A Sketch for a Quantum Theory of Gravity V Quantized Cosmology, valid over expanding radius range, $10^{-13}\ m \rightarrow 7.4 \times 10^{25}\ m$

James G. Gilson j.g.gilson@qmul.ac.uk *

January 27, 2005

Abstract

This article is essentially a fourth appendix to the paper *A Sketch for a Quantum Theory of Gravity* containing a demonstration that the quantum theory for gravity obtained in the four earlier articles is consistent with the mathematical structure of the Friedman Cosmological models. In the previous article a specific quantized oscillating model was obtained using the epoch dependent gravitation function $G(t^* \cos(\chi_G))$ and the mass of the universe, M_U derived from quantum theory in those articles. However, that model was restricted to an unspecified range of radius values vaguely called large. This was necessary because of problems with the evaluation of a key integral. In this article the restriction to range is lifted to give a model valid over a very large range of values for the epoch changing universe radius.

1 A Second Quantum Friedman Cosmology

The papers: *A Sketch for a Quantum Theory of Gravity, A Sketch ...II, A Sketch ...III, A Sketch ...IV* will be referred to as A, B, C, D in this paper.

The quantum theory for gravity developed in the earlier four papers was formed from quantum theory based on concepts from Bohr's work, from Sommerfeld's work, from a theory for quantum coupling constants and a *time* dependent formula for Newton's gravitation constant G. The author's theory for the quantum coupling constants depends on quantum and relativity concepts introduced via an idea called *projection quantization*. Projection quantization is more familiar in quantum studies in relation to angular momentum. However, it is used in this work in relation to the ordinary geometry of spatial lengths and spatial angles and effectively supplies a bridge between length quantization and relativity length contraction. It comes into the quantization of gravitation via the formula,

$$r^* \cos(\chi_G(N_G)) = N_G l''_p \quad (1.1)$$
$$l''_p = l_p / \cos^2(\chi_G(n_G)) \quad (1.2)$$

r^* is to be identified with the present day radius of the universe and l_p is the crossed Compton wave length of the proton $\hbar/(m_p c)$. The quantity $r'^* = r^* \cos(\chi_G(N_G))$ is the radius of the very large radius gravitational orbit that any proton exists on as a consequence of being part of the universe and subject to the gravitational influence of the rest of the universe. The gravitational orbit of the general proton under the gravitational attraction of the rest of the universe is exactly analogous to the orbit of an electron in the electromagnetic orbit of a hydrogen-like atom under the Coulomb potential of its nucleus. The analogue of the quantity $\cos(\chi_G(N_G)) = N_G \alpha_G$ is the quantum electromagnetic quantity $\cos(\chi_{137}) = 137\alpha$ in the case of hydrogen$_{137}$. This is all explained in great detail in the earlier papers of this series. In the paper D, the Friedman cosmological equations from general relativity were used to derive a *quantum* cosmological model and it was shown that the numerical value of the cosmological constant could be chosen together with identifications involving the measured value of G to give very accurate model agreement with present day astronomical measurement. The steps involved in deriving this quantum model universe depended on performing an integration of the Friedman equations which was carried out under the assumption that $\cos((\chi_G(N_G))) = 1$. This assumption is in fact very accurately correct and increasingly so for values of $N_G > 137$ as can be seen by using the general coupling constant formula which gives $\cos(\chi) = 137\alpha \approx 1 - 0.000263$. Thus the model obtained in D could

*School of Mathematical Sciences, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom.

