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# Solutions of a Cosmological Schrödinger Equation for Exact Gravitational Waves based on a Friedman Dust Universe with Einstein's Lambda

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## Abstract

In an earlier paper, it was shown that the cosmological model that was introduced in a sequence of three earlier papers under the title *A Dust Universe Solution to the Dark Energy Problem*, originally described by the Friedman equations, can be expressed as a solution to a non-linear Schrödinger equation. In this paper, a large collection of solutions to this Schrödinger equation are found and discussed in the context of relaxing the uniform mass density condition usually employed in cosmology theory. The surprising result is obtained that this non-linear equation can have its many solutions *linearly superposed* to obtain solution of the cosmology theory problem of great generality and applicability.

# 1 Introduction

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [32] together with applications of those papers to the cosmological constant problem, the cosmological coincidence problem and other subsidiary cosmological problems.

The conclusions arrived at in those papers was that the dark energy *substance* is physical material with a positive density, as is usual, but with a negative gravity, -G, characteristic and is twice as abundant as has usually been considered to be the case. References to equations in those papers will be prefaced with the letter $A$, $B$ and $C$ respectively. The work in $A$, $B$ and $C$, and the application here have origins in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7],[25],[3]). Other useful sources of information are ([17],[3],[30],[27],[29],[28]) with the measurement essentials coming from references ([1],[2],[11],[37]). Further references will be mentioned as necessary. The application of the cosmological model introduced in the papers $A$ [23], $B$,[24] and $C$ [32], in paper $E$, ([36]), is to the extensively discussed and analysed *Cosmological Coincidence Problem*. In the paper $D$, [34], it was shown that the quantum vacuum polarisation idea can be seen to play a central role in the Friedman dust universe model introduced by the author. In the paper, [40], it was shown that the Friedman equation structure can be converted into a *non-linear* Schrödinger equation structure. Here, this aspect is further developed by supplementing the solutions to this time only equation with a dependence on a three dimensional space position vector, $\mathbf{r}$, so that the equation remains consistent with its cosmological origin. This step then enables finding cosmological models that are not restricted to having a mass density that is certainly time dependent but otherwise remains constant over all *three dimensional* position space at every definite time. It is convenient here to give a *very brief* reminder of the structure of Schrödinger theory in relation to the Friedman equations. The two Friedman equations from general relativity and the Schrödinger equation from quantum theory have the following three forms,

$$\begin{aligned}
8\pi G\rho r^2/3 &= \dot{r}^2 + (k - \Lambda r^2/3)c^2 && (1.1)\\
-8\pi G P r/c^2 &= 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2 && (1.2)\\
i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} &= -\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r})\Psi(\mathbf{r},t) && (1.3)\\
E_n\Psi_n(\mathbf{r},t) &= i\hbar\frac{\partial\Psi_n(\mathbf{r},t)}{\partial t} && (1.4)\\
\nabla &= \mathbf{i}\partial/\partial x + \mathbf{j}\partial/\partial y + \mathbf{k}\partial/\partial z && (1.5)\\
\rho_Q(\mathbf{r},t) &= \Psi(\mathbf{r},t)\Psi^*(\mathbf{r},t) && (1.6)\\
\Psi(\mathbf{r},t) &= \sum_n \int c_n\Psi_n(\mathbf{r},t). && (1.7)
\end{aligned}$$

The non-linear Schrödinger equation that was obtained in reference [40] has the form

$$\begin{aligned}
i\hbar\partial\Psi_{nl,\rho}(t)/\partial t &= (V_C(t))\Psi_{nl,\rho}(t) && (1.8)\\
V_C(t) &= -(3i\hbar/2)H(t) && (1.9)
\end{aligned}$$

and can be compared with the general linear Schrödinger equation at (1.3). The non-linearity of the cosmological version is indicated by the feedback potential $V_C(t)$, (1.9) replacing the external potential at (1.3). The state vector $\Psi_{nl,\rho}(t)$ in the cosmology version initially has no dependence on local position denoted by the three vector, $\mathbf{r}$, as in the quantum version, (1.3). This deficiency will be rectified in the following section.

## 2 Position Variable Cosmology Schrödinger equation

Before starting this section, it is necessary to make some remarks about the dimensionality of the usual physical position coordinate vector, $\mathbf{r} = \mathrm{x}\mathbf{i} + \mathrm{y}\mathbf{j} + \mathrm{z}\mathbf{k}$. This is often taken to have the dimension, $m$, physical length. The relativistic metric used in this theory is of the form

$$ds^2 = c^2dt^2 - r^2(t)(d\grave{x}^2 + d\grave{x}^2 + d\grave{x}^2). \qquad (2.1)$$

