A Sketch for a Quantum Theory of Gravity II
Enhancement of the Sketch

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Abstract

This article is essentially an appendix to the paper *A Sketch for a Quantum Theory of Gravity* containing some elaborations and re-identifications of the age and radius of the universe. These changes involve a detailed following up of a change in the definition, $T \to t^*$ of the age of the universe, T as suggested in the paper dated 24th November 2004.

1 Orbital Angular Momentum

In the paper *A Sketch for a Quantum Theory of Gravity*, which I will refer to as A in this paper, a possible nominal age for the universe was identified from a new formula for the gravitational constant as

$$T = \xi \tau_p \tag{1.1}$$

$$\tau_{m_p} = \hbar/(m_p c^2) \tag{1.2}$$

where $\tau_{m_p} = \hbar/(m_p c^2) = l_p/c$ is the Compton time interval for a proton.

There is freedom in making this identification because of some dimensionless multipliers appearing in the formula for G. Expressed in terms of T or R the formula for G is

$$G = \alpha\hbar^2/(T m_p^2 m_e c) \tag{1.3}$$

$$= \alpha\hbar^2/(R m_p^2 m_e). \tag{1.4}$$

It became apparent that a better correspondence with *the age of the universe* values suggested from measurement would occur if the α appearing in the numerator of G were incorporated with T to give a transformed value $T^* = \alpha^{-1}T$. Thus $T^* > T$ and has the same order of magnitude as suggested from the measurement arena. The gravitation constant G in terms of the starred variables becomes

$$G = \hbar^2/(T^* m_p^2 m_e c) \tag{1.5}$$

$$= \hbar^2/(R^* m_p^2 m_e). \tag{1.6}$$

This redefinition was suggested at the end of the paper A and it was indicated that other quantities would need to be changed in order for a consistent theory to incorporate the starred definitions. It turns out that some simple changes have to be made to the quantum orbital theory to achieve this consistency for the starred version. The basic change is to replace the classical radius of the proton, $r_p = \alpha l_p$ in equation ($1.17A$) with its Compton wave length, l_p as here in equations (1.7), (2.3), (2.4) and (2.5). This gives a starred version for, r^*_{G,Z_G} for example, replacing the original r_{G,Z_G} displayed at equation ($2.18A$) in this case. All the cases are listed below. From an inspection of the last equality in equation ($2.18A$), it is then *hindsight* obvious that this is how it should have been from the start! Below is a list of the changes to original variables consequent upon making the change $r_p \to l_p$.

$$r_p \to l_p \tag{1.7}$$

$$T \to T^* = \alpha^{-1}T \tag{1.8}$$

$$R \to R^* = \alpha^{-1}R \tag{1.9}$$

$$H \to H^* = (T^*)^{-1} \tag{1.10}$$

$$M_U \to m^*_U = M_U \tag{1.11}$$

$$\lambdabar_{G,0} \to \lambdabar^*_{G,0} = \alpha^{-1}\lambdabar_{G,0} \tag{1.12}$$

$$r_{G,Z_G} \to r^*_{G,Z_G} = l_p/(Z_G\alpha_G^2) \tag{1.13}$$

I now wish to make an altogether more subtle change in the *interpretational* aspects of this theoretical structure. This will involve some additional notation and changes to some physical variables. Generally the numerical effect of these changes will be negligible and the theory could be left without these additional changes. This would mean that we are taking what might be called a course grained view of the universe and we would not be looking at the detailed structure. However, the enhanced version that emerges is *logically* more satisfactory and the link and similarity between the small scale quantum aspects and the large scale gravitational aspects becomes more clarified. To see why such changes are desirable we can examine the wave capture diagram fig.1

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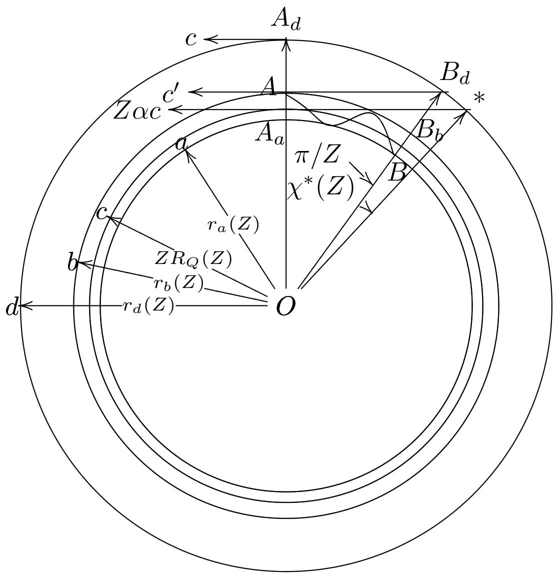
Figure 1
Bohr Orbit Hydrogen-Z Configuration