be said to be reliable for $N_G > 137$ or equivalently for $r^* > 137 l_p \approx 2.9 \times 10^{-14} < 10 \times 10^{-14} = 10^{-13}$ meters. It follows that for a range of radius values ranging from the subatomic 10^{-13} m to the ultimate radius of the universe the first model is valid. However, it still remains very interesting to explore just what the effect on the model would be by taking the projection cosine $\cos(\chi_G(n_G))$ not equal to unity but rather taking it to have an acceptable dependence on r^* and then performing the integration in that case. It is also desirable to carry through the more general case just in case there are unexpected nasty surprises such as singularities that would have been missed in the simple case. It turns out that this can be done as will now be shown. Firstly, for ease of reference here at (1.4) is the integral that has to be performed,

$$\frac{cdt}{dr} = \pm \frac{1}{(a(k,\chi_G(N_G)) + br^2)^{1/2}} \quad (1.3)$$

$$\int_{t_0}^{t_R} cdt = \pm \int_{R_0}^{R} \frac{dr}{(a(k,\chi_G(N_G)) + br^2)^{1/2}} \quad (1.4)$$

$$a(k,\chi_G(N_G)) = (2\cos^4(\chi_G(N_G)) - k),$$

$$a(k,\chi_G(N_G)) \approx (2\cos^2(\chi_G(N_G)) - k) \quad (1.5)$$

as $\chi_G(N_G))$ is very small. Using the power 2 rather than 4 only makes a very small difference to the constant function $B(R_\Lambda)$ that appears at formula (1.15). I shall restrict the discussion to what is called the closed universe case $k = 1$ so that the first need is to find an r variable function to replace the quantity $a(1,\chi_G(N_G))$. This can be achieved as follows,

$$a(1,\chi_G(N_G)) \approx (2\cos^2(\chi_G(N_G)) - 1)$$

$$\approx \cos(2\chi_G(N_G)) \quad (1.6)$$

$$\approx \cos(2\pi/N_G) \approx 1 - 2(\pi/N_G)^2 \quad (1.7)$$

$$N_G \approx r^*/l_p \quad (1.8)$$

$$a(1,\chi_G(N_G)) \approx 1 - 2(\pi l_p/r^*)^2. \quad (1.9)$$

Four approximation steps are used in getting to the final form (1.9). Each of these approximations are accurate for all N_G and very accurate indeed for $N_G > 137$. Thus one can have great confidence in the final formula (1.9) as usable in the integral 1.4. The integral that now has to be performed is

$$\int_{t_0}^{t_R} cdt = \pm \int_{R_0}^{R} \frac{dr}{(1 - 2(\pi l_p/r)^2 + br^2)^{1/2}}. \quad (1.10)$$

The integral (1.10) can be carried through after multiplying numerator and denominator by $2r$ and using the transformation (1.11) to give the result (1.12) together with definitions (1.13) → (1.17).

$$r(t) \rightarrow w(t) = r^2(t) + 1/(2b) \quad (1.11)$$

$$r^2(t) = R_\Lambda B(R_\Lambda)\sin\phi(t) + R_\Lambda^2/2 \quad (1.12)$$

$$\phi(t) = \frac{\pm 2ct}{R_\Lambda} + \phi_0(R_\Lambda) \quad (1.13)$$

$$\phi_0(R_\Lambda) = \sin^{-1}(-R_\Lambda/(2B(R_\Lambda))) \quad (1.14)$$

$$B(R_\Lambda) = ((R_\Lambda/2)^2 - 2(\pi l_p)^2)^{1/2} \quad (1.15)$$

$$R_\Lambda = (|\Lambda|/3)^{-1/2} \quad (1.16)$$

$$\Lambda = 3b < 0 \quad (1.17)$$

The value for Hubble's constant at epoch t is given by

$$H(t) = \frac{\dot r(t)}{r(t)} = \frac{cB(R_\Lambda)\ cos\phi(t)}{r^2(t)} \quad (1.18)$$

Let us now eliminate the sine and cosine functions between (1.12) and (1.18) using $\cos^2\phi) + \sin^2(\phi) - 1 = 0$ to obtain the zero valued function FQ(t) of t,

$$FQ(t) = (r^2(t) - R_\Lambda^2/2)^2c^2 + (H(t)r^2(t)R_\Lambda)^2 - (R_\Lambda B(R_\Lambda))^2 \quad (1.19)$$