In this work up to date, I have taken the scale factor $r(t)$ to represent the *physical* radius of the universe at epoch time $t$ so that it has the dimension $m$, physical length. If as usual, $c$ has the physical dimensions $ms^{-1}$ and $t$

has the physical dimension, $s$, then ds in the metric will have the dimension $m$ and so the vector, $\grave{\mathbf{r}} = \grave{x}\mathbf{i} + \grave{y}\mathbf{j} + \grave{z}\mathbf{k}$, will be dimensionless and this is indicated by the above grave accent. The theory I am working with here is non-linear and attempting to use dimensioned position coordinates can lead to dimensionality chaos. Thus from now on, I shall usually work with the dimensionless position coordinates and use the grave sign to indicate this. Consistent with this policy it is useful it define the dimensionless quantities using the fundamental length $R_\Lambda$ as follows and starting with a dimensionless radius for the universe, $\grave{r}(t)$,

$$\grave{r}(t) = r(t)/R_\Lambda \tag{2.2}$$

$$\grave{x} = x/R_\Lambda \tag{2.3}$$

$$\grave{y} = y/R_\Lambda \tag{2.4}$$

$$\grave{z} = z/R_\Lambda. \tag{2.5}$$

I shall also use the grave accent to indicate that *a function* is dimensionless as with $\grave{f}(r)$. My strategy in the following work is firstly, to introduce space dependence, $\grave{\mathbf{r}}$, into the cosmological Schrödinger equation (1.8) and then , secondly to show that the introduction of an $\grave{\mathbf{r}}$ dependence can be made consistent with the original Friedman equations structure without damaging their validity as a rigorous solution to Einstein's field equations. Firstly, I rewrite the purely time dependent equation (1.8) assuming an extra dependence on $\grave{\mathbf{r}}$ in the original state vector $\Psi_{nl,\rho}(t)$, while leaving the feedback term unchanged.

$$i\hbar\partial\Psi_{nl,\rho}(t,\grave{\mathbf{r}})/\partial t = (V_C(t))\Psi_{nl,\rho}(t,\grave{\mathbf{r}}) \tag{2.6}$$

$$V_C(t) = -(3i\hbar/2)H(t). \tag{2.7}$$

The first question that arises is, *can this step be done consistently*? The answer to this is in the affirmative as can be shown as follows. Rewrite (2.6) as equation (2.8) and followed by the time integration at (2.9) and then inverting the logarithm at (2.10)

$$\partial \ln \Psi_{nl,\rho}(t,\grave{\mathbf{r}})/\partial t = -(3/2)H(t) \tag{2.8}$$

$$\ln(\Psi_{nl,\rho}(t,\grave{\mathbf{r}})/\Psi_{nl,\rho}(t_0,\grave{\mathbf{r}})) = -(3/2)\int_0^t H(t')dt' \tag{2.9}$$

$$\Psi_{nl,\rho}(t,\grave{\mathbf{r}}) = \Psi_{nl,\rho}(t_0,\grave{\mathbf{r}})\exp\left(-\frac{3}{2}\int_{t_0}^t H(t')dt'\right) \tag{2.10}$$

$$\Psi_{nl,\rho}(t_0,\grave{\mathbf{r}}) = \Psi_{nl,\rho}(t_0)\grave{f}(\grave{\mathbf{r}}) \tag{2.11}$$

Thus introducing a dimensionless function, $\grave{\mathbf{f}}$, with $\grave{\mathbf{r}}$ dependence presents no problems. It means just multiplying the original time only dependent wave function, $\Psi_{nl,\rho}(t_0)$, with the purely space dependent function, $\grave{f}(\grave{\mathbf{r}})$. This also partially justifies not including any space variation in the Hubble function, $H(t)$. However, this last point will be fully justified when the affect on the purely time dependent Friedman equations, (1.1) and (1.2), is examined in the next paragraph. I should be remarked that the function $\grave{f}(\grave{\mathbf{r}})$ can be a complex valued function in the context of quantum theory wave function structure. This fact will be seen to be useful as the story unfolds.

The relation of the cosmological Schrödinger equation and the Friedman equations clearly has to be mutual consistency. A threat to this consistency is the obvious difference between the purely time dependent mass density function $\rho(t)$ in the Friedman set and the now proposed space time variability through $\grave{\mathbf{r}}$ in the Schrödinger equation wave function, (2.10). In using the original Friedman equations, (1.1) and (1.2), it has been common practice to assume that $\rho(t)$ is a purely time dependent mass density chosen as a working approximation to a correct more general time and space dependant version and so rendering difficult mathematics viable though less physically accurate. This was my starting position when I wrote the first paper, $A$, in this sequence of papers. However, having found the non-linear Schrödinger equation (1.8) it has become clear that the common practice position with regard to $\rho(t)$ needs some modification. My view now is that $\rho(t)$ is a correct quantity in its own right, giving information about the cosmology structure as a global entity. Its definition is repeated below,

$$
\begin{aligned}
\rho(t) &= M_U/V_U(t) \qquad (2.12)\\
M_U &= \rho(t)V_U(t), \qquad (2.13)
\end{aligned}
$$

