A Sketch for a Quantum Theory of Gravity
Rest Mass Induced by Graviton Motion

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Abstract

The numerical quantum electronic structure for the energies of the states of the hydrogen like atoms as given by Sommerfeld in 1915-16 is studied and is shown to present a scheme that is able to express a unique *observer* point of view. The perspective of this observer is essentially how he, if *fixed* on a trapped electron, would see his and his electron's state of containment within the full atomic quantum state. This particular internal view of a quantum state is then shown to have strong analytic powers in the extremely different numerical scale of gravitation theory. This unexpected analogy becomes possible when it is recognised that a basic part of gravitation theory can be expressed in terms completely analogous to the quantum relativistic electromagnetic structure involved in Sommerfeld's formula for quantum state energy. The introduction section brings together the essentials for the understanding of the electromagnetic side of this analogy and contains a condensation of some earlier work. The second section *Gravitation Analogy* develops a theory for gravitation which closely follows the quantum form to give an exact analogy to the Sommerfeld quantum theory of hydrogen atomic states.

1 Introduction

One of the great discoveries of the 20th Century was Sommerfeld's formula for the energy values of the quantum states of the hydrogen like atoms. The energy values involved were those assumed by a negatively charged electron orbiting a nucleus composed of Z positively charged protons, the charged nucleus being the source of a classical Coulomb electric potential. In the context of Schrödinger wave mechanics which is usually regarded as non-relativistic [35], it is surprising that this *relativistic* formula should appear at all and that its predictions are so accurate is very remarkable. Sommerfeld's [1] formula for the energy of hydrogen like atoms with a Z-valued positive charge Coulomb central field is,

$$E_{n,j,Z} = m_0c^2 \left(1+\gamma^2\right)^{-1/2} \tag{1.1}$$

where

$$\gamma = \frac{Z\alpha}{n-j-1/2+((j+1/2)^2-(Z\alpha)^2)^{1/2}} \tag{1.2}$$

with n what is usually called the principal quantum number and j with its possible half integral values is the quantum number for total angular momentum.

The expression (1.1) is not the last word in energy specification for hydrogen like atoms because it does not include the Lamb shift complications. However, it is extraordinarily accurate and shows how the energies depend on the charge number Z. The energies of the first Bohr orbits for various values of Z are given by $n=1, j=1/2$ and $1 \le Z \le 137$ and are

$$E_{1,1/2,Z} = m_0c^2(1-(Z\alpha)^2)^{1/2}. \tag{1.3}$$

We are particularly interested in the speeds $v_{B,Z}$ with which the trapped electron moves in these orbits for the various values of Z. The centrifugal equilibrium equation expresses a relation between the Bohr orbit radius $r_{B,Z}$ and the Bohr orbit velocity $v_{B,Z}$ and is

$$Ze^2/4\pi\epsilon_0 r_{B,Z} = m_0v_{B,Z}^2. \tag{1.4}$$

Thus (1.4) supplies a definition for the velocities in the first Bohr orbits in terms of the first Bohr radii in the form

$$\begin{aligned} v_{B,Z} &= (Ze^2/4\pi\epsilon_0 r_{B,Z}m_0)^{1/2} && (1.5)\\ &= (Zl_cc^2/r_{B,Z})^{1/2} = Z\alpha c, && (1.6)\end{aligned}$$

the last equality following from the well known result for the circular orbits $r_{B,Z} = r_{B,1}/Z$. Thus in particular,

$$v_{B,1} = \alpha c \tag{1.7}$$

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and

$$v_{B,137} = 137\alpha c. \tag{1.8}$$

Sommerfeld's formula (1.1) for the special case of the first Bohr orbits can easily be derived ([1], [47], [18]) as will be shown in the next paragraph.

Suppose that a hydrogen-Z atom with its single circling electron is at rest in some frame of reference S_0, the electron being in the first Bohr orbit. In this frame of reference, the energy $E(Z)$ of the electron will be given by Sommerfeld's formula for a first Bohr orbit. However, for the moment let us simply call the energy $E(Z)$ and assume we do not know formula (1.1). Consider the view of the situation that an observer in frame of reference S moving with velocity $v_{B,Z}$ relative to the first frame such that this velocity is also in the same direction instantaneously as that of the direction of circling electron. At the instant of coincidence of these velocities in magnitude and direction the observer in S will have the electron at rest in his frame so he will assesses its total energy as being its rest energy plus its potential energy in the field of the passing centre of force on the nucleus of the hydrogen-Z atom. This potential energy is that of one negatively charged electron in the Coulomb field of the Z positively charged nucleus. This potential energy is given by $V = -(Ze)e/(4\pi\epsilon_0 r_{B,Z})$. Thus this observer assesses the total energy E_T of the electron to be $E_T = m_0c^2 - (Ze)e/(4\pi\epsilon_0 r_{B,Z})$. This might be called the electron orientated point of view. Now consider what might be called the hydrogen first Bohr state point of view. We are assuming that the energy of the electron in this state is given by $E(Z) = M(Z)c^2$, $M(Z)$ being the equivalent mass of this state. $M(Z)$ is *rest mass* for this state in the frame S_0 in which the *state* is at rest. The observer in S will see this mass as a rest mass $M(Z)$ with a charge Ze moving with velocity $-v_{B,Z}$. Thus from this point of view he will assess the energy in motion as $M(Z)c^2/(1-(v_{B,Z}/c)^2)^{1/2}$ and see it as the nuclear charge Ze moving in the Coulomb potential $V' = -e(Ze)/(4\pi\epsilon_0 r_{B,Z})$ due to his local captured electron. Thus his assessment of the total energy from the nuclear state in motion point of view is $E'_T = M(Z)c^2/(1-(v_{B,Z}/c)^2)^{1/2} - e(Ze)/(4\pi\epsilon_0 r_{B,Z})$. The two points of view are two different ways that the observer in S can perceive the overall situation. Both of the energies E_T and E'_T are the energies of the trapped electron in hydrogen-Z as seen in the frame of reference S albeit from the two points of view. Thus they should be numerically equal. The two potential energies from the two points of view are also equal. Thus we have the result

$$\begin{aligned} m_0c^2 &= M(Z)c^2/(1-(v_{B,Z}/c)^2)^{1/2} \\ &= M(Z)c^2/\sin(\chi^*(Z)). \end{aligned} \tag{1.9}$$