which originally arose from the electrical-quantum consideration of the orbital structure of H_{137} in relation to the fine structure constant. The most obvious aspect of this diagram are the circular boundaries of radii r_a, r_c, r_b and r_d. There is another possible boundary missing from the diagram of radius smaller than the shown four that we can call $r_i = r_c \cos(\chi^*(137))$ in the case of H_{137}. Thus all the wave structure lies between radii r_i and r_d and the numerical difference between these two radii is $r_d - r_i = r_d(1 - \cos^2(\chi^*(137))) = r_d \sin^2(\chi^*(137)) \approx r_d(1/137)^2 \approx r_d \times 10^{-4}$. The whole of the geometrical structure between these two extremes would vanish in the thickness of the single circular circumference, if the diagram had not been greatly distorted to make it readable. The philosophy of the work in this article is that the gravitation orbits for a proton, almost straight lines for a free proton, follow the same pattern as in the wave capture diagram with the additional condition that the outer and inner radii for the gravitation orbits differ by vastly smaller amount than the electromagnetic case. This is why I suggest that the new set of changes that are to be implemented are logically and conceptually important but numerically negligible. However, the thickness of a very thin line on a small disc representing the whole universe could represent 10^8 human life spans. The two inner radii r_d and r_b are important for QED but need not concern us in the gravitation regime for reasons that will become clear.

2 Enhancement Variables

The first step in the enhancement is to consider the formulae $(2.6A)$ and $(2.7A)$ for the gravitation constant G in terms of the time like parameter T^* and abandon the *interpretation* of cT^* as the radius of the universe. Then reinterpret $R^* = cT^*$ as the nearby quantity the gravitational equivalent of the QED radius of $r_{B,137}$, of first Bohr orbit of H_{137}. This equivalent is a quantity, $r^*_{G,N_G} = ct^*_{G,N_G}$, a starred version of what was r_{G,N_G} in A originally. The result of this change is a *functional* form for $G(r'^*)$, the gravitational constant in terms of r'^*.

$$G(r^*_{G,N_G}) = \hbar^2/(t^*_{G,N_G} m_p^2 m_e c) \quad (2.1)$$
$$= \hbar^2/(r^*_{G,N_G} m_p^2 m_e). \quad (2.2)$$

Equation (2.2) is to be our fundamental equation connecting the quantum theory for gravitational orbits to the value of the relativistic cosmological quantity G. The numerical value of G will now be regarded as *defined* through equation (2.2). To show how the radius or the age of the of the universe now comes into the structure we have to examine the other important circular quantum orbits within which the circle of radius r^*_{G,N_G} is to be found, the analogues of r_i and r_d from H_{137} theory. They are defined in ascending size order with r^*_{G,N_G} central in magnitude as

$$r''^* = r^*_{G,N_G} \cos(\chi_G) = \frac{N_G l_p}{\cos(\chi_G)} \quad (2.3)$$
$$r'^* = r^*_{G,N_G} = \frac{N_G l_p}{\cos^2(\chi_G)} \quad (2.4)$$
$$r^* = \frac{r^*_{G,N_G}}{\cos(\chi_G)} = \frac{N_G l_p}{\cos^3(\chi_G)}. \quad (2.5)$$

The notation,

$$l_p = \hbar/(m_p c) \quad (2.6)$$
$$l'_p = \hbar/(m_p c \cos(\chi_G)) \quad (2.7)$$
$$l''_p = \hbar/(m_p c \cos^2(\chi_G))), \quad (2.8)$$

will be used. The speed on the r'^* is given by $r'^*\omega = N_G \alpha_G c$ so that the speed v^* on the r^* orbit will be given by

$$v''^* = r''^*\omega = r''^* N_G c \alpha_G / r'^* = c'' \quad (2.9)$$
$$v'^* = r'^*\omega = r'^* N_G c \alpha_G / r'^* = c' \quad (2.10)$$
$$v^* = r^*\omega = r^* N_G c \alpha_G / r'^* = c \quad (2.11)$$
$$c' = c \cos(\chi_G) \quad (2.12)$$
$$c'' = c \cos^2(\chi_G). \quad (2.13)$$

Thus we can take these inner and outer boundaries r''^* and r^* within which the main orbit of radius r'^* lies to represent an annulus fixed to and rotating with the geometrically extended object. The object has a mean position on the radius r'^*. The outer boundary of this annular platform has a transverse velocity of the speed of light and so is naturally to be regarded as the maximum outer boundary for the whole rotating system. This then is good reason to identify r^* as the radius of the universe. This does not mean that the universe is rotating. It rather means that the maximum separation for gravitationally coupled systems coincides numerically with the radius of the universe, r^*.