where $M_U$ is the total conserved positively gravitational mass of the universe and $V_U(t)$ is the volume of the universe at epoch time $t$. If $\rho(t)$, does have a definite meaning in its own right and is not just an approximation to a better space dependent version then it can be retained with its self identity as before. This special significance of $\rho(t)$ is effectively retained by keeping it but multiplied by the space dependant contribution as in (2.11). From the existence of a possible true space and time dependent version from Schrödinger theory it can be seen that the definition for the mass, $M_U$, of the universe that appears in (2.12) with the space dependent density, should be

$$\begin{aligned}
M_U(t) &= {R_\Lambda}^3 \int\int\int_{V_U(t)} \rho(t, \grave{\mathbf{r}}) d\grave{x} d\grave{y} d\grave{z} && (2.14)\\
&= {R_\Lambda}^3 \int\int\int_{V_U(t_0)} \rho(t_0, \grave{\mathbf{r}}) d\grave{x} d\grave{y} d\grave{z} && (2.15)\\
&= M_U = \ \textit{a constant} && (2.16)\\
\rho(t_0, \grave{\mathbf{r}}) &= \Psi_{nl,\rho}(t_0, \grave{\mathbf{r}}) \Psi^*_{nl,\rho}(t_0, \grave{\mathbf{r}}) = \Psi_{nl,\rho}(t_0) \Psi^*_{nl,\rho}(t_0) \grave{f}(\grave{\mathbf{r}}) f^*(\grave{\mathbf{r}}) && (2.17)\\
&= \rho(t_0) \grave{f}(\grave{\mathbf{r}}) \grave{f}^*(\grave{\mathbf{r}}), && (2.18)
\end{aligned}$$

where $V_U(t)$ is the volume of the universe at time $t$, the time dependent spherical volume over which the integration is taken at time $t$ and equations, (2.14), (2.15) and (2.16), holding because the total mass within the universe is a constant over time. In other words $M_U$ is a time conserved quantity or within the universe's changing boundary, density movement should satisfy the equation of continuity which in the usual coordinates is

$$\partial \rho(t, \mathbf{r}) / \partial t = -\nabla(\mathbf{v}(t, \mathbf{r}) \rho(t, \mathbf{r})). \qquad (2.19)$$

From equations (2.15) and (2.18), we get

$$\begin{aligned}
M_U(t_0) &= {R_\Lambda}^3 \int\int\int_{V_U(t_0)} \rho(t_0, \grave{\mathbf{r}}) d\grave{x} d\grave{y} d\grave{z} && (2.20)\\
&= \rho(t_0) {R_\Lambda}^3 \int\int\int_{V_U(t_0)} \grave{f}(\grave{\mathbf{r}}) \grave{f}^*(\grave{\mathbf{r}}) d\grave{x} d\grave{y} d\grave{z}. && (2.21)
\end{aligned}$$

Thus once the function, $\grave{f}(\grave{\mathbf{r}})$, is chosen, it appears that we can find the constant value of the mass of the universe, $M_U$. However, this appearance is deceptive because there is the complication that to get a constant valued numerical value from this equation we have to have a constant valued volume to integrate over while $V_U(t_0)$ depends on $t_0$ and so is in a sense time variable. It is necessary to have a value for $M_U$ so that the value of the dimensioned length multiplier $b = (R_\lambda/c)^{2/3}(2M_U G)^{1/3}$ in the radius of the universe can be considered known,

$$\begin{aligned}
r(t) &= b \sinh^{2/3}(\pm 3ct/(2R_\Lambda)) && (2.22)\\
b &= (R_\Lambda/c)^{2/3} C^{1/3} && (2.23)\\
R_\Lambda &= (3/\Lambda)^{1/2} && (2.24)\\
C &= 2M_U G. && (2.25)
\end{aligned}$$

Thus we seem to be left with the only options of finding the value of $M_U$ from experiment or just accept that it is an arbitrary dimensioned constant until some alternative route to finding its value is found. The numerical value of $M_U$ makes no difference to the theoretical structure of the theory, it only effects the numerical value of Rindler's constant, $C$, and any quantity in which this constant appears as a numerical multiplier which beside $r(t)$ the velocity of expansion $v(t)$ and the acceleration, $a(t)$, are involved. However, *importantly* for the non-linear Schrödinger equation, $H(t)$, does *not* involve the value of $M_U$,

$$H(t) = \dot{r}(t)/r(t) = (c/R_\Lambda)\coth(3ct/(2R_\Lambda)). \tag{2.26}$$

Because the integral in (2.21) is over the volume of the universe $V_U(t_0)$ which is given by