α can be given the form

$$\alpha \quad = \cos(\chi^*(Z))/Z, \tag{1.10}$$

so that

$$\begin{aligned} v_{B,Z} &= Z\alpha c \\ &= c\cos(\chi^*(Z)). \end{aligned} \tag{1.11}$$

Hence (1.9) becomes

$$E(Z) = M(Z)c^2 = m_0c^2\sin(\chi^*(Z)) \tag{1.12}$$

which agrees with (1.3). If the Compton wave length l_m of a rest mass m is defined generally as $\hbar/(mc)$, then (1.12) gives

$$l_{m_0} = l_{M(Z)}\sin(\chi^*(Z)). \tag{1.13}$$

The maximum value for Z, $Z = 137$, represents choosing the last member of the hydrogen like set of atoms for analysis, H_{137}. This particular atom is a long way outside present day condition possibilities. It is a theoretical structure but it has led to important theoretical discoveries, notably the derivation of a theoretical formula for α ([33],[2],[3],[4],[16],[23], [24],[25],[28],[21],[22],[20]). Here I shall show that it also has potential for steering us towards an understanding of *rest mass* in terms of gravitation. This aspect will be pursued in the next section. However before that, I shall here pull together some earlier results that will be required in that study. We shall need definitions for some standard Schrödinger quantum theory quantities. The basic ones are, the classical electron radius which here will be denoted by r_e but which was denoted in earlier work by l_c, the Compton wavelength of the electron, λ_C and its divided by 2π varient, $\lambdabar_C$ here denoted by $2l_0$. Less basic are the radii of the first Bohr orbits $r_{B,Z}$ and the velocities in those orbits $v_{B,Z}$. These definitions are now set out in the order introduced above together with representations for α and the general formula for dimensionless coupling constants.

$$r_e = e^2/(4\pi\epsilon_0 m_0c^2) = l_c \tag{1.14}$$

$$\lambda_C = h/(m_0c) \tag{1.15}$$

$$\lambdabar_C = \hbar/(m_0c) = 2l_0 = l_{m_e} \tag{1.16}$$

$$r_{B,Z} = l_c/(Z\alpha^2) \tag{1.17}$$

$$v_{B,Z} = Z\alpha c \tag{1.18}$$

$$1 \le Z \le 137 \tag{1.19}$$

$$\alpha = r_e/\lambdabar_C = l_c/2l_0 = l_{m_e}/r_{B,1} \tag{1.20}$$

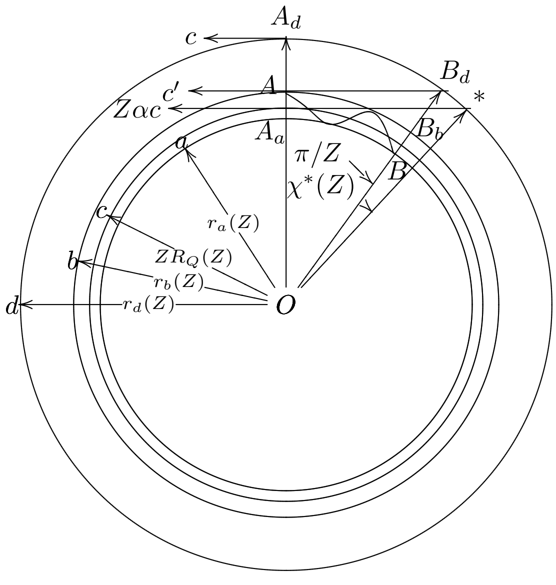
$$\alpha = \cos(\chi^*(Z))/Z = a\ const. \tag{1.21}$$

$$\alpha(n_1,n_2) = \frac{n_2\cos(\pi/n_1)\tan(\pi/(n_1n_2))}{\pi} \tag{1.22}$$

$$\alpha = \frac{29\cos(\pi/137)\tan(\pi/(29\times 137))}{\pi} \tag{1.23}$$

In relation to the table above, a remark about the relation between $\alpha(n_1, n_2)$ and the integer parameter n_1 will be seen to have important significance in the work to follow. The representation of the fine structure *constant* α with emphasis on the constant characteristic given by (1.20) depends on the many valued parameter Z. This representation is very useful in bringing together the physical description for the 137 members of the hydrogen like family of atoms. As we have seen above it is the member of that set H_{137} with the first Bohr orbit described by the one orbiting electron at the maximum speed $v_{B,137} = 137\alpha(137, 29)c$ that can be used to explain important fundamental physics issues. This speed $v_{B,137}$ is the speed that gives the lowest rest mass $M(137)$ for the rest mass generation process (1.12) for the electron rest mass m_0. Thus in the quantum electromagnetic context $M(137)$ can be taken to be the most *fundamental* particle-like mass or smallest mass unit and the smallness of its value depends on the closeness of the light speed multiplier quantity $137\alpha(137, 29)$ to unity. In the gravitation analogy context to follow this mass value $M(137)$ will be shown to have an analogy m_G greatly smaller numerically and so much as to be *measurably* indistinguishable from a photon in a limiting situation.

Figure 1
Bohr Orbit Hydrogen-Z Configuration



Before discussing what this all has to do with gravity let us consider the physical implication of equations (1.9) and (1.12). Both of these equations represent a simple relation between the rest mass $M(Z)$ of an atomic state and the rest mass m_0 of an electron and of course there are many such relations according to the Z value, that is to say, the atomic system chosen. Equation(1.9) clearly means that the electron's rest mass can be fully expressed as the relativistic energy of a *particle* of rest mass $M(Z)$ in motion with the velocity $v_{B,Z}$. From the (1.12) equation, the ratio of the mass $M(Z)$ to the electron's rest mass is in general a relatively small quantity because the orbital velocity $v_{B,Z}$ can have values very close to the velocity of light. The largest value that the orbital velocity $v_{B,Z}$ can have is that which occurs in the case of Hydrogen-137 when $Z = 137$ and in this case (1.12) gives the ratio of the mass $M(137)$ to the mass m_0 of the electron to be the last displayed result below.

$$v_{B,137}/c = 137\alpha, \quad (1.24)$$
$$= \cos(\chi^*(137)), \quad (1.25)$$
$$M(137)/m_0 = 0.02292\ldots\ldots \quad (1.26)$$

