A Sketch for a Quantum Theory of Gravity IV
A Quantized Oscillating Friedman Cosmology

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Abstract

This article is essentially a third appendix to the paper *A Sketch for a Quantum Theory of Gravity* containing a demonstration that the quantum theory for gravity obtained in the four earlier articles is consistent with the mathematical structure of the Friedman Cosmological models. A specific quantized oscillating model is obtained using the epoch dependent gravitation function $G(t^* \cos(\chi_G))$ and the mass of the universe, M_U derived from quantum theory in those articles.

1 The Friedman metric

The paper *A Sketch for a Quantum Theory of Gravity* will be referred to as A, the paper *A Sketch for a Quantum Theory of Gravity II* will be referred to as B, and *A Sketch for a Quantum Theory of Gravity III* will be referred to as C in this paper.

The quantum theory for gravity developed in the earlier four papers was formed from quantum theory based on concepts from Bohr's work, from Sommerfeld's work, from a theory for quantum coupling constants and a *time* dependent formula for Newton's gravitation constant G. Thus here the term constant for G is a misnomer. However, the two quantities radius and mass, of the universe come out of that theory together with the conclusion that the graviton has a rest mass negligibly greater than zero. An explanation for the rest mass value for the proton as being due to graviton kinematics also comes out of that theory. However, apart from the input of the formula for $G(t)$, no general relativity theory in the form of field equations or metric is used as input. Thus the main question that arises is how does this all reflect on relativistic cosmology theory. It will be shown here that the quantum theory structure integrates perfectly with standard Friedman cosmology theory.

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The standard cosmological Friedman metrics are of the form

$$\begin{aligned} ds^2 &= (cdt)^2 - R^2(t)\{\frac{dr^2}{1-kr^2} \\ &+ r^2 d\theta^2 + r^2\sin^2(\theta)d\phi^2\} \end{aligned} \tag{1.1}$$

where $R(t)$ is the length dimensioned scale function of time, t and $k = +1, 0, -1$ is the curvature parameter with the three indicated allowable values. The rather simplistic meanings of these three cases are universes that are *closed*, *open and finite*, *open and infinite* respectively, See Rindler[11] page 367 for more detail about the parameter k. The scale factor, R(t), often called the expansion factor is the only function that needs to be found from the general relativity field equations to produce a cosmological theory. However, the form and physical structure of the theory generated depends on input to the field equation. The quantum theory of gravity from papers A, B and C has given the main requirements for input into the field equations. They are the mass of the universe $M_U(N_G)$, at the present epoch as represented by the key quantum number N_G or by t^*, the gravitation constant G as a function $G(r'^*)$ of the radius r'^* of a general proton's *gravitional orbit* and how this orbit with radius, r'^*, depends on the radius of the universe, $r*$. This input information is displayed next for easy reference

$$G(t'^*) = \hbar^2/(m_p^2 m_e t'^* c) \tag{1.2}$$

$$M_U(N_G) = \frac{N_G^2 m_e}{\cos(\chi_G(N_G))} \tag{1.3}$$

The first move is to solve the general relativity field equations to find the two equations that are often called the *Friedman* equation and the *second* equation for $r(t)$ as a function of the time t. These equations have the forms (1.4) and (1.5),

$$8\pi G\rho r^2/3 = \dot{r}^2 + (k - \Lambda r^2/3)c^2 \quad (1.4)$$
$$-8\pi G P' r/c^2 = 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2, \quad (1.5)$$
$$P' = c^2\rho/3 \quad (1.6)$$

where the quantity G on the left hand side of these equations is the usual constant gravitation constant, G. k is the curvature parameter that can take on the values $-1, 0, +1$ and Λ is the cosmological constant. The last equation here (1.6) is the assumption that the density function ρ is capable of exerting the pressure P'. The three equations (1.4), (1.5) and (1.6) form the usual basis for the non-quantal cosmologies that are discussed in great detail, in Rindler's book[11]. Other assumptions can be made at this point which will render these equation more suitable for describing a cosmology based on the quantum gravity theory that has been outlined on the earlier papers of this sequence. This will be dealt with in the next section.