$$\begin{aligned}
\frac{M_U}{V_U(t_0)} &= \left(\frac{3}{8\pi G}\right)\left(\frac{c}{R_\Lambda}\right)^2 \sinh^{-2}\left(\frac{3ct_0}{2R_\Lambda}\right) = \rho(t_0) \\
&= \left(\frac{\rho_\Lambda^\dagger}{2}\right)\sinh^{-2}\left(\frac{3ct_0}{2R_\Lambda}\right) = \rho(t_c) = \frac{M_U}{V_U(t_c)} && (2.27) \\
M_U(t_0) &= \rho(t_0){R_\Lambda}^3 \int\int\int_{V_U(t_0)} \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})d\grave{x}d\grave{y}d\grave{z} && (2.28) \\
M_U(t_0) &= M_U(t_0)\frac{{R_\Lambda}^3}{V_U(t_0)} \int\int\int_{V_U(t_0)} \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})d\grave{x}d\grave{y}d\grave{z} && (2.29) \\
1 &= \frac{{R_\Lambda}^3}{V_U(t_0)} \int\int\int_{V_U(t_0)} \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})d\grave{x}d\grave{y}d\grave{z} && (2.30)
\end{aligned}$$

the relation, (2.30), gives a normalisation condition over physical space on a probability function density of space position variability, $\rho_{space}(\mathbf{r}) = \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})$, following by cancellation of the mass of the universe $M_U$ in the previous equation, which apparently holds from some definite time, $t_0$, at least. Thus the function $\rho_{space}(\mathbf{r})$ is just what is needed to describe the probability for finding mass at position $\mathbf{r}$, in the Schrödinger equation cosmology context at time, $t_0$. However, consistency demands that equation (2.30) holds, at least, for some specific time $t_0$. Thus we need to check out that such a time exists. From equation (2.27), we see much that we knew all along but, usefully, we see the value for the volume of the universe at time $t_c$, the time when deceleration changes to acceleration, is the obviously very constant value,

$$V_U(t_c) = \frac{M_U}{\rho_\lambda^\dagger} = \left(\frac{4\pi M_U G}{3}\right)\left(\frac{R_\Lambda}{c}\right)^2 \tag{2.31}$$

that we need to evaluate the apparently time dependent multiples integrals such as

$$1 \;=\; \frac{{R_\Lambda}^3}{V_U(t_c)}\int\int\int_{V_U(t_c)} \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})d\grave{x}d\grave{y}d\grave{z}. \tag{2.32}$$

Thus we seem very near a prescription for a usable cosmological Schrödinger equation. However, given a space dependent solution like (2.10) it is likely that the part $-\frac{\hbar^2}{2m}\nabla^2\Psi(\grave{r},t)$ of the quantum version at (1.3) would occur and this might render the cosmological quantum version not consistent with cosmology. It is by no means certain that such a complication would necessarily occur and not be handleable but certainly it can be avoided by playing safe and imposing the condition on this term as being zero as follows,

$$\frac{\hbar^2}{2m}\nabla^2\Psi(t,\grave{\mathbf{r}}) = 0. \tag{2.33}$$

This implies that the function $\grave{f}(\grave{\mathbf{r}})$ from equation (2.11) also satifies the Laplace equation,

$$\nabla^2\grave{f}(\grave{\mathbf{r}}) = 0. \tag{2.34}$$

The *Laplace* equation has a very large number of solutions. Thus there are many possible space dependent versions for the wave function, $\Psi(t,\grave{\mathbf{r}})$. Furthermore, I shall show that in spite of the cosmological Schrödinger being non-linear, the many solutions of the Laplace equation can be *linearly* superposed to produce yet more solutions. Thus although the condition (2.33) reduces the number of possibilities that might be considered for the space dependent wave function it leaves us more than enough solutions to think about for a very long time. It does have another advantage that could turn out to be important concerning a possible quantum conjugate momentum, $\hat{\mathbf{p}}_C$, for the space variable $\mathbf{r}$. this can be defined as

$$\hat{\mathbf{p}}_C = \frac{\hbar\partial}{\partial\mathbf{r}} = \hbar\nabla. \tag{2.35}$$

and this momentum exists as a result of the Laplace equation (2.33) and automatically takes the form after operating on the wave function as follows

$$\hat{\mathbf{p}}_C \Psi(t, \grave{\mathbf{r}}) = \hbar \nabla \wedge \mathbf{g}(t, \grave{\mathbf{r}}), \tag{2.36}$$

where $\mathbf{g}(t, \grave{\mathbf{r}})$ is some definite vector function of $t$ and $\grave{\mathbf{r}}$.