The numerical value of the radius of the universe at epoch now, $t^\dagger$, identified from the quantum theory for the gravitation constant, $r^*(t^\dagger) = r(t^\dagger)$, and the experimental value for Hubble's constant $H(t^\dagger)$ at time $t^\dagger$ can be used in this equation to give an equation, $FQ(t^\dagger) = 0$ from which the value of R_Λ and hence the value of Λ can be obtained.

$$FQ(t^\dagger) = (r^2(t^\dagger) - R_\Lambda^2/2)^2c^2 + (H(t^\dagger)r^2(t^\dagger)R_\Lambda)^2 - (R_\Lambda B(R_\Lambda))^2 \quad (1.20)$$

$$r(t^\dagger) = 6.535 \times 10^{25}\ m \quad (1.21)$$

$$H(t^\dagger) = 2.1056 \times 10^{-18}\ s^{-1} \quad (1.22)$$

$$R_\Lambda = 7.355566 \times 10^{25}\ m \quad (1.23)$$

$$\Lambda = -5.54484054 \times 10^{-52}\ m^{-2} \quad (1.24)$$

To find the numerical values of the time now, $t^\dagger$; $r^* = r(t^\dagger)$, and the time at maximum radius t_{max} we can use the formulae (1.25) and (1.26),

$$t^\dagger = R_\Lambda(\sin^{-1}(\frac{r^{*2} - R_\Lambda^2/2}{R_\Lambda B(R_\Lambda)}) - \phi_0(R_\Lambda))/(2c) \quad (1.25)$$

$$t_{max} = R_\Lambda(\pi/2 - \phi_0(R_\Lambda))/(2c) \quad (1.26)$$

$$r_{max} = r(t_{max}) = 7.355566 \times 10^{25} \quad (1.27)$$

$$H(t^\dagger) = 2.1056 \times 10^{-18}\ s^{-1} \quad (1.28)$$

$$R_\Lambda = 7.355566 \times 10^{25}\ m \quad (1.29)$$

$$\Lambda = -5.54484054 \times 10^{-52}\ m^{-2} \quad (1.30)$$

Finally here is a list of the main numerical values and ratios arising from this second quantum cosmology model each one proceeded with the corresponding value obtained in the original $\chi_G(N_G)$ model and distinguished by a prime.

$$\Lambda' = -5.5438\cdots \times 10^{-52}\ m^{-2} \quad (1.31)$$
$$\Lambda = -5.5448\cdots \times 10^{-52}\ m^{-2} \quad (1.32)$$
$$R'_\Lambda = 7.356244\cdots \times 10^{25}\ m \quad (1.33)$$
$$R_\Lambda = 7.355566\cdots \times 10^{25}\ m \quad (1.34)$$
$$r'_{max} = R'_\Lambda\ m \quad (1.35)$$
$$r_{max} = R_\Lambda\ m \quad (1.36)$$
$$t'_{max} = 3.85438724\cdots \times 10^{17}\ s \quad (1.37)$$
$$t_{max} = 3.85403596\cdots \times 10^{17}\ s \quad (1.38)$$
$$r'_{max}/r'_\dagger = 1.1255867\ldots \quad (1.39)$$
$$r_{max}/r_\dagger = 1.1255648\ldots \quad (1.40)$$
$$t'_{max}/t'_\dagger = 1.4359549\ldots \quad (1.41)$$
$$t_{max}/t_\dagger = 1.4359054\ldots \quad (1.42)$$
$$T'_U = 4.8888727\cdots \times 10^{10}\ yrs \quad (1.43)$$
$$T_U = 2.4421080\cdots \times 10^{10}\ yrs \quad (1.44)$$
$$t'^\dagger = 2.68419786\cdots \times 10^{17}\ s \quad (1.45)$$
$$t^\dagger = 2.68404281\cdots \times 10^{17}\ s \quad (1.46)$$