Hence equation (1.9) essentially expresses the rest mass m_0 of an electron in terms of the sub-mass $M(137)$, about a 200th part of the electron's mass, in motion. Motion here is generating rest mass by the standard relativistic kinematics-dynamic process. Thus this analysis gives a model for the internal dynamics of the electron in terms of sub-particle in motion structure. This, of course, is the trend of theoretical physics matter analysis that has occupied most of the last century, the decomposition of a system into successively smaller mass sub-systems. The puzzling question is, does this descending proliferation go on without limit or are there truly fundamental masses that will ultimately be encountered? However, though that question cannot now be answered every step downwards is progress but within the context of Sommerfeld quantum theory we are here at a base level because $M(137)$ is the smallest valued mass state to occur. It will be shown that the downward structure picture can be taken very much further when we examine the gravitation analogy. There is the well known but curious property of mass in quantum systems that very large masses are often very localized and very small masses can be distributed over large regions. This aspect is reflected in the way mass appears in the Compton wave lengths such as $\lambda\!\!\!^{-}$ for example in equation (1.15) where a small mass in the denominator gives a large wave length and vice versa. This aspect is related to quantum uncertainty and we shall see that it has important consequences in the analogy to be developed in the next section. There is another feature of the rest mass generation formula (1.9) that will be seen to play an important part in the gravitation story to come. Rest mass is the most important energy idea that emerged from relativity theory. Here we can see it has the special role of supplying a *theoretical* observer reference frame from which a particular energy aspect of the electromagnetic system takes on a very simple form. Of course, an observer cannot sit on an electron but it seems that the electron *feels* its quantum electronic states relative to its own rest mass platform as though it is disconnected from such issues as total system rest mass weighting to the actual space background or to the wider system in which its states may be imbedded. The actual mass structure or the centre of gravity of the massive nucleus of an atom such as that of H_{137} does not come into the derivation of the relation between state rest mass $M(Z)$ and kinetically generated rest mass m_0 of the orbiting electron. If H_{137} were found to exist physically, besides the 137 protons in its nucleus, one might expect it to further contain a large number of neutrons so that altogether it would be very massive. This all implies the possible generality of a theoretical *unique observer fixed on the particle* viewpoint which is blind to system embedding. The valuable consequence that follows from this is that we can study the electron along with its *internal* states as though it is in a stand alone condition and just in a low velocity unbound state. Perhaps this is merely the recognition of a principle that *the internal states of a particle are relative to the particle's rest frame*! I shall refer to this as *the internal relativity principle for a particle*. Essentially, this principle allows role reversal for nucleus and bound rotating particle. This aspect will also be exploited in the gravitation context. One may well ask where within the H_{137} atom is the generating state mass $M(137)$ to be found. The answer to this is within a transverse speed of light horizon determined by a rotating radius vector $r_d = c/\omega$ rotating with the orbiting electron such that $r_{B,137}\omega = v_{B,137}$. The values of r_d and $r_{B,137}$ are very close in value, $r_{B,137}/r_d \approx \cos(\pi/137) \approx 1$. See figure 1, where the orbiting electron is represented as an extended wave confined in the angular segment $\pi/137$ between the radii $r_a(137)$ and $r_b(137)$ its mean position being at radius, $r_c = r_{B,137}$. The state rest mass $M(137)$ shares the same rest frame in which the centre of force is at rest.

2 Gravitation Analogue

The clue that suggests how to quantize gravity comes from one of the three very large pure numbers that have been known about for many years and were extensively studied and used by (Eddington, Dirac, Jordon, Dick, Hayakawa, Carter [38]) and others ([5],[6],[1],[7],[8],[9]) in their theoretical work. Previously, I have denoted this number by R_{PE} indicating that it is the ratio of the electromagnet potential energy of a proton electron pair to the gravitational potential energy of an electron proton pair. This ratio clearly does not depend on the separation of the two particle as it would appear in the numerator and denominator of this ratio and so would cancel. Here I shall denote it by $\xi = R_{PE}$. Its definition is

$$\xi = \alpha\hbar c/(Gm_p m_e) = R_{PE}, \tag{2.1}$$

where G is the constant of gravitation m_p is the rest mass of a proton and $m_e = m_0$ is the rest mass of an electron. α is the dimensionless coupling constant for the electromagnetic field. Thus we can identify the *dimensionless electromagnetic quantum* value for gravitational coupling as α_G, say, so that

$$\alpha_G = Gm_p m_e/(\hbar c) \tag{2.2}$$

and then

$$\xi = \alpha/\alpha_G. \tag{2.3}$$

For the purpose of developing this model for a quantum theory of gravity, I shall make the working assumption that all quantum related coupling constants are related to or are members of the set $Q_C = \{\alpha(n_1, n_2)\}$ defined by equation (1.22). As all members of this set are dimensionless, we can then expect to find α_G within this set. Thus I shall represent α_G as

$$\alpha_G = \alpha(N_G, N'_G), \tag{2.4}$$

where the pair of integers N_G, N'_G are to be determined. If we compare this equation with equation (1.12) we see that the electromagnetic analogue for the maximum value of N_G is 137 and for N'_G it is 29. Using equation (1.22), the ξ ratio (2.2) can be given the more detailed form,

$$\xi = \alpha(137, 29)/\alpha(N_G, N'_G). \tag{2.5}$$

We can now usefully use a *generalization* of a formula for the gravitation constant G. The original was suggested by Ross McPherson [34] based on his observation of a near numerical coincidence between ξ and the quantum frequency of the proton's rest mass multiplied by 10^{16}. The generalized form to be used here will be expressed in terms of different physical constants and will have the usual dimensional form $k_g^{-1} m^3 s^{-2}$ but will be expressed as a function of a time t_n. It has the definition,

$$G(t_n) = \alpha\hbar^2 H(t_n)/(m_p^2 m_e c), \tag{2.6}$$

$$H(t_n) = 1/(t_n + t_0), \tag{2.7}$$

$$t_n + t_0 = T = 1.59 \times 10^{15}\ s. \tag{2.8}$$

$H(t_n)$ is Hubble's constant expressed as a function of a time, t_n measured in seconds from a time $t_0 > 0$ near the beginning of the universe. T is a nominal time for which the universe is assumed to have existed. m_p is the rest mass of a proton and m_e is the rest mass of an electron. From (2.2), (2.6) and (2.8) we can now derive

$$\alpha_G(t_n) = \alpha\hbar/(R(t_n)m_p c) \quad (2.9)$$
$$= r_p/R(t_n). \quad (2.10)$$