The wave motion followed by the dark mass dark energy time relation process can help to identify the effect that introducing position dependence has on the hyperspace vacuum. Space dependence implies the need to see this process as also space dependent. The density functions for the dark mass, dark energy and the ratio, $r_{\Lambda,DM}(t)$, of dark energy to dark mass as functions of the time only global process are respectively represented by

$$\rho(t) = (3/(8\pi G))(c/R_\Lambda)^2 \sinh^{-2}(3ct/(2R_\Lambda)) \tag{2.37}$$

$$\rho^\dagger_\Lambda = (3/(4\pi G))(c/R_\Lambda)^2 \tag{2.38}$$

$$r_{\Lambda,DM}(t) = \rho^\dagger_\Lambda/\rho(t) = 2\sinh^2(3ct/(2R_\Lambda)) \tag{2.39}$$

$$r_{\Lambda,DM}(\pm t_c) = 2\sinh^2(\pm 3ct_c/(2R_\Lambda)) = 1. \tag{2.40}$$

The *space time* dependent version for (2.37) is given simply by multiplying both sides of this equation by the space dependant contribution $\grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})$ giving

$$\rho(t, \mathbf{r}) = (3/(8\pi G))(c/R_\Lambda)^2 \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}}) \sinh^{-2}(3ct/(2R_\Lambda)) \tag{2.41}$$

$$= (\Lambda c^2/(8\pi G))\grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}}) \sinh^{-2}(3ct/(2R_\Lambda)). \tag{2.42}$$

From equation (2.42), it follows that the cosmological constant $\Lambda$ and the space dependence function can be taken together to define a *local space dependent cosmological function,* $\Lambda(\mathbf{r})$, associated with any specific solution of the Laplace equation as follows

$$\Lambda(\mathbf{r}) = \Lambda \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}}) \tag{2.43}$$

$$= \Lambda \grave{f}(\mathbf{r}/R_\Lambda)\grave{f}^*(\mathbf{r}/R_\Lambda). \tag{2.44}$$

It follows from this definition, that the mean value of the cosmological function is equal to $\Lambda$ for all solutions of the Laplace equation. In other words, the cosmological *function* is centred on Einstein's cosmological constant.

The linearity superposition of the various solutions of the Cosmological Schrödinger equation (1.8) to produce more solutions follows from (2.10) as in the following. Suppose we have two arbitrarily chosen spatially different solutions of this equation labelled with subscripts 1 and 2 as in

$$\Psi_{nl,\rho,1}(t,\grave{\mathbf{r}}) = \Psi_{nl,\rho,1}(t_0,\grave{\mathbf{r}})\exp\left(-\frac{3}{2}\int_{t_0}^{t} H(t')dt'\right) \tag{2.45}$$

$$\Psi_{nl,\rho,2}(t,\grave{\mathbf{r}}) = \Psi_{nl,\rho,2}(t_0,\grave{\mathbf{r}})\exp\left(-\frac{3}{2}\int_{t_0}^{t} H(t')dt'\right) \tag{2.46}$$

$$\begin{aligned}\Psi_{spp}(t,\grave{\mathbf{r}}) &= c_1\Psi_{nl,\rho,1}(t,\grave{\mathbf{r}}) + c_2\Psi_{nl,\rho,2}(t,\grave{\mathbf{r}}) \\ &= \Psi_{spp}(t_0,\grave{\mathbf{r}})\exp\left(-\frac{3}{2}\int_{t_0}^{t} H(t')dt'\right),\end{aligned} \tag{2.47}$$

where $c_1$ and $c_2$ are arbitrary constants and the subscript *spp* means superposed. It follows from (2.47) that any number of solutions of the cosmological Schrödiner equation can be linearly superposed to produce yet further solutions. Thus, altogether, there is vast scope to produce solutions with almost any space form whatsoever. The common feature of all the solutions is that that they all related to the common cosmological platform defined *by and with* the same time variation structure of the space constant density function of the Friedman equations. The final prescription for finding solutions to the cosmological Schrodinger equation involve the following three steps. Find any solution, $f$, to the three dimensional Laplace equation and involve in this solution one initially multiplicative arbitrary constant, $A_0$. Form the space-time wave function for this solution, $\Psi(t,\mathbf{r})$. Find the value of $A_0$ by using the probability normalisation condition and integration over the Hermitian square of $f$ over the volume of the universe at time, $t_c$. The wave function will then be completely determined. The probability density is also now fully determined via the definition $\rho_C(t,r) = \Psi(t,\mathbf{r})\Psi^*(t,\mathbf{r})$. The result will be a probability density function over space and time which is compatible with the Friedman equations from general relativity. The steps will be demonstrated in the next subsection for one typical case.

