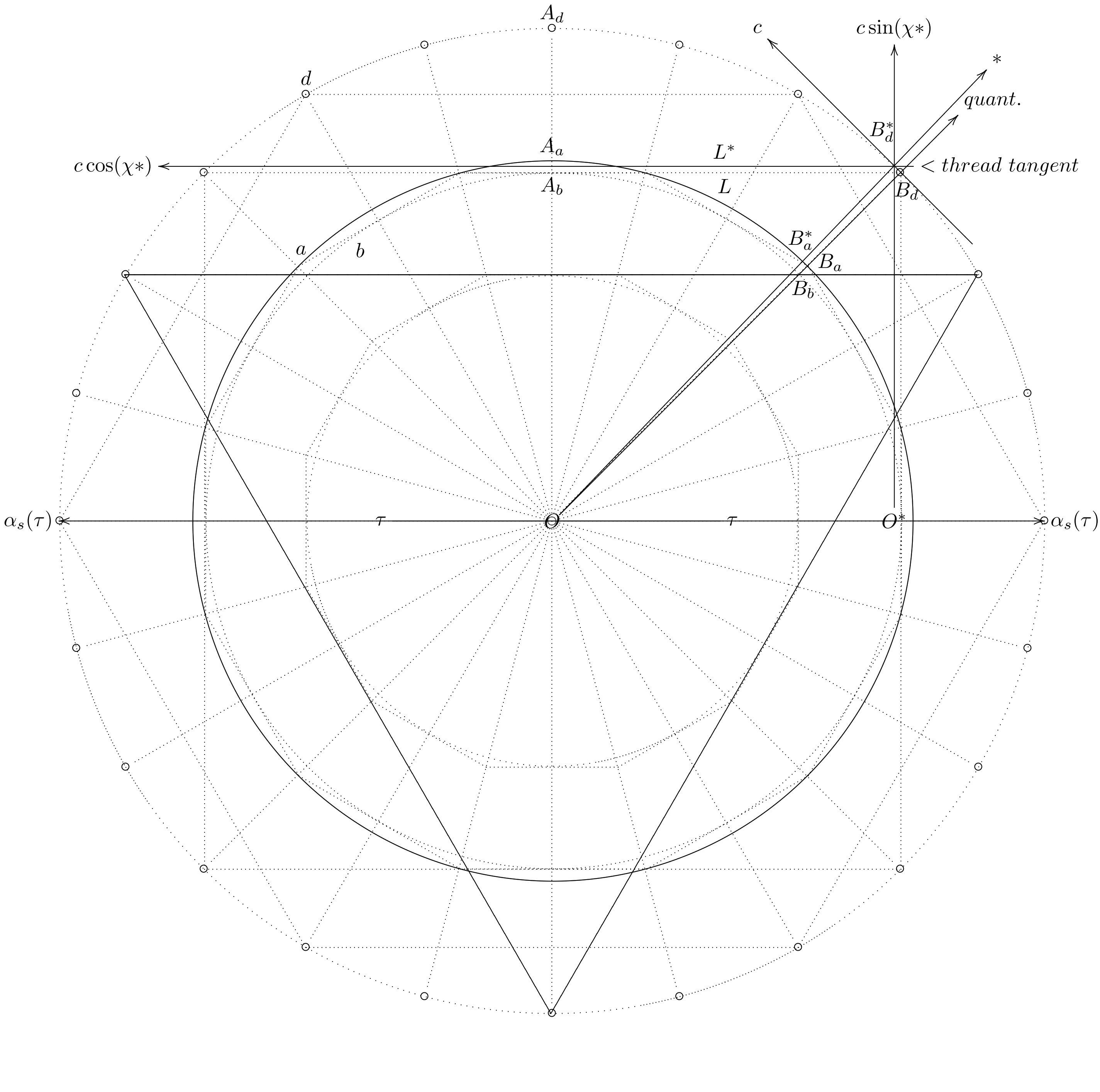
The first of these formula equation (2.10) for the general coupling constant element $\alpha(n_1,n_2)$ is the form in which I first introduced it in reference [4] at which time the important connection with the set $\Pi_{\alpha,i}$ of π siblings had not become apparent. This general connection of the quantum coupling constants with π was anticipated by R.P. Feynman in a remarkable intuitional leap some 40 years ago as can be seen from remarks in his book[19].

It will be shown in this paper how very deeply the special circle invariant number π and its siblings set elements $\pi_i(n)$ are involved in the physics and mathematics of the coupling between quantum systems. An analysis of this special coupling set will be carried through here making use of the single diagram Fig 1.