$R(t_n) = cT(t_n)$ is a nominal radius for the universe now and r_p is the classical radius of the proton. We note that the dimensionless gravitational coupling constant $\alpha_G(t_n)$ depends on the time t_n through the dependence of the radius of the universe through the time t_n. Equation (2.9) enables us to complete the analogy between quantum theory and gravitation theory. If we compare equation (2.10) with equation (1.20) while assuming that α_G is the gravitational analogue α, it follows that r_p is the gravitational analogue of $r_e = l_c$ and a radial length R_0 related to $R(t_n)$ is the gravitational analogue of $\lambda\!\!\!^{-}_C = 2l_0$. The rest mass of a graviton m_G, say, is naturally defined as the rest mass which has a Compton wave length $R_0 = \hbar_G/(m_G c)$ so that we can write down the gravitational analogue for equation (1.12) as

$$m_G(Z_G) = m_p \sin(\chi^*_G(Z_G)). \quad (2.11)$$

Below is set out the list for gravitational quantities analogous to the quantum quantities defined in the list (1.13)-(1.22)

$$r_p = e^2/(4\pi\epsilon_0 m_p c^2) = \alpha l_{m_p} \quad (2.12)$$
$$\lambda\!\!\!^{-}_{G,0} = \hbar_G/(m_G c) \quad (2.13)$$
$$\hbar_G = \xi\hbar \quad (2.14)$$
$$r_{G,Z_G} = r_p/(Z_G\alpha_G^2) \quad (2.15)$$
$$v_{G,Z_G} = Z_G\alpha_G c \quad (2.16)$$
$$1 \le Z_G \le N_G \quad (2.17)$$
$$\alpha_G(t_n) = \frac{r_p}{\lambda\!\!\!^{-}_G} = \frac{r_p}{R(t_n)} = \frac{l_{m_p}}{R(t_n)\alpha^{-1}} \quad (2.18)$$
$$\alpha_G(t_n) = \cos(\chi^*_G(Z_G))/Z_G \quad (2.19)$$
$$\alpha_G(n_1,n_2) = \frac{n_2\cos(\pi/n_1)\tan(\pi/(n_1 n_2))}{\pi} \quad (2.20)$$
$$\alpha_G = \frac{N'_G\cos(\pi/N_G)\tan(\pi/(N'_G N_G))}{\pi} \quad (2.21)$$

The analogue of the electron in motion on a first Bohr orbit of H_{137} is, what I have indicated above by G, a proton in motion on an orbit of radius r_{G,Z_G} with a velocity of v_{G,Z_G} and coupled to a central force via the dimensionless coupling constant α_G. Thus the analogue of the relativistic rest mass generation equation (1.12) for the proton's rest mass is

$$m_p c^2 = m_G(Z_G)c^2/(1-(v_{G,Z_G}/c)^2)^{1/2}$$
$$= m_G(Z_G)c^2/\sin(\chi^*_G(Z_G)). \quad (2.22)$$

α_G can be given the form

$$\alpha_G = \cos(\chi^*_G(Z_G))/Z_G. \quad (2.23)$$

The analogy between the structure of the first Bohr orbit of an electron in H_{137} coupled by α to an electromagnetic force centre and the gravitational orbit of a proton coupled by α_G to a gravitational force *centre* is exact. However, the scales of the analogous systems are greatly different. Below is a list of the approximate numerical values and scale relations for key parameters.

$$\alpha/\alpha_G \approx 2.269 \times 10^{39} \quad (2.24)$$
$$N_G/137 \approx 2.2697 \times 10^{39} \quad (2.25)$$
$$M(137)/m_e \approx 0.02292 \quad (2.26)$$
$$m_G/m_e \approx 0.00137568 \quad (2.27)$$
$$M(137) \approx 2.0878919 \times 10^{-32} \quad (2.28)$$
$$m_G \approx 1.2531594783 \times 10^{-33} \quad (2.29)$$
$$m_e \approx 9.10938188 \times 10^{-31} \quad (2.30)$$
$$m_p \approx 1.67262158 \times 10^{-27} \quad (2.31)$$
$$r_{B,137} \approx 3.8626073592 \times 10^{-13} \quad (2.32)$$
$$r_{G,Z_G} \approx 4.7721275373 \times 10^{23} \quad (2.33)$$
$$v_{B,137} \approx 2.9971370158 \times 10^{8} \quad (2.34)$$
$$v_{G,Z_G} \approx 2.9979245799 \times 10^{8} \quad (2.35)$$
$$c = 2.99792458 \times 10^{8} \quad (2.36)$$

3 Conclusions

As suggested in the title, I regard the material presented here as a *sketch for a quantum theory of gravitation*. However, this material is detailed in that it shows a way in which the very small quantum theory domain of atomic systems can be integrated with the very large cosmology domain within a single theory. The fundamental theories that are called upon to render this construction possible are Sommerfeld's relativistic formula for quantum states together with this author's theory for quantum coupling constants.

A physical result that emerges from this construction is a relativistic dynamical way in which the rest mass of the proton can be seen as formed from the kinematic's of what I have called here, a *graviton*, a particle with the very small rest mass, m_G. As the theory has been developed here, the rest mass of this particle is about a 1/1000th part of the rest mass of an electron. However, the construction here is limited by experimental knowledge of the true values of the gravitational and other constants and the consequent difficulty of reliably being able to work out the arithmetic of the very large numbers that are involved. It seems that with more reliably accurate information it will be possible to assert that m_G has a *limiting* value zero. Almost putting it on a par with the photon or neutrino. Further more it's orbits are very large radius circles which are limiting straight lines, if the universe is sufficiently large, again showing a close relation with the photon or neutrino. The angular velocity of the electronic states which are analogous to the m_G states have spin 1/2 so perhaps it is the neutrino with its spin 1/2 that is the key to the understanding of quantum gravity.