## 2.1 A Simple Example

I shall finish this paper with the simplest nontrivial example giving a universe that involves a varying space and time density. One of the simplest solutions, $f(\grave{\mathbf{r}})$, to the Laplace equation (2.34) is the sum of three variable complex numbers and just one arbitrary dimensionless constant, $A_0$,

$$\begin{aligned} f(\grave{\mathbf{r}}) &= A_0((\grave{x} + i\grave{y}) + (\grave{y} + i\grave{z}) + (\grave{z} + i\grave{x})) \\ &= A_0(\grave{x} + \grave{y} + \grave{z})(1 + i). \end{aligned} \tag{2.48}$$

$$\grave{f}^*(\grave{\mathbf{r}}) = A_0(\grave{x} + \grave{y} + \grave{z})(1 - i) \tag{2.49}$$

$$\grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}}) = 2A_0^2(\grave{x} + \grave{y} + \grave{z})^2 \tag{2.50}$$

$$= 2(A_0/R_\Lambda)^2(x + y + z)^2 = F(\mathbf{r}),\ \mathit{say}. \tag{2.51}$$

The definition (2.51) displays the formula in terms of the physical space coordinates, $x, y, z$. The normalisation condition on the probability density, (2.30), at time $t_c$ requires the following two results

$$1 = \frac{1}{V_U(t_c)} \int\int\int_{V_U(t_c)} \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}}) dxdydz \tag{2.52}$$

$$V_U(t_c) = \left(\frac{4\pi M_U G}{3}\right)\left(\frac{R_\Lambda}{c}\right)^2. \tag{2.53}$$

We need to evaluate the triple integral over the physical coordinates to find the value of the arbitrary constant $A_0$. This will be done in spherical polar coordinates with some condensations of notation used for the sin and cos functions,

$$x = r\sin(\theta)\cos(\phi) = rS_\theta C_\phi \tag{2.54}$$

$$y = r\sin(\theta)\sin(\phi) = rS_\theta S_\phi \tag{2.55}$$

$$z = r\cos(\theta) = rC_\theta \tag{2.56}$$

$$dxdydz = r^2 dr S_\theta d\theta d\phi \tag{2.57}$$

$$0 < \theta \le \pi, \quad 0 < \phi \le 2\pi, \quad 0 < r \le r(t_c) \tag{2.58}$$

$$r(t_c) = (M_U G(R_\Lambda/c)^2)^{1/3} \tag{2.59}$$

Thus the function, $F(\mathbf{r}) = \grave{f}(\grave{\mathbf{r}})\grave{f}^*(\grave{\mathbf{r}})$, in the triple integral becomes

$$F(\mathbf{r}) = 2(A_0/R_\Lambda)^2(x + y + z)^2 \tag{2.60}$$

$$= 2(A_0 r/R_\Lambda)^2(S_\theta C_\phi + S_\theta S_\phi + C_\theta)^2 \tag{2.61}$$

$$= 2(A_0 r/R_\Lambda)^2((1 + 2(S_\theta C_\phi S_\theta S_\phi + S_\theta C_\phi C_\theta + S_\theta S_\phi C_\theta)) \tag{2.62}$$

$$= 2(A_0 r/R_\Lambda)^2((1 + 2(S_\theta S_\theta S_\phi C_\phi + S_\theta C_\theta C_\phi + S_\theta C_\theta S_\phi)) \tag{2.63}$$

Introducing the further notation

$$\begin{aligned}
i_r &= r^4 dr \qquad I_r = \int_0^{r(t_c)} i_r = r^5(t_c)/5 && (2.64)\\
i_{\theta,1} &= S_\theta^3 d\theta, \qquad I_1 = \int_0^{\pi} i_{\theta,1} = \frac{4}{3} && (2.65)\\
i_{\phi,1} &= S_\phi C_\phi d\phi, \quad I_2 = \int_0^{2\pi} i_{\phi,1} = 0 && (2.66)\\
i_{\theta,2} &= S_\theta^2 C_\theta d\theta, \quad I_3 = \int_0^{\pi} i_{\theta,2} = 0 && (2.67)\\
i_{\phi,2} &= C_\phi d\phi, \qquad I_4 = \int_0^{2\pi} i_{\phi,2} = 0 && (2.68)\\
i_{\phi,3} &= S_\phi d\phi, \qquad I_5 = \int_0^{2\pi} i_{\phi,3} = 0 && (2.69)
\end{aligned}$$

the integral element and the integral can be expressed as

$$\begin{aligned}
dI &= 2(A_0 r/R_\Lambda)^2((1 + 2(S_\theta S_\theta S_\phi C_\phi + S_\theta C_\theta C_\phi + S_\theta C_\theta S_\phi))r^2 dr S_\theta d\theta d\phi \\
&= 2(A_0/R_\Lambda)^2 i_r(d\theta d\phi + 2(i_{\theta,1} i_{\phi,1} + i_{\theta,2} i_{\phi,2} + i_{\theta,2} i_{\phi,3})) && (2.70)
\end{aligned}$$

$$I(t_c) = (4/5)\left(\frac{A_0\pi}{R_\Lambda}\right)^2 r^5(t_c). \qquad (2.71)$$