2 Conclusions

The formula (1.12) together with the value for the constant maximum radius R_Λ gives a model for a *quantum* cosmology that agrees with the experimental data to high accuracy. R_Λ is calculated from the theory and in turn determines the value of the cosmological constant, Λ. The version for the model developed here *has* taken into account the mostly near zero projection angle, χ_G, in contrast to the original version developed in paper D. This more in depth study has not shown up any dangerous physical or mathematical complications such as *singularities*. Importantly, there is no big bang. The main feature is that the model is smoothly cyclical or simple harmonic with a period of approximately 2.442108×10^{10} years in contrast with the $4.48888727 \times 10^{10}$ years from the first model, apparently twice as long to complete a full cycle. This is in fact not an indication of a great difference between the two models. It partly arises because we are not comparing like with like. The radius in the first model depends on a sine function whereas in the second model the radius squared depend on a similar sine function at twice the frequency. Effectively, this means that the second sine wave appears as the modulus of the first sine wave so that over the period the negative values of the first sine wave are sign inverted. Thus one complete cycle of the first model corresponds to two complete cycles of the second model. One may view the second model as having the physical advantage of only giving positive values for r over its complete cycle unlike the situation for the first model that allows negative r for its second half cycle. This factor 2 is a consequence of different mathematical representations of two similar systems. The physical difference associated with this aspect is more reflected in the fact that this factor is not exactly 2. The observation that the ratio of maximum radius to radius now is $r_{max}/r(t^\dagger) = 1.1255648$ whereas the ratio time at maximum radius to time now is $t_{max}/t^\dagger = 1.4359054$ is much the same as in the first model with the same implication that radius-wise now we are relatively quit near to the turn round point when contraction begins but time-wise that turn round is some way in the future. The rest of the numerical comparisons in list (1.31)→ (1.46) show close *similarity* for the two models. However, one should recognise that the small differences in the decimal parts of the various parameter tend rather to hide the great differences in value that are really involved. For example, the numerical difference between $t'^\dagger$ and $t^\dagger$ is $1.5505 \times 10^{13}\ s \approx 5463$ *life-spans*. From a human perspective this is a long time. Perhaps the most interesting result from these quantum models is that there is no *Big Bang* and conservation of energy still rules.

3 Acknowledgements

I am greatly indebted to Professors Clive Kilmister and Wolfgang Rindler for help, encouragement and inspiration over many years.

References

[1] Sommerfeld, A. 1916 Annalen der Physik **51**, 1

[2] Dirac, P. A. M. 1974 Proc. R. Soc., A333, 439

[3] Narlikar, J. V. 1993 Introduction to Cosmology, CUP

[4] Gilson, J.G. 1996, Calculating the fine structure constant , Physics Essays, **9** , 2 June, 342-353 .

[5] Eddington, A.S. 1946, Fundamental Theory, Cambridge University Press.

[6] Kilmister, C. W. 1994 , Eddington's search for a Fundamental Theory, CUP.

[7] Taylor, B. N. Units and Fundamental Constants in Physics and Chemistry, Subvolume b(Springer Verlag, 1991), Ref. p. 3-131.

[8] W. Rindler, 1961, Am. J. Phys. **29**, 365

[9] Soshichi Uchii at www.bun.kyoto-u.ac.jp/%7Esuchii/mach.pr.html

[10] Gilson, J.G. 1999, The fine structure constant, www.fine-structure-constant.org/

[11] Rindler, W. 2001, Relativity: Special, General and Cosmological, Oxford University Press

[12] Mach, H. 1893, The Science of Mechanics, Chicago, IL: Open Court

[13] Misner, C. W.; Thorne, K. S.; and Wheeler, J. A. 1973, Gravitation. Boston, San Francisco, CA: W. H. Freeman

[14] J. G. Gilson 2004, arxiv.org/PS_cache/physics/pdf /0409/0409010.pdf