2 Expansion Process

In the quantum system developed earlier, mass is generated over time *inside* the expanding universe. In paper B, I showed how *conservation of energy* for the quantum expansion process can be seen to hold by regarding any increase δM of mass within the outer expanding boundary over some time δt, say, as arising from mass enveloped from outside the expanding boundary over the time δt, $\delta M = c\delta t 4\pi R^2 \rho$ where ρ is the mass density just outside the expanding boundary. It follows that the process occurring in this system is not to be thought of as *continuous creation*. It is necessary for this development that there are two mass densities involved ρ_i and ρ_o, ρ_i inside the moving spherical boundary and ρ_o outside the moving boundary. In principle they can exert positive pressures P_i, P_o away from their regions of location, if they are physically suitably constituted by formulae such as $P_i = c^2\rho_i/3$ and $P_o = c^2\rho_o/3$. Thus the effective pressure exerted outward from the expanding universe on to its expanding boundary will be $P_{o,e} = P_i - P_o$ if the contributions from both densities are taken into account. The mass density distribution, ρ_o outside the expanding universe, I shall call the universe's mass halo. It is not observationally accessible from within the universe. The expanding universe consumes it own halo. On the basis of these remarks about inside and outside mass densities, we can give a clear qualitative explanation of the nature of the expansion process. It can be regarded as a spherically expanding change of state or phase transition in which the material outside the moving spherical boundary in state S_o is continuously consumed to reappear within the boundary in state S_i. The state inside and the state outside having characteristics reflected in the form of equation of state. I shall take it to be the case that the external state is that of a continuous mass density satisfying $P_o = c^2\rho_o/3$ and the internal state is that of a *dust* distribution with a mass density $\rho_i = \rho_o$, the same as the external density at the boundary but unable to exert pressure, $P_i = 0$. The change of state does *not* involve a change of density. Thus the total effective outwards pressure will be the negative pressure $P_{o,e} = -P_o$ so that we need to replace equations ((1.4)), (1.5) and (1.6) with the prime on P' now dropped,

$$8\pi G\rho r^2/3 = \dot{r}^2 + (k - \Lambda r^2/3)c^2 \quad (2.1)$$
$$-8\pi G P r/c^2 = 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2, \quad (2.2)$$
$$P = -c^2\rho/3 \quad (2.3)$$

3 The Quantum Model

I shall confine the discussion in this paper to the cases when the cosmological constant is negative, that is $\Lambda = -|\Lambda|$. Using a radius variable version of the gravitation constant, $G(r') \leftarrow G(r'/c)$, the mass of the universe $M_U = \rho 4\pi r^3/3$ and the quantization projection formula, (3.2) for R, the quantity $8\pi G(R\cos(\chi))\rho R^2/3$ on the left hand side of (1.4) can be expressed in the form,

$$8\pi G(R\cos(\chi))\rho R^2/3 = 2(c\cos^2(\chi))^2. \quad (3.1)$$
$$R\cos(\chi) = N_G l''_p. \quad (3.2)$$

Using (3.1) in the Friedman equation (1.4), that equation becomes

$$2(c\cos^2(\chi))^2 = \dot{r}^2 + (k - \Lambda r^2/3)c^2 \quad (3.3)$$
$$-8\pi G P r/c^2 = 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2, \quad (3.4)$$

or

$$\dot{r}^2 = (a(k, \chi_G(N_G)) + br^2)c^2 \quad (3.5)$$
$$a(k, \chi_G(N_G)) = (2\cos^4(\chi_G(N_G)) - k) \quad (3.6)$$
$$b = \Lambda/3 = -|\Lambda|/3 \quad (3.7)$$