The theory given here suggests that the rest mass of the proton originates from the graviton kinematics. Possibly this is extendable to some extent to all the hadrons though other considerations may well be involved. Clearly the very complex rest mass relations between the many elementary particles is barely touched by this rest mass generation result for the proton. However it could be a useful start to revelations in that context. It has been shown that an orbiting proton moving on a very large radius path is the gravitational analogue of the orbiting electron in the Sommerfeld quantum electronic theory. Included, is the possibility that the radius of the proton's path is so large that it is indistinguishable from a straight line. The structure given here enables us to identify the gravitational potential analogous to the hydrogen$_{137}$ electric central potential $V(r) = 137\alpha\hbar c/r$ to which a proton is subjected generally and is the kinematic source of its rest mass m_p. This is given by

$$V_G(r_G) = N_G\alpha_G\hbar_G c/r_G, \qquad (3.1)$$
$$= \xi N_G G m_p m_e/r_G. \qquad (3.2)$$

From (3.2) it can be seen that the central force that holds the proton in its orbit arises from the gravitational attraction of a total mass, $\xi N_G m_e$, comprised of N_G electronic sub-masses of strength ξm_e in analogy with the case of H_{137} where the central force is composed of a total charge, $137e$, comprised of 137 electronic sub-charges of strength e. Thus if we appeal to the particle *internal relativity principle* for a proton and its internal gravitational states we see that the proton's rest mass arises from a centre of force at a distance R_G from its position. This centre of force is on a reference frame which will also be host for the internal proton rest mass gravitational generating mass m_G. In keeping with the usual *classical* gravitational potential theory, this centre of force can be thought off as a mass distribution spread over a finite sphere within the radius R_G. R_G is the radius of the universe so here we have the situation of greatly distant masses determining the local rest mass m_p of the proton. This is a clear indication of the appropriateness of Mach's Principle ([47], [37],[32],[36]) in the context of this theoretical construction. This contrasts sharply with work on Mach's principle in relation to the local standard of zero rotation ([47], [40], [41], [42], [43], [44], [45]) which does not seem to be directly related to the local inertia problem. The key result that connects the quantum domain with the cosmological domain is the relation between the graviton mass $m_G(t_n)$ and the radius of the universe $R(t_n)$ at time t_n, now. They are both dependent on time and are related by a rest length radius, R_0, of the universe coinciding with the crossed Compton wavelength $\lambda\!\!\!^{-}_{G,0}$ of the mass m_G and $\lambda\!\!\!^{-}_G(t_n) = R(t_n)$.

$$R(t_n) = R_0(1 - (v_{G,Z_G}/c)^2)^{1/2} \qquad (3.3)$$

In view of equation (2.18), the reader may prefer to define the nominal radius of the universe as the larger value $R^*(t_n) = \alpha^{-1}R(t_n)$ together with an appropriate T^* and H^*. T^* would then have the same order of magnitude as *adopted* in reference [38] page 738. There is another significant advantages of adopting the starred definions above because with them there is a starred potential function,

$$V_G^*(r_G^*) = N_G\alpha^{-1}\alpha_G\hbar_G c/r_G^*, \qquad (3.4)$$
$$= \xi N_G G m_p(\alpha^{-1}m_e)/r_G^*, \qquad (3.5)$$

that replaces the (3.1,3.2) form and in which the quantity of mass causing the gravitation potential for the very large radius orbit on which the circling proton moves can be seen to be given by $M_U = \xi N_G\alpha^{-1}m_e$. This turns out to have the same order of magnitude, $10.68 \times 10^{53}\ k_g$, as the amount of matter in the universe also as suggested in the reference [38] page 738. This enhancement option will be considered in an appendix to follow in the next section.

4 Enhancment Appendix

This section is essentially an appendix containing some elaborations and re-identifications of the age and radius of the universe and other additions. These changes involve a detailed following up of a change in the definition, $T \to t^*$ of the age of the universe, T as suggested at the end of the previous section.

In earlier sections of this paper, a possible nominal age for the universe was identified from a new formula for the gravitational constant as

$$T = \xi\tau_p \quad (4.1)$$
$$\tau_{m_p} = \hbar/(m_p c^2) \quad (4.2)$$

where $\tau_{m_p} = \hbar/(m_p c^2) = l_p/c$ is the Compton time interval for a proton.

There is freedom in making this identification because of some dimensionless multipliers appearing in the formula for G. Expressed in terms of T or R the formula for G is

$$G = \alpha\hbar^2/(T m_p^2 m_e c) \quad (4.3)$$
$$= \alpha\hbar^2/(R m_p^2 m_e). \quad (4.4)$$

It became apparent that a better correspondence with *the age of the universe* values suggested from measurement would occur if the α appearing in the numerator of G were incorporated with T to give a transformed value $T^* = \alpha^{-1}T$. Thus $T^* > T$ and has the same order of magnitude as suggested from the measurement arena. The gravitation constant G in terms of the starred variables becomes

$$G = \hbar^2/(T^* m_p^2 m_e c) \quad (4.5)$$
$$= \hbar^2/(R^* m_p^2 m_e). \quad (4.6)$$

This redefinition was suggested at the end of the last section and it was indicated that other quantities would need to be changed in order for a consistent theory to incorporate the starred definitions. It turns out that some simple changes have to be made to the quantum orbital theory to achieve this consistency for the starred version. The basic change is to replace the classical radius of the proton, $r_p = \alpha l_p$ in equation (1.17) with its Compton wave length, l_p as here in equations (4.7), (5.3), (5.4) and (5.5). This gives a starred version for, r^*_{G,Z_G} for example, replacing the original r_{G,Z_G} displayed at equation (2.18) in this case. All the cases are listed below. From an inspection of the last equality in equation (2.18), it is then *hindsight* obvious that this is how it should have been from the start! Below is a list of the changes to original variables consequent upon making the change $r_p \rightarrow l_p$.

$$r_p \rightarrow l_p \quad (4.7)$$
$$T \rightarrow T^* = \alpha^{-1}T \quad (4.8)$$
$$R \rightarrow R^* = \alpha^{-1}R \quad (4.9)$$
$$H \rightarrow H^* = (T^*)^{-1} \quad (4.10)$$
$$M_U \rightarrow m^*_U = M_U \quad (4.11)$$
$$\lambdabar_{G,0} \rightarrow \lambdabar^*_{G,0} = \alpha^{-1}\lambdabar_{G,0} \quad (4.12)$$
$$r_{G,Z_G} \rightarrow r^*_{G,Z_G} = l_p/(Z_G\alpha_G^2) \quad (4.13)$$