The last expression for $I(t_c)$ is all that is left after integration. The normalisation condition at time $t_c$ using (2.31) can now be used to find the numerical value of $A_0$ by

$$\begin{aligned}
1 &= I/V_U(t_c) = (4/5)\left(\frac{A_0\pi}{R_\Lambda^2}\right)^2 r^5(t_c)\left(\frac{3c^2}{4\pi M_U G}\right) && (2.72)\\
&= \frac{3A_0^2\pi}{5}\left(\frac{M_U G}{c^2 R_\Lambda}\right)^{2/3} && (2.73)\\
A_0 &= \left(\frac{5}{3\pi}\right)^{1/2}\left(\frac{c^2 R_\Lambda}{M_U G}\right)^{1/3} && (2.74)
\end{aligned}$$

Thus the full solution for the wave function, the probability density and all the constants involved is as follows:

$$\Psi_{nl,\rho}(t,\grave{\mathbf{r}}) = \Psi_{nl,\rho}(t_c,\grave{\mathbf{r}}) \exp\left(-\frac{3}{2}\int_{t_c}^{t} H(t')dt'\right) \tag{2.75}$$

$$\Psi_{nl,\rho}(t_c,\grave{\mathbf{r}}) = \Psi_{nl,\rho}(t_c)\grave{f}(\grave{\mathbf{r}}) \tag{2.76}$$

$$\Psi_{nl,\rho}(t_c) = (\rho_\Lambda^\dagger)^{1/2} \tag{2.77}$$

$$\grave{f}(\grave{\mathbf{r}}) = (A_0/R_\Lambda)(x+y+z)(1+i) \tag{2.78}$$

$$\rho(t,\mathbf{r}) = 2(A_0/R_\Lambda)^2\rho_\Lambda^\dagger(x+y+z)^2 \exp\left(-3\int_{t_c}^{t} H(t')dt'\right) \tag{2.79}$$

$$\rho_\Lambda^\dagger = \frac{\Lambda c^2}{4\pi G} \tag{2.80}$$

$$A_0 = \left(\frac{5}{3\pi}\right)^{1/2}\left(\frac{c^2 R_\Lambda}{M_U G}\right)^{1/3}. \tag{2.81}$$

## 3 Conclusions

In an earlier paper, it was shown that a non-linear Shrödinger equation can be obtained from the Friedman cosmology equations which is entirely consistent with those equations. Here, the time evolution of this Schrödinger equation is examined in relation to conservation of the universe's total positive gravitational mass. This leads to the identification of a wave function for cosmology states with a definite time evolution and consequently also to a probability density for cosmology. This cosmological probability density can depend on spatial variability in addition to just the time variability of the Friedman equation structure. Consistency of the new Schrödinger equation with its originating Friedman set is achieved by restricting solutions to the condition that they satisfy the Laplace equation in hyperspace. It becomes clear that, even with this restriction, a multiple infinity of solutions remain available and applicable. The structure of this theory seems to confirm the view often expressed about the *quantum vacuum* that it is a bubbling cauldron of activity in the form of random quantum transitions, such as pair production and annihilation, between short lived virtual states of fundamental particles. The expansion of the universe can be explained in such terms as a spherical advancing and evolving wave of quantum *before and after measurement type conditions* in reverse through the expanding boundary. Just outside the expanding boundary, the vacuum chaotic states as described by the *wave function*, resourced by the multiplicity of solutions of the Laplace equation, are progressively converted from chaos to a definite gravitational form sufficient to describe the mass density that has taken up residence within the expanded boundary. The universe expansion colonises surrounding hyperspace so as to accommodate within its boundary its *conserved positive* gravitational mass with more territory and in a quantum form that can hold non-transient positive gravitational mass. Outside the universe the solution holds but remains a linear superposition of many varied chaotic transient states with mass density value centred on the value of twice Einstein's dark energy mass density $\rho_{\Lambda}^{\dagger}$.

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## References

[1] R. A. Knop et al. arxiv.org/abs/astro-ph/0309368
New Constraints on $\Omega_M$, $\Omega_\Lambda$ and $\omega$ from
an independent Set (Hubble) of Eleven High-Redshift
Supernovae, Observed with HST

[2] Adam G. Riess et al xxx.lanl.gov/abs/astro-ph/0402512
Type 1a Supernovae Discoveries at $z > 1$
From The Hubble Space Telescope: Evidence for Past
Deceleration and constraints on Dark energy Evolution

[3] Berry 1978, Principles of cosmology and gravitation, CUP

[4] Gilson, J.G. 1991, Oscillations of a Polarizable Vacuum,
Journal of Applied Mathematics and Stochastic Analysis,
**4**, 11, 95–110.

[5] Gilson, J.G. 1994, Vacuum Polarisation and
The Fine Structure Constant, Speculations in Science
and Technology , **17**, 3 , 201-204.

[6] Gilson, J.G. 1996, Calculating the fine structure constant,
Physics Essays, **9** , 2 June, 342-353.

[7] Eddington, A.S. 1946, Fundamental Theory, Cambridge University Press.

[8] Kilmister, C.W. 1992, Philosophica, **50**, 55.