Let us first calculate the orbital angular momentum for an electron in the first Bohr orbit of H_{137}. From equations (1.17A) and (1.18A) this is

$$r_{B,Z}v_{B,Z}m_0 = Z\alpha c m_0 l_c/(Z\alpha^2) \quad (2.14)$$
$$= m_0 c l_c/\alpha = m_0 c l_{m_0} \quad (2.15)$$
$$= \hbar \quad (2.16)$$

The orbital angular momentum for the corresponding gravitation orbit of a proton is from equations (2.4) and (2.10) and using the starred system

$$r^*_{G,Z_G}v_{G,Z_G}m_p = \frac{Z_G\alpha_G c m_p l_p}{(Z_G\alpha_G^2)} \quad (2.17)$$
$$= \frac{m_p c l_p}{\alpha_G} \quad (2.18)$$
$$= \frac{\hbar}{\alpha_G} = \frac{N_G\hbar}{\cos(\chi_G(Z_G))} \quad (2.19)$$

The starred version of the gravitational potential in which a proton moves has the form on the mean orbit and on the outer boundary as given by

$$V_G^*(r_G'^*) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r_G'^*} \quad (2.20)$$
$$= m_p c'^2 \quad (2.21)$$
$$V_G^*(r_G^*) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r_G^*} \quad (2.22)$$
$$= m_p c'^2\cos(\alpha_G) \quad (2.23)$$
$$r_G'^* < r_G^* \quad (2.24)$$
$$V_G^*(r_G'^*) > V_G^*(r_G^*) \quad (2.25)$$

The *gravitation orbit* of a proton here means a free proton at rest or in motion under gravitational influence alone. The numerical value of the orbital velocity in the orbit given by the quantum number $Z_G = N_G$ for a given $\alpha_G(N_G, N_G')$ is the maximum quantum number and so it gives the highest speed in orbit, $v_{G,N_G} = \cos(\chi_G^*(N_G))c < c$. In general this speed will be very close to but definitely *less* than c. Thus when a proton is encountered experimentally apparently not moving at high speed relative to the laboratory it does not mean that such a proton has not got the high velocity in its gravitational orbit being studied here. This is because the observer's laboratory rest frame can in general have a small velocity relative to the proton. In other words, any proton can have a high gravitational velocity relative to its distant rest mass generating graviton's rest frame.

3 Numerical Values for Radius, Age and Mass r_G^*, t_G^* *and* M_U of Universe

The nominal time for the existence of the universe, its age, will from now on be denoted by t^* and defined by $t^* = r_G^*/c$ where r_G^* is taken to be the current radius of the universe. It is evident that these theoretically obtained values give a very good agreement with the experimentally assessed values and also have the theoretical advantage of greatly clarifying detailed aspects of the structure. The mass of the universe can be obtained by considering the magnitude of the mass that induces the gravitational potential in which the proton moves, or more appropriately one might say, *exists*. From equation (2.20) this potential is

$$V_G^*(r_G'^*) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r_G'^*} \quad (3.1)$$
$$= m_p c'^2 \quad (3.2)$$
$$= \frac{G M_u m_p}{r_G'^*} \quad (3.3)$$
$$M_U = N_G m_e \alpha_G^{-1}$$
$$= N_G^2 m_e/\cos(\chi_G(N_G) \quad (3.4)$$
$$N_G \approx 3.11\times 10^{41} \quad (3.5)$$
$$\alpha_G^{-1} \approx 3.11\times 10^{41} \quad (3.6)$$
$$M_U \approx 8.81\times 10^{52} kg \quad (3.7)$$
$$M_{U,m} \approx 5.68\times 10^{53} kg \quad (3.8)$$
$$t^* \approx 2.18\times 10^{17} s \quad (3.9)$$
$$t_m^* \approx 3.1536\times 10^{17} s \quad (3.10)$$

$M_{U,m}$ is the value for the mass and t_m^* is the age of the universe from observation and measurements as suggested in reference [11] and also admitted as to not being *canonical*.

The three quantities r_G^*, t_G^* *and* M_U together with the rest mass of the graviton, m_G, all depend on the value of the integer parameter N_G and when I write of *theoretical* values, I imply that the actual numerical values can only be found if the value of the integer N_G can be input into the calculation. There is no theoretical way to evaluate N_G any more than there is to evaluate 137 at this time in the physics science story. There is a second parameter N'_G involved but again, unlike in the QED case where it is important, in gravitation theory it does not appear to play a significant part. It can be shown why this is the case as follows.