I now wish to make an altogether more subtle change in the *interpretational* aspects of this theoretical structure. This will involve some additional notation and changes to some physical variables. Generally the numerical effect of these changes will be negligible and the theory could be left without these additional changes. This would mean that we are taking what might be called a course grained view of the universe and we would not be looking at the detailed structure. However, the enhanced version that emerges is *logically* more satisfactory and the link and similarity between the small scale quantum aspects and the large scale gravitational aspects becomes more clarified. To see why such changes are desirable we can examine the wave capture diagram fig.1

which originally arose from the electrical-quantum consideration of the orbital structure of H_{137} in relation to the fine structure constant. The most obvious aspect of this diagram are the circular boundaries of radii r_a, r_c, r_b and r_d. There is another possible boundary missing from the diagram of radius smaller than the shown four that we can call $r_i = r_c\cos(\chi^*(137))$ in the case of H_{137}. Thus all the wave structure lies between radii r_i and r_d and the numerical difference between these two radii is $r_d - r_i = r_d(1-\cos^2(\chi^*(137))) = r_d\sin^2(\chi^*(137)) \approx r_d(1/137)^2 \approx r_d \times 10^{-4}$. The whole of the geometrical structure between these two extremes would vanish in the thickness of the single circular circumference, if the diagram had not been greatly distorted to make it readable. The philosophy of the work in this article is that the gravitation orbits for a proton, almost straight lines for a free proton, follow the same pattern as in the wave capture diagram with the additional condition that the outer and inner radii for the gravitation orbits differ by vastly smaller amount than the electromagnetic case. This is why I suggest that the new set of changes that are to be implemented are logically and conceptually important but numerically negligible. However, the thickness of a very thin line on a small disc representing the whole universe could represent 10^8 human life spans. The two inner radii r_d and r_b are important for QED but need not concern us in the gravitation regime for reasons that will become clear.

5 Enhancement Variables

The first step in the enhancement is to consider the formulae (2.6) and (2.7) for the gravitation constant G in terms of the time like parameter T^* and abandon the *interpretation* of cT^* as the radius of the universe. Then reinterpret $R^* = cT^*$ as the nearby quantity the gravitational equivalent of the QED radius of $r_{B,137}$, of first Bohr orbit of H_{137}. This equivalent is a quantity, $r^*_{G,N_G} = ct^*_{G,N_G}$, a starred version of what was r_{G,N_G} in A originally. The result of this change is a *functional* form for $G(r'^*)$, the gravitational constant in terms of r'^*.

$$G(r^*_{G,N_G}) = \hbar^2/(t^*_{G,N_G} m_p^2 m_e c) \quad (5.1)$$
$$= \hbar^2/(r^*_{G,N_G} m_p^2 m_e). \quad (5.2)$$

Equation (5.2) is to be our fundamental equation connecting the quantum theory for gravitational orbits to the value of the relativistic cosmological quantity G. The numerical value of G will now be regarded as *defined* through equation (5.2). To show how the radius or the age of the of the universe now comes into the structure we have to examine the other important circular quantum orbits within which the circle of radius r^*_{G,N_G} is to be found, the analogues of r_i and r_d from H_{137} theory. They are defined in ascending size order with r^*_{G,N_G} central in magnitude as

$$r''^* = r^*_{G,N_G}\cos(\chi_G) = \frac{N_G l_p}{\cos(\chi_G)} \quad (5.3)$$
$$r'^* = r^*_{G,N_G} = \frac{N_G l_p}{\cos^2(\chi_G)} \quad (5.4)$$
$$r^* = \frac{r^*_{G,N_G}}{\cos(\chi_G)} = \frac{N_G l_p}{\cos^3(\chi_G)}. \quad (5.5)$$

The notation,

$$l_p = \hbar/(m_p c) \quad (5.6)$$
$$l'_p = \hbar/(m_p c\cos(\chi_G)) \quad (5.7)$$
$$l''_p = \hbar/(m_p c\cos^2(\chi_G))), \quad (5.8)$$

will be used. The speed on the r'^* is given by $r'^*\omega = N_G\alpha_G c$ so that the speed v^* on the r^* orbit will be given by

$$v''^* = r''^*\omega = r''^* N_G c\alpha_G/r'^* = c'' \quad (5.9)$$
$$v'^* = r'^*\omega = r'^* N_G c\alpha_G/r'^* = c' \quad (5.10)$$
$$v^* = r^*\omega = r^* N_G c\alpha_G/r'^* = c \quad (5.11)$$
$$c' = c\cos(\chi_G) \quad (5.12)$$
$$c'' = c\cos^2(\chi_G). \quad (5.13)$$

Thus we can take these inner and outer boundaries r''^* and r^* within which the main orbit of radius r'^* lies to represent an annulus fixed to and rotating with the geometrically extended object. The object has a mean position on the radius r'^*. The outer boundary of this annular platform has a transverse velocity of the speed of light and so is naturally to be regarded as the maximum outer boundary for the whole rotating system. This then is good reason to identify r^* as the radius of the universe. This does not mean that the universe is rotating. It rather means that the maximum separation for gravitationally coupled systems coincides numerically with the radius of the universe, r^*.

Let us first calculate the orbital angular momentum for an electron in the first Bohr orbit of H_{137}. From equations (1.17) and (1.18) this is

$$r_{B,Z} v_{B,Z} m_0 = Z\alpha c m_0 l_c/(Z\alpha^2) \quad (5.14)$$
$$= m_0 c l_c/\alpha = m_0 c l_{m_0} \quad (5.15)$$
$$= \hbar \quad (5.16)$$

The orbital angular momentum for the corresponding gravitation orbit of a proton is from equations (5.4) and (5.10) and using the starred system

$$r^*_{G,Z_G} v_{G,Z_G} m_p = \frac{Z_G\alpha_G c m_p l_p}{(Z_G\alpha_G^2)} \quad (5.17)$$
$$= \frac{m_p c l_p}{\alpha_G} \quad (5.18)$$
$$= \frac{\hbar}{\alpha_G} = \frac{N_G\hbar}{\cos(\chi_G(Z_G))} \quad (5.19)$$

The starred version of the gravitational potential in which a proton moves has the form on the mean orbit and on the outer boundary as given by

$$V^*_G(r'^*_G) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r'^*_G} \quad (5.20)$$
$$= m_p c'^2 \quad (5.21)$$
$$V^*_G(r^*_G) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r^*_G} \quad (5.22)$$
$$= m_p c'^2\cos(\alpha_G) \quad (5.23)$$
$$r'^*_G < r^*_G \quad (5.24)$$
$$V^*_G(r'^*_G) > V^*_G(r^*_G) \quad (5.25)$$

The *gravitation orbit* of a proton here means a free proton at rest or in motion under gravitational influence alone. The numerical value of the orbital velocity in the orbit given by the quantum number $Z_G = N_G$ for a given $\alpha_G(N_G, N'_G)$ is the maximum quantum number and so it gives the highest speed in orbit, $v_{G,N_G} = \cos(\chi^*_G(N_G))c < c$. In general this speed will be very close to but definitely *less* than c. Thus when a proton is encountered experimentally apparently not moving at high speed relative to the laboratory it does not mean that such a proton has not got the high velocity in its gravitational orbit being studied here. This is because the observer's laboratory rest frame can in general have a small velocity relative to the proton. In other words, any proton can have a high gravitational velocity relative to its distant rest mass generating graviton's rest frame.