[9] Bastin, T., Kilmister, C. W. 1995, Combinatorial Physics World Scientific Ltd.

[10] Kilmister, C. W. 1994 , Eddington's search for a Fundamental Theory, CUP.

[11] Peter, J. Mohr, Barry, N. Taylor, 1998,
Recommended Values of the fundamental Physical Constants,
Journal of Physical and Chemical Reference Data, AIP

[12] Gilson, J. G. 1997, Relativistic Wave Packing
and Quantization, Speculations in Science and Technology,
**20** Number 1, March, 21-31

[13] Dirac, P. A. M. 1931, Proc. R. Soc. London, **A133**, 60.

[14] Gilson, J.G. 2007, www.fine-structure-constant.org
The fine structure constant

[15] McPherson R., Stoney Scale and Large Number
Coincidences, Apeiron, Vol. 14, No. 3, July, 2007

[16] Rindler, W. 2006, Relativity: Special, General
and Cosmological, Second Edition, Oxford University Press

[17] Misner, C. W.; Thorne, K. S.; and Wheeler, J. A. 1973,
Gravitation, Boston, San Francisco, CA: W. H. Freeman

[18] J. G. Gilson, 2004, Physical Interpretations of
Relativity Theory Conference IX
London, Imperial College, September, 2004
Mach's Principle II

[19] J. G. Gilson, A Sketch for a Quantum Theory of Gravity:
Rest Mass Induced by Graviton Motion, May/June 2006,
Vol. 17, No. 3, Galilean Electrodynamics

[20] J. G. Gilson, arxiv.org/PS_cache/physics/pdf/0411/0411085v2.pdf
A Sketch for a Quantum Theory of Gravity:
Rest Mass Induced by Graviton Motion

[21] J. G. Gilson, arxiv.org/PS_cache/physics/pdf/0504/0504106v1.pdf
Dirac's Large Number Hypothesis
and Quantized Friedman Cosmologies

[22] Narlikar, J. V., 1993, Introduction to Cosmology, CUP

[23] Gilson, J.G. 2005, A Dust Universe Solution to the Dark Energy Problem, Vol. 1, *Aether, Spacetime and Cosmology*,
PIRT publications, 2007,
arxiv.org/PS_cache/physics/pdf/0512/0512166v2.pdf

[24] Gilson, PIRT Conference 2006, Existence of Negative Gravity Material, Identification of Dark Energy,
arxiv.org/abs/physics/0603226

[25] G. Lemaître, Ann. Soc. Sci. de Bruxelles
Vol. A47, 49, 1927

[26] Ronald J. Adler, James D. Bjorken and James M. Overduin 2005,
Finite cosmology and a CMB cold spot, SLAC-PUB-11778

[27] Mandl, F., 1980, Statistical Physics, John Wiley

[28] Rizvi 2005, Lecture 25, PHY-302,
http://hepwww.ph.qmw.ac.uk/∼rizvi/npa/NPA-25.pdf

[29] Nicolay J. Hammer, 2006
www.mpa-garching.mpg.de/lectures/ADSEM/SS06_Hammer.pdf

[30] E. M. Purcell, R. V. Pound, 1951, Phys. Rev.,**81, 279**

[31] Gilson J. G., 2006, www.maths.qmul.ac.uk/∼ jgg/darkenergy.pdf
Presentation to PIRT Conference 2006

[32] Gilson J. G., 2007, Thermodynamics of a Dust Universe,
Energy density, Temperature, Pressure and Entropy
for Cosmic Microwave Background http://arxiv.org/abs/0704.2998

[33] Beck, C., Mackey, M. C. http://xxx.arxiv.org/abs/astro-ph/0406504

[34] Gilson J. G., 2007, Reconciliation of Zero-Point and Dark Energies in a Friedman Dust Universe with Einstein's Lambda, http://arxiv.org/abs/0704.2998

[35] Rudnick L. et al, 2007, WMP Cold Spot, Apj in press

[36] Gilson J. G., 2007, Cosmological Coincidence Problem in an Einstein Universe and in a Friedman Dust Universe with Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*, PIRT publications, 2008

[37] Freedman W. L. and Turner N. S., 2008, Observatories of the Carnegie Institute Washington, Measuring and Understanding the Universe

[38] Gilson J. G., 2007, Expanding Boundary Pressure Process. All pervading Dark Energy Aether in a Friedman Dust Universe with Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*, PIRT publications, 2008

[39] Gilson J. G., 2007, Fundamental Dark Mass, Dark Energy Time Relation in a Friedman Dust Universe and in a Newtonian Universe with Einstein's Lambda, Vol. 2, *Aether, Spacetime and Cosmology*, PIRT publications, 2008

[40] Gilson J. G., 2008, . A quantum Theory Friendly Cosmology Exact Gravitational Waves in a Friedman Dust Universe with Einstein's Lambda, PIRT Conference, 2008