3 Wave Capture and Strong Coupling

This diagram contains a geometric-kinematic image implying length ratios giving the non-dimensional possible numerical values for the strong coupling constant α_s at various energies. The members represented are:- $\alpha(2,1)$, $\alpha(4,3)$, $\alpha(3,4)$ and $\alpha(6,2)$ in decreasing numerical order and are the four with the largest numerical values. The largest one $\alpha(2,1) = 1/\pi = \alpha_s(\tau)$ can be identified with the energy of the τ meason and is associated with the degenerate *on x-axis 2-sided* polygonal diameter of the horizon circle d.

3.1 Strong Coupling Wave Capture Diagram



This section contains explanations as to why the quantum coupling constants take the numerical geometrical and kinematical form implied by the members of the set of two integer dependent functions $C_Q = \{\alpha(n_1, n_2)\}$. We shall use the quantum strong coupling constant context to develop this explanation because the geometry at this particular coupling strength case can be exhibited using the *realistic* scale diagram Figure 4. This diagram just displays kinematic and geometrical representations and properties of possible values for the quantum *strong* coupling constant α_s and only involves values associated with the cyclical symmetry group of order 12. However, the physical and mathematical ideas involved are the same for all possible coupling elements of the coupling set so that this account covers the characteristic structure and its explanation for all members of the set. The great advantage of working with the higher valued elements of the coupling set lies in the physical scale diagrammatic realism of fig 1. A Realistic diagram is only possible for the few largest members of the set C_Q because visually resolvable size angles are involved such as $\pi/24$ and larger whereas for the countably infinite number of remaining elements of the set, which includes the fine structure constant value with an angle $\chi = \pi/137$ involved, visualisation only becomes possible by using unrealistic distorted diagrams. The whole procedure involved in the description and calculation of quantum couplings in general, can be the summarized as finding the *strengths* of couplings involved in the localized capture and holding in orbit of finite *length* waves in a quantized form which is consistent with representations of a cyclical group and special relativity. Values of the members of the set of coupling strengths C_Q can be read off from diagrams like Figure 4 so that such diagrams can be regarded as representing the geometry and kinematics involved in coupling values in quantum mechanics in general. There is a clear *mathematical* correlation between members of the set $C_Q = \{\alpha(n_1, n_2)\}$ and diagrams $D(n_1, n_2)$ (such as the part $D(4, 3)$, the square, one of elements of figure 1) which in generally can also be designated by an integer pair (n_1, n_2). However, strictly speaking diagrams such as described by a diagram $D(n_1, n_2)$ display the *physics* of the strongest value of coupling that an element $\alpha(n_1, n_2)$ for fixed (n_1, n_2) can be responsible for, the strongest value being $n_1\alpha(n_1, n_2)$. Thus from the more physical interpretational and visualisation point of view the set of diagrams $\{D(n_1, n_2)\}$ correlates more directly with the set $\{n_1\alpha(n_1, n_2)\}$ of strongest values and more usefully than it does with the set $\{\alpha(n_1, n_2)\}$ of fundamental coupling constants.

The first clue to the finding of the set of functions C_Q was the recognition that a finite sized extended object moving on a circular orbit could be *viewed* as occupying an angular segment of the orbit and this segment could have a quantized size such as π/n_1 where n_1 is an integer. Such an angular quantization can be regarded as coming from a radial quantization such as $r = n_1 l_b$ where l_b is an eigen-length associated with the quantum number n_1. Thus a circle with such a radius would have a circumference $C = 2\pi r = 2n_1\pi l_b$ and so the circumference C becomes quantized with quantum number $2n_1$ and naturally occurring circumferential quantum length πl_b. Further, the total circular angle 2π is then naturally quantized as $2\pi = 2n_1(\pi/n_1)$ where (π/n_1) is the angle subtended by the circumference length quantum πl_b at the circle centre. This now all seems very obvious and simple but it was only first noticed in the context of evaluating the fine structure constant by the present author[3] in 1994. A second clue that, had it been noticed, *might* have led to a much earlier finding of the formula for the fine structure constant has been with us since the time of Eddington. This is the general knowledge that alpha is slightly less than 1/137. As far as I know, all experiments over the intervening years between Eddington's[5,7] time and now led to the confirmation of that fact. Thus it is mathematically certain that $\cos(\chi^*)/137$ for some value of χ^* would give the correct numerical value for α because the cosine is always numerically less than or equal to one. This seems to have been missed by everyone including the present author. If the cosine divided by 137 form had been explored by someone, after comparing this form with the current experimental value available, that someone *might* have noticed that $\chi^* \approx \pi/137$ and then the angular quantisation would have been unearthed. However, that did not happen, and that was not the route taken by the present author in deriving the function set C_Q. This clearly need not inhibit us here, where we are looking for the quickest route forward, so we shall assume that all the quantum coupling constants can be expressed exactly as $\alpha = \cos(\chi^*)/n_1$ or *approximately* as $\alpha_{app} = \cos(\pi/n_1)/n_1$. The set of approximate coupling constants will be denote by C_{Qapp}. Extending this idea from the fine structure constant to the other quantum coupling constants is quite a jump but we shall see that it is a justifiable jump. The coupling constant approximation