Equation (3.5) can be written in a form suitable for integration as

$$\frac{cdt}{dr} = \pm 1/(a(k,\chi_G(N_G)) + br^2)^{1/2} \quad (3.8)$$

$$\int_{t_0}^{t_R} cdt = \pm \int_{R_0}^{R} \frac{dr}{(a(k,\chi_G(N_G)) + br^2)^{1/2}} \quad (3.9)$$

The function of r, $a(k,\chi_G(N_G))$, in the denominator of equation (3.8) makes for some difficulty in evaluating the integral because it is not a simple function. However, for larger values of r, $a(k,\chi_G(N_G))$ is indistinguishable from the pure constant $a(k,0)$ because the angle $\chi_G(N_G)$ is very small for large r so that the integration is easily performed in that case to give a relation between r and t which is physically very good provided we do not interpret it for the smaller values of r. This is the course that will be taken in this article while the more involved situation for smaller values of r will be dealt with in a later article. Thus under this restriction our integral becomes

$$\int_{t_0}^{t_R} cdt = \pm \int_{R_0}^{R} \frac{dr}{(a(k,0) + br^2)^{1/2}} \quad (3.10)$$

$$a(k,0) = 2 - k \quad (3.11)$$

$$b = -|\Lambda/3| \quad (3.12)$$

$$k = (-1, 0, +1) \quad (3.13)$$

which is a standard form with the possible values from integration,

$$R(t) = a(k,0)^{1/2} R_\Lambda \sin(\pm c(t - t_0)/R_\Lambda + \sin^{-1}(R_0/(a(k,0)^{1/2} R_\Lambda))) \quad (3.14)$$

$$R_\Lambda = |3/\Lambda|^{1/2} \quad (3.15)$$

$$R(t) = R_0, \ when\ t = t_0 \quad (3.16)$$

As this solution is rigorously cyclical, while not forgetting that the true physical case would involve the function $a(k,\chi_G(N_G))$ rather than $a(k,0)$, we might just as well discuss this for the simplest case $R(0) = R_0$, $a(1,0) = 1$ and with only the plus sign in the sine function. Then

$$R(t) = R_\Lambda \sin(ct/R_\Lambda) \quad (3.17)$$

$$\dot{R}(t) = c\cos(ct/R_\Lambda) \quad (3.18)$$

$$R_\Lambda = |3/\Lambda|^{1/2} \quad (3.19)$$

$$R(t) = 0, \ when\ t = t_0 = 0 \quad (3.20)$$

and we see that everything depends on the value assigned to Λ as this will determine the maximum value for $R(t)$ which is given by R_Λ through (3.19) according to (3.17). Thus the cosmological constant assumes prime importance for this quantum cosmology but it still remains a numerical value that does not come out of the theory but remains rather a value that needs to be found from experiment or observation and then input into the theoretical construction. The value of Λ in this construction is directly related to the ultimate radius of the universe R_Λ. A list of values for important parameters can be found in the Wheeler book[13] on page 738, (Box 27.4). I shall use some of this information to determine the viability of this quantum model as representing the physically observed or assumed values for describing the universe after considering the second Friedman equation and some physically measurable characteristics.

If we take the difference of (1.4) and (1.5) and take (2.1) into account we obtain successively,

$$\ddot{R}(t)/R = -4\pi G(\rho/3 + P/c^2) - |\Lambda|c^2/3 \quad (3.21)$$

$$= -|\Lambda|c^2/3 \quad (3.22)$$

$$= -c^2/R_\Lambda^2 \quad (3.23)$$

$$\ddot{R}(t) = -\omega^2 R(t) \quad (3.24)$$

$$\omega = c/R_\Lambda = c|3/\Lambda|^{-1/2}. \quad (3.25)$$

Hence we have a simple harmonic universe with angular frequency parameter ω determined by the cosmological constant Λ through (3.24). Thus the acceleration of this universe is negative for positive $R(t)$ and positive for negative $R(t)$. The values of the three functions of cosmic time H, Ω, q, Hubble's constant, the dimensionless density parameter and the dimensionless deceleration parameter that are used to test theory against observation are,