6 Numerical Values for Radius, Age and Mass r^*_G, t^*_G *and* M_U of Universe

The nominal time for the existence of the universe, its age, will from now on be denoted by t^* and defined by $t^* = r^*_G/c$ where r^*_G is taken to be the current radius of the universe. It is evident that these theoretically obtained values give a very good agreement with the experimentally assessed values and also have the theoretical advantage of greatly clarifying detailed aspects of the structure. The mass of the universe can be obtained by considering the magnitude of the mass that induces the gravitational potential in which the proton moves, or more appropriately one might say, *exists*. From equation (5.20) this potential is

$$V^*_G(r'^*_G) = \frac{N_G G m_e m_p \alpha_G^{-1}}{r'^*_G} \quad (6.1)$$

$$= m_p c'^2 \quad (6.2)$$

$$= \frac{G M_u m_p}{r'^*_G} \quad (6.3)$$

$$M_U = N_G m_e \alpha_G^{-1}$$

$$= N_G^2 m_e / \cos(\chi_G(N_G) \quad (6.4)$$

$$N_G \approx 3.11 \times 10^{41} \quad (6.5)$$

$$\alpha_G^{-1} \approx 3.11 \times 10^{41} \quad (6.6)$$

$$M_U \approx 8.81 \times 10^{52} kg \quad (6.7)$$

$$M_{U,m} \approx 5.68 \times 10^{53} kg \quad (6.8)$$

$$t^* \approx 2.18 \times 10^{17} s \quad (6.9)$$

$$t^*_m \approx 3.1536 \times 10^{17} s \quad (6.10)$$

$M_{U,m}$ is the value for the mass and t^*_m is the age of the universe from observation and measurements as suggested in reference [38] and also admitted as to not being *canonical*.

The three quantities r^*_G, t^*_G *and* M_U together with the rest mass of the graviton, m_G, all depend on the value of the integer parameter N_G and when I write of *theoretical* values, I imply that the actual numerical values can only be found if the value of the integer N_G can be input into the calculation. There is no theoretical way to evaluate N_G any more than there is to evaluate 137 at this time in the physics science story. There is a second parameter N'_G involved but again, unlike in the QED case where it is important, in gravitation theory it does not appear to play a significant part. It can be shown why this is the case as follows.

The QED difference between α^{-1} and 137 and the gravitational equivalents have the approximate values

$$\alpha^{-1}(137, 29) - 137 \approx 0.036 \quad (6.11)$$

$$\alpha_G^{-1}(N_G, N'_G) - N_G \approx \frac{\pi^3}{N_G^2}\left(\frac{1}{2} - \frac{1}{3{N'_G}^2}\right) \quad (6.12)$$

$$\rightarrow \frac{\pi^3}{2N_G^2} \ \ as \ \ N'_G \rightarrow \infty \quad (6.13)$$

$$\alpha_G^{-1}(10^{42}, \infty) - 10^{42} \approx 10^{-84} \quad (6.14)$$

$$\cos(\chi_G(N_G)) \rightarrow 1 \ as \ N_G \rightarrow \infty \quad (6.15)$$

$$\chi_G(N_G) \rightarrow 0 \ as \ N_G \rightarrow \infty \quad (6.16)$$

From equations (2.22) and (2.23) we get the relation for the graviton rest mass,

$$m_G = m_p(1 - (c'/c)^2)^{1/2} \quad (6.17)$$

$$= m_p(1 - \cos^2((\chi_G(N_G)))^{1/2} \quad (6.18)$$

$$= m_p \sin(\chi_G(N_G)) \quad (6.19)$$

$$\rightarrow 0 \ as \ N_G \rightarrow \infty \quad (6.20)$$

$$r^* = N_G l_p / \cos^3(\chi_G) \quad (6.21)$$

$$t^* = N_G \tau_p / \cos^3(\chi_G) \quad (6.22)$$

$$\tau_p = l_p / c \quad (6.23)$$

$$\cos(\chi_G(N_G)) r^* = N_G l''_p. \quad (6.24)$$

From equation (6.11), it can be seen that the inverse fine structure constant differs from the integer 137 at the second decimal place whereas from equation (6.14), its gravitational equivalent α_G^{-1} differs from the integer $N_G \approx 10^{42}$ at the $84th$ decimal place, they are essentially equal and from equation (6.22) it can be seen that t^* is essentially equal to $N_G \tau_p$ where τ_p is a constant eigenfunction *time* increment associated with the eigenvalue number N_G. Thus the digitally changing quantity N_G can be regarded as a digital parameter determining the age of the universe or we can write $N_G(t^*)$ implying the converse. It makes little difference if you care to regard time as the primary mover for change or regard N_G as determining the time t^* or the age of the universe. Thus in this gravitational theory the quantum integer eigenvalue N_G is of primary importance. The whole story is contained in the concept of *projection quantization*. This concept is the bridge between quantum theory quantization and relativity theory length contraction. One example is displayed at equation (6.24) for the case of the radius of the universe expressed in terms of the eigen-length l_p''. This is just one case among the five others for the quantum orbits in terms of their own l_p versions and the key state integer N_G. From equations (6.4) and 6.22, it is clear that the amount of mass within this universe is increasing proportionally to the square of its age. The question then arises as to where this mass is coming from and is conservation of energy being violated in this model? There is a simple answer to this question though it may not be everyone's satisfaction. If we are to talk about expansion at all we do need to have some idea about what the expansion is taking place into. Let us consider the most obvious and simple situation and suppose that the expansion is taking place into a larger containing space and for simplicity assume it is euclidean, E_{con}, say. In fact, we have no idea of the actual geometry that might be involved so that this assumption will have to do here. Suppose that our expanding universe is centred at the origin of E_{con}. I see no reason why this containing space should be completely empty. It contains an expanding universe but suppose further the centre of that expanding universe is surrounded with a mass distribution that extend in all direction just as a consequence of its containing that universe. At small values for the age of the universe t^*, its radius r^* or expanding boundary is within the mass distribution which could itself extend to infinity in E_{con}. Let the density of this *halo* of electronic sized masses be given by, the inverse linear, in R function, $\sigma_G(R) = 3m_e/(\cos(\chi_G(N_G(R/c))4\pi l_p''^2 R)$, where R is the distance from the centre of coordinates in E_{con}. At time t^* the universe will have expanded to the volume size $V(r^*) = 4\pi(r^*)^3/3$ so that it will then contain mass of amount $\sigma_G(r^*)V(r^*) = M_U(t^*)$ as given by equation (6.4). A similar argument could be adduced for more geometrically exotic container spaces. Thus conservation of energy is not threatened.