$$\alpha_{app} = \cos(\pi/n_1)/n_1 \text{..........................(1)}$$

has been and can be obtained from orthodox Schrödinger quantum theory[4] in the case of the fine structure constant so that at least in that context it is an *established* easily confirmed item of science theory. In the following sections, we shall assume that such an approximation is a valid basis for the electro-weak and the strong coupling constants. In the present article, we shall concentrate on the examination, explanation and derivation of the *exact* formulae for α_s, the strong coupling constant as this typifies the general situation for the fundamental coupling constants and the diagram Fig 1 is immediately applicable. Let us now consider the wave capture diagram Fig 1 just in terms of the outer circle d, the transverse horizon where the point B_d moves at the speed of light, and the contained square with top right hand corner instantaneously at B_d a point on the outer circle and initially disregard all the rest of the diagram. This would be the situation when only being aware of the angular quantization involved in analysing the approximation α_{app} with in this case the angular quantum $\pi/4$ being the smallest angle we need to take into account. The motion of the centre of the top side of the square is now clearly towards the left hand side with a speed $v' = c\cos(\pi/4)$. The wave capture diagram is a geometrical-kinematical representation that must be interpreted as being what we or an observer actually *sees* at some specific instant of observation time and if the top of the square is identified with being a physically extended linear object according to special relativity we are seeing a contracted version of a quantity with rest length $L_0 = (1-(v'/c)^2)^{-1/2} A_b B_d$ where L, say, is the viewed length $A_b B_d$. However, $L = OB_d \sin(\pi/4)$ and $v'/c = \cos(\pi/4)$. Therefore $L_0 = OB_d = r_d$, the horizon distance or the distance of the outer circle from its center. That is to say, in this approximation, the trapped wave in motion sits in the angle $\pi/4$ exactly lying along $A_b B_d$ consistently with the Lorentz length contraction formula for a rod of rest length $L_0 = r_d$ with the speed $v' = c\cos(\pi/4)$. We now notice that $L_0 \cos(\pi/4) = OA_b$. That is to say the projection of the horizon radius through the quantum angle $\pi/4$ gives the radius of the circle b. If we now denote the circumference of the circle b by $8\pi l_b$, where πl_b is the length of the quantum $arc(A_b B_b)$, then $OA_b(\pi/4) = OB_d \cos(\pi/4)(\pi/4) = \pi l_b$, or $OA_b = 4 l_b$. This is projection quantization well known in orthodox quantum theory in connection with *angular momentum*. However, it appears here in relation to *space geometry*. Thus the smaller circle radius is quantized with quantum length l_b and quantum number $n_1 = 4$. It is thus clear that the ratio of lengths $l_b/r_d = \cos(\pi/4)/4 = \cos(\pi/n_1)/n_1$, where $n_1 = 4$. That is the ratio of the special quantum length l_b to the horizon distance r_d gives the member of the quantum coupling set C_{Qapp} of approximations to C_Q for the case of the square in rotational motion when $n_1 = 4$. This does not show why the elements of the set C_{Qapp} are the approximate strengths of the coupling of the trapped wave involved. This aspect will be addressed later. A Similar case can be made out for the appearance in the diagram of elements of the set $C_{Qapp} = \{\cos(\pi/n_1)/n_1\}$ associated with the triangle and hexagon with side numbers $n_1 = 3$ and $n_1 = 6$ respectively. The *two*-sided polygon with $n_1 = 2$ corresponds with a set member which seems to have the value $\cos(\pi/2)/2 = 0$ is represented by the degenerate polygon its two sides being the diameter *twice* of the horizon circle at $\alpha_s(\tau)$. However, we shall show that the *true* value, which should be associated with this degenerate polygon and which will emerge later, is very important. The only case that can be used to check out the effectiveness of the approximate formula to generate physical values for coupling constants is the fine structure constant case when $n_1 = 137$. This is the only coupling constant for which the value is known with great accuracy. The formula gives the value

$$\cos(\pi/137)/137 = 0.0072973510109,$$

whereas the latest CODATA[11] recommended experimental measured value is

$$\alpha_{exp} = 0.007297352533.$$

Thus there is agreement for the first eight decimal places using the approximate formula. This is very good by all normal standards of accuracy but clearly it is desirable, if possible, to do better and how that can be done will be shown in the following sections.

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