# A Sketch for a Quantum Theory of Gravity III
# Dirac's Large Number Hypothesis

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## Abstract

This article is essentially a second appendix to the paper *A Sketch for a Quantum Theory of Gravity* containing some further elaboration and applications.


## 1 The Three Large Numbers

The paper *A Sketch for a Quantum Theory of Gravity*, will referred to as $A$ in this paper and the paper *A Sketch for a Quantum Theory of Gravity II* will referred to as $B$ in this paper

The three large numbers that appeared in the works of Dirac[2], Eddington[5] and others[6] have been extensively discussed their meanings, possible relations between them and their significance has been studied in great detail by many workers[13]. Many attempts have been made to explain their physical philosophical position in the natural world. Generally they have been regarded as presenting us with a great mystery that needs to be resolved, if we are ever to understand the relation between the very small world of atoms and the very large cosmological world. The quantum-gravitation theory in the developing series of articles *sketch for a Quantum Theory of Gravity* does explain just how and why they occur and how they are related. The noted three large numbers are the numerical values of the quantities that are often called *the relative strength of the electrical and gravitational forces between an electron and a proton,* $\xi$ ; *the ratio of the length scale associated with the universe,* $c/H$, [3], and *the classical radius of the electron,* $r_e$; and lastly, *N, the number of particles in the universe*. They are listed below in the order just mentioned.

$$\begin{aligned} \xi &= \alpha\hbar c/Gm_pm_e \approx 2.27\times 10^{39} \\ &\approx 10^{40} \text{ !} \qquad (1.1) \\ (c/H)/r_e &\approx 10^{40} \qquad (1.2) \\ N &\approx 10^{80} \qquad (1.3) \end{aligned}$$

where $H$ is Hubble's constant. The first of these $\xi$ can be calculated from the given formula in the first entry to equation (1.1) and on the basis of present day measured values for the constants $\alpha$, $m_p$, $m_e$ and $G$ it assumes a value of order $10^{39}$. The first three of these constants are from quantum measurements and are known very accurately. The fourth $G$ is from astronomical and other macroscopic measurements and its value is not known to anything like the same order of accuracy as the quantum constants. Nevertheless the calculated value of $\xi$ is the value cited on the first line of equation (1.1). I can only assume that the suggested order of magnitude often quoted, $10^{40}$, and displayed in the second line with a ! is wishful thinking in attempting to see a close relation between $\xi$ and $(c/H)/r_e$. There seems little reason to expect these quantities to be equal just because they have the same large order of magnitude. On the same tack, I cannot see that there is any reason to expect that $N$ is the numerical square of $(c/H)/r_e$. However, it will next be shown that the orders of magnitude of these quantities are derivable from the quantum-gravitation theory in $A$ and $B$. Thus demonstrating that their relationships are indeed physical and not accidental as some workers believe. Dirac's large number hypothesis, LNH, is essentially the conclusion that these three large number valued relations between quantum and cosmological quantities are not accidental. They rather represent relations that hold for all time throughout the whole history of the universe. On the main interpretation of (LNH), it is implied that $G$ must vary inversely with epoch. Using the quantity $\xi$, we get the approximate value of $N_G$ from experimental numerical values together with gravitation theory as

$$\begin{aligned} N_G &= \alpha^{-1}\xi\cos(\chi_G(N_G)) \qquad (1.4) \\ &\approx 3.11\times 10^{41}. \qquad (1.5) \\ \xi &= \alpha N_G/\cos(\chi_G(N_G)). \qquad (1.6) \end{aligned}$$

The factor $\cos(\chi_G(N_G))$ on the right hand side of equations (1.4) and (1.6) does not detract from the

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evaluation of $N_G$ because it is so close to unity as to be measurably indistinguishable from it. This is because of the largeness of $N_G$. Equation (1.4) is just the inverse of equation (1.6) under the same understanding about the factor $\cos(\chi_G(N_G))$. It shows how the observed large value order of $\xi$ is determined by the key state integer or eigen-number, $N_G$. Let us now consider the second of the three observed large numbers $(c/H)/r_e$. Within the theory we have, from equations (3.22$A$) and (2.5$A$), its definition is,

$$(c/H^*)/r_e = m_e c^2 t^*/(\alpha\hbar) \quad (1.7)$$
$$= \frac{m_e N_G}{\alpha m_p \cos^3(\chi_G(N_G))}$$
$$= 3.36 \times 10^{40} \quad (1.8)$$
$$\frac{m_e}{\alpha m_p \cos^3(\chi_G(N_G))} \approx 7.46 \times 10^{-2} \approx 10^{-1} \quad (1.9)$$

Thus here again we find that the largeness of $(c/H)/r_e$ is determined by the key quantum integer $N_G$, the order of largeness of $(c/H)/r_e$ is brought down into line with $\xi$ by the relatively small factor in equation (1.9). Let us now consider the last large number $N$, the number of particles in the universe. This is related to the mass of the universe $M_U$ from the theory and is given by equation (3.4$A$),

$$M_U = N_G m_e \alpha_G^{-1} \quad (1.10)$$
$$= \frac{N_G^2 m_e}{\cos(\chi_G(N_G)} \quad (1.11)$$
$$M_U/m_p = \frac{N_G^2 m_e}{m_p \cos(\chi_G(N_G)}$$
$$\approx 5.27 \times 10^{79}$$
$$\approx 10^{80} \quad (1.12)$$
$$\frac{m_e}{m_p \cos(\chi_G(N_G))} \approx 5.4 \times 10^{-4}. \quad (1.13)$$

Collecting the three results together with the best value that can be assessed from theory and experiment for the current value of $N_G$, we have

$$\xi = \frac{\alpha N_G}{\cos(\chi_G(N_G)} \quad (1.14)$$
$$(c/H)/r_e = \frac{m_e N_G}{\alpha m_p \cos^3(\chi_G(N_G))} \quad (1.15)$$
$$M_U/m_p = \frac{m_e N_G^2}{m_p \cos(\chi_G(N_G)} = N_p \quad (1.16)$$
$$N_G \approx 3.11 \times 10^{41}. \quad (1.17)$$

Here also we get the clear representation for the number $N_p$ of *protonic* sized rest masses in the universe as determined by the square of the key state quantum number $N_G$.

All three large numbers can thus be identified as definite functions of the microscopic constants $\alpha$, $m_e$ and $m_p$ together with $c$ and the key quantum cosmological state quantum number $N_G$. This state parameter is the source of the large numbers and the basis of Dirac's large number hypothesis.

The model quantum system developed in *A Sketch for A Quantum Theory of Gravity* conforms exactly to the Dirac large number hypothesis and at the same time fills in the detailed construction for the quantum theory necessary to fulfil that role. All the coefficients of the powers of $N_G$ involved with the large numbers are obtained as explicit functions of microscopic, measurable and recorded[7] experimental numerical values of these physical parameters.

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