Related material can be found in references, ([10], [11], [12], [13], [14], [15], [19], [29], [30], [31], [39], [46]).

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References

[1] Sommerfeld, A. 1916 Annalen der Physik **51**, 1

[2] Gilson, J.G. 1991, Oscillations of a Polarizable Vacuum , Journal of Applied Mathematics and Stochastic Analysis , **4**, 11, 95–110.

[3] Gilson, J.G. 1994, Vacuum Polarisation and The Fine Structure Constant, Speculations in Science and Technology , **17**, 3 , 201-204.

[4] Gilson, J.G. 1996, Calculating the fine structure constant , Physics Essays, **9** , 2 June, 342-353 .

[5] Eddington, A.S. 1946, Fundamental Theory, Cambridge University Press.

[6] Kilmister, C.W. 1992, Philosophica, **50**, 55 .

[7] Bastin, T. 1966, Philosophica Gandensia, **4**, 77.

[8] Bastin, T., Kilmister, C. W. 1995 , Combinatorial Physics , World Scientific Ltd.

[9] Kilmister, C. W. 1994 , Eddington's search for a Fundamental Theory, CUP.

[10] Thirring, W.E. 1958 , Principles of Quantum Electrodynamics, Academic Press Inc.

[11] Feynman, R. P.1961, Quantun Electrodyanamics, Reprint Series , W.A. Benjamin,Inc, NY.

[12] Bogoliubov N. N., Shirkoov, D. V. 1959, Introduction to the Theory of Quantized Fields Volume III , Interscience New York.

[13] Feynman, R. P. 1961, Theory of Fundamental Processes, Reprint Series, W.A. Benjamin, Inc, New York.

[14] F. J. Dyson 1949, The S-Matrix in Quantum Electrodynamics, Phys. Rev., 75, 1736.

[15] Schwinger, J. 1953, The Theory of Quantized Fields II, Phys. Rev., 91, 713.

[16] Barrow, J.D. and Tipler, F. J 1986, The Anthropic Cosmological Principle, OUP.

[17] Taylor, B. N. Units and Fundamental Constants in Physics and Chemistry, Subvolume b(Springer Verlag, 1991), Ref. p. 3-131.

[18] Gilson, J. G. 1997, Relativistic Wave Packing and Quantization , Speculations in Science and Technology , **20** Number 1 March, 21-31

[19] Gray, I. R. 1994, Symmetrical Unification from Electromagnetostatics, Part II, Physics Essays , **7**, 4, 464-474.

[20] Schönfeld, E. 1990, Electron and Fine-Structure Constant, Metrologia , **27**, 117-125.

[21] Cohen, E. R. and Taylor, B. N. 1993, The Fundamental Physical Constants, Physics Today (August), BG9-15.

[22] Aoki, H. 1989, Rep. Prog. Phys., **50**, 655-730.

[23] Klitzing, Von. Donda, G. and Pepper, M. 1980, Phys. Rev. Lett. , **45**, 494-7.

[24] Kinoshita, T. 1996, Rep. Prog. Phys., **59**, 1459-1492.

[25] Max Born, Atomic Physics (Blackie and Son Ltd, 1953), p. 164 .

[26] Feynman, R. P. 1988 , QED The Strange Theory of Light and Matter, Princeton University Press.

[27] Dirac, P. A. M (1931) , Proc. R. Soc. London, **A133**, 60.

[28] A. Wyler 1969 , C. R. Acad. Sc. Paris, t. **269** Série , 743-745

[29] G. Cavalleri, Lettere Al Nuovo Cimento , **43**(6), 16 Luglio, 1985.

[30] W. Rindler, 1961, Am. J. Phys. **29**, 365

[31] S. Weinberg, 1981, Einstein Symposium on Gauge Theory and Uniification of physical Forces, Addison Wesley

[32] Soshichi Uchii at www.bun.kyoto-u.ac.jp/%7Esuchii/mach.pr.html

[33] Gilson, J.G. 1999, The fine structure constant, www.fine-structure-constant.org/

[34] McPherson R. 2004, Electrifying Gravity, www.fine-structure-constant.org/ross.pdf

[35] Rindler, W. 2001, Relativity: Special, General and Cosmological, Oxford University Press

[36] Mach, H. 1893, The Science of Mechanics, Chicago, IL: Open Court

[37] Barbour, J. and Pfister, H. 1995, Mach's Principle: From Newton's Bucket to Quantum Gravity. Boston, MA: Birkhuser

[38] Misner, C. W.; Thorne, K. S.; and Wheeler, J. A. 1973, Gravitation. Boston, San Francisco, CA: W. H. Freeman

[39] Wolfgang Pauli 1958, Theory of Relativity, 130–134, Pergamon Press

[40] Abraham Pais, 1990, Subtle is the Lord: the Science and Life of Albert Einstein, p 214

[41] J. S. Bell 1987, Speakable and Unspeakable in Quantum Mechanics, p 67, CUP

[42] Dover 1980, The Principle of Relativity, Dover

[43] G. Cavalleri 1987, Nuovo Cimento 53B, p 415

[44] O. Gron 1975, AJP, Vol. 43, No. 10, p 869

[45] C. Berenda 1942, Phys. Rev. 62, p 280

[46] M. Weiss 1988, www.physics.adelaide.edu.au/∼ dkoks/Faq/Relativity/SR/rigi$d$_disk.html

[47] J. G. Gilson 2004, arxiv.org/PS_cache/physics/pdf /0409/0409010.pdf