$$H = \dot{R}/R = c\cot(ct/R_\Lambda)/R_\Lambda \quad (3.26)$$

$$\Omega = 8\pi G\rho/(3H^2) = 2(\cos(\chi_G)R_\Lambda/R)^2 \quad (3.27)$$

$$q = -\ddot{R}/(RH^2) = \tan^2(ct/R_\Lambda). \quad (3.28)$$

The quantum model cosmology could not be more simple than that described by the function $R(t)$ above where the cosmological constant Λ is a free input parameter not determined through the present theory. Thus all that remains to be shown is that it can be assigned a value that gives good agreement for the values of other cosmological quantities that have been measured at the present epoch. The most important of these is the cosmological present day radius of the universe which is derived in the quantum gravitation theory to be given by $ct^* = 2.18c \times 10^{17} m$. However the present day age of the universe can not now be taken to be $t^* = 2.18 \times 10^{17} s$ because the relation between time and radius has now been shown to have the more involved form (3.17). In order to make this distinction between possible representations of ages of the universe let us now denote the cosmological quantum age or epoch by $t^\dagger$ and regard t^* as a *formal age* or just the time a light ray at velocity c would take to transverse the radius if that kept constant at the value, r^*. The most important physical quantities of the present day universe at their present day epoch values are, Hubble's constant $H(t^\dagger)$ and $R(t^\dagger)$ given below with their measured values. We can solve the two equations (3.29) and (3.30) to obtain the values for R_Λ, Λ and $t^\dagger$ given at equations (3.31), (3.32) and (3.33).

$$R(t^\dagger) = r^* = 2.18c \times 10^{17} = 6.535 \times 10^{25}\ m \quad (3.29)$$

$$H(t^\dagger) = \dot{R}/R = c\cot(ct^\dagger/R_\Lambda)/R_\Lambda = 2.1056 \times 10^{-18}\ s^{-1} \quad (3.30)$$

$$t^\dagger = 2.684 \times 10^{17}\ s \quad (3.31)$$

$$R_\Lambda = 7.356 \times 10^{25}\ m \quad (3.32)$$

$$\Lambda = 5.544 \times 10^{-52}\ m^{-2} \quad (3.33)$$

$$t_\Lambda = R_\Lambda\pi/(2c) = 3.864 \times 10^{17}\ s \quad (3.34)$$

$$R_\Lambda/R(t^\dagger) = 1.126 \quad (3.35)$$

$$t_\Lambda/t^\dagger = 1.436 \quad (3.36)$$

$$T_U = 2\pi/\omega = 4.9 \times 10^{10}\ yr \quad (3.37)$$

4 Conclusions

The simple formula (3.17) together with the value for the constant maximum radius R_Λ gives a model for a *quantum* cosmology that agrees with the experimental information available with high accuracy. The numerical value for R_Λ depends only on the value assumed for the cosmological constant Λ. Thus here there is no question of whether the cosmological constant is important or not, its value and existence is fundamental and crucial to this quantum version of cosmology. The version for the model developed here has not taken into account the mostly near zero angle χ_G which will have more importance for small values of epoch. This will be dealt with in a later paper. However, the approximate model that has been developed here does give all the essential feature of a quantum model for larger epoch values. The main feature is that the model is cyclical or simple harmonic with a period of approximately 4.9×10^{10} years. It is interesting that the that the ratio of maximum radius to radius now is $R_\Lambda/R(t^\dagger) = 1.126$ whereas the ratio time at maximum radius to time now is $t_\Lambda/t^\dagger = 1.436$. This means that radius-wise now we are relatively quit near to the turn round point when contraction begins but time-wise that turn round is some way in the future.

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