The QED difference between α^{-1} and 137 and the gravitational equivalents have the approximate values

$$\alpha^{-1}(137, 29) - 137 \approx 0.036 \tag{3.11}$$

$$\alpha_G^{-1}(N_G, N'_G) - N_G \approx \frac{\pi^3}{N_G^2}\left(\frac{1}{2} - \frac{1}{3{N'_G}^2}\right) \tag{3.12}$$

$$\rightarrow \frac{\pi^3}{2N_G^2} \quad as \quad N'_G \rightarrow \infty \tag{3.13}$$

$$\alpha_G^{-1}(10^{42}, \infty) - 10^{42} \approx 10^{-84} \tag{3.14}$$

$$\cos(\chi_G(N_G)) \rightarrow 1 \; as \; N_G \rightarrow \infty \tag{3.15}$$

$$\chi_G(N_G) \rightarrow 0 \; as \; N_G \rightarrow \infty \tag{3.16}$$

From equations $(2.22A)$ and $(2.23A)$ we get the relation for the graviton rest mass,

$$m_G = m_p(1 - (c'/c)^2)^{1/2} \tag{3.17}$$

$$= m_p(1 - \cos^2((\chi_G(N_G)))^{1/2} \tag{3.18}$$

$$= m_p \sin(\chi_G(N_G)) \tag{3.19}$$

$$\rightarrow 0 \; as \; N_G \rightarrow \infty \tag{3.20}$$

$$r^* = N_G l_p / \cos^3(\chi_G) \tag{3.21}$$

$$t^* = N_G \tau_p / \cos^3(\chi_G) \tag{3.22}$$

$$\tau_p = l_p / c \tag{3.23}$$

$$\cos(\chi_G(N_G)) r^* = N_G l''_p. \tag{3.24}$$

From equation (3.11), it can be seen that the inverse fine structure constant differs from the integer 137 at the second decimal place whereas from equation (3.14), its gravitational equivalent α_G^{-1} differs from the integer $N_G \approx 10^{42}$ at the $84th$ decimal place, they are essentially equal and from equation (3.22) it can be seen that t^* is essentially equal to $N_G \tau_p$ where τ_p is a constant eigenfunction *time* increment associated with the eigenvalue number N_G. Thus the digitally changing quantity N_G can be regarded as a digital parameter determining the age of the universe or we can write $N_G(t^*)$ implying the converse. It makes little difference if you care to regard time as the primary mover for change or regard N_G as determining the time t^* or the age of the universe. Thus in this gravitational theory the quantum integer eigenvalue N_G is of primary importance. The whole story is contained in the concept of *projection quantization*. This concept is the bridge between quantum theory quantization and relativity theory length contraction. One example is displayed at equation (3.24) for the case of the radius of the universe expressed in terms of the eigen-length l''_p. This is just one case among the five others for the quantum orbits in terms of their own l_p versions and the key state integer N_G. From equations (3.4) and 3.22, it is clear that the amount of mass within this universe is increasing proportionally to the square of its age. The question then arises as to where this mass is coming from and is conservation of energy being violated in this model? There is a simple answer to this question though it may not be everyone's satisfaction. If we are to talk about expansion at all we do need to have some idea about what the expansion is taking place into. Let us consider the most obvious and simple situation and suppose that the expansion is taking place into a larger containing space and for simplicity assume it is euclidean, E_{con}, say. In fact, we have no idea of the actual geometry that might be involved so that this assumption will have to do here. Suppose that our expanding universe is centred at the origin of E_{con}. I see no reason why this containing space should be completely empty. It contains an expanding universe but suppose further the centre of that expanding universe is surrounded with a mass distribution that extend in all direction just as a consequence of its containing that universe. At small values for the age of the universe t^*, its radius r^* or expanding boundary is within the mass distribution which could itself extend to infinity in E_{con}. Let the density of this *halo* of electronic sized masses be given by, the inverse linear, in R function, $\sigma_G(R) = 3m_e/(\cos(\chi_G(N_G(R/c))4\pi {l''_p}^2 R)$, where R is the distance from the centre of coordinates in E_{con}. At time t^* the universe will have expanded to the volume size $V(r^*) = 4\pi(r^*)^3/3$ so that it will then contain mass of amount $\sigma_G(r^*)V(r^*) = M_U(t^*)$ as given by equation (3.4). A similar argument could be adduced for more geometrically exotic container spaces. Thus conservation of energy is not threatened.

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