Fundamental Dark Mass, Dark Energy Time Relation in a Friedman Dust Universe and in a Newtonian Universe with Einstein's Lambda

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Abstract

In this paper, it is shown that the cosmological model that was introduced in a sequence of three earlier papers under the title *A Dust Universe Solution to the Dark Energy Problem* can be used to recognise a fundamental time dependent relational process between dark energy and dark mass. It is shown that the formalism for this process can also be obtained from Newtonian gravitational theory with only the additional assumption that Newtonian space contains a constant universal dark energy density distribution dependant on Einstein's Lambda, Λ. It thus seems that the process is independent of general relativity and applies in more contexts than just the expansion of the entire universe. It is suggested that the process can be thought of as a local space and time packaging for dark mass going through part transmutations into locally condensed visible material. The process involves a contracting and then expanding sphere of conserved dark matter. At two stages in the process at special times before and after a singularity at time zero, the spherical package goes through a condition of gravitational neutrality of very low mass density which could be identified as cosmological voids. The process is an embodiment of the principle of equivalence.

Keywords: Dust Universe, Dark Energy, Dark Mass, Friedman Equations, Zero-Point Energy, Cosmological Voids, Coincidence Problem

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1 Introduction

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [32]. The conclusions arrived at in those papers was that the dark energy *substance* is physical material with a positive density, as is usual, but with a negative gravity, -G, characteristic and is twice as abundant as has usually been considered to be the case. References to equations in those papers will be prefaced with the letter A, B and C respectively. The work in A, B and C, and the application here have origins in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7],[25],[3]). Other useful sources of information are ([17],[3],[30],[27],[29],[28]) with the measurement essentials coming from references ([1],[2],[11]). Further references will be mentioned as necessary.

Dark Mass Dark Energy Relation

The work to be described in this paper is an application of the cosmological model introduced in the papers *A Dust Universe Solution to the Dark Energy Problem* [23], *Existence of Negative Gravity Material. Identification of Dark Energy* [24] and *Thermodynamics of a Dust Universe* [32]. Further references will be mentioned as necessary. Application of the cosmological model introduced in the papers A [23], B,[24] and C, [32], is to be found in the paper D, ([34]), to the extensively discussed and analysed *Cosmological Constant Problem*. Here a relation between dark mass and dark energy over time will be obtained and analysed.

2 Cosmological Vacuum Polarisation

Consider the result for gravitational vacuum polarisation derived in paper (D)

$$G\rho_\Lambda = G_-\Gamma_B(t) + G_+\Delta_B(t) \quad (2.1)$$
$$0 = G_-\Gamma_Z(t) + G_+\Delta_Z(t), \quad (2.2)$$

where $G_- = -G$ and $G_+ = G$. The upper case Greek functions $\Gamma_B(t)$, $\Delta_B(t)$, $\Gamma_Z(t)$ and $\Delta_Z(t)$ are defined from the equations of state for Δ and Γ substances which together are assumed to form all the time conserved material of the universe,

$$
\begin{aligned}
P_{\Delta B}/c^2 = \rho_{\Delta B,\nu_c}(t)\omega_\Delta(t) &= \Delta_B(t) \quad (2.3)\\
P_{\Gamma B}/c^2 = \rho_{\Gamma B,\nu_c}(t)\omega_\Gamma(t) &= \Gamma_B(t) \quad (2.4)\\
P_{\Delta Z}/c^2 = \rho_{\Delta Z,\nu_c}(t)\omega_\Delta(t) &= \Delta_Z(t) \quad (2.5)\\
P_{\Gamma Z}/c^2 = \rho_{\Gamma Z,\nu_c}(t)\omega_\Gamma(t) &= \Gamma_Z(t). \quad (2.6)
\end{aligned}
$$

The Z subscript above denotes zero-point values. Let us now consider the Einstein cosmological constant, Λ, in relation to the Friedman equations,

$$
\begin{aligned}
8\pi G\rho r^2/3 &= \dot{r}^2 + (k - \Lambda r^2/3)c^2 \quad (2.7)\\
-8\pi G P r/c^2 &= 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2. \quad (2.8)
\end{aligned}
$$

Einstein introduced a physical explanation for his Λ term by associating it with a density of what is nowadays called *dark energy* in the form of an additional *mass* density, ρ_Λ, where $\rho_\Lambda = \Lambda c^2/(8\pi G)$. Thus with this density the Friedman equations can be written with the Hubble function of epoch time $H(t)$ as,

$$
\begin{aligned}
8\pi G\rho r^2/3 &= \dot{r}^2 + (k - 8\pi G\rho_\Lambda r^2/3)c^2 \quad (2.9)\\
-8\pi G P r/c^2 &= 2\ddot{r} + \dot{r}^2/r + (kc^2/r - 8\pi G\rho_\Lambda r) \quad (2.10)\\
H(t) &= \dot{r}(t)/r(t) = (c/(R_\Lambda))\coth(3ct/(2R_\lambda)). \quad (2.11)
\end{aligned}
$$

Thus the first friedman equation can be expressed as

$$
\begin{aligned}
8\pi G(\rho + \rho_\Lambda)/3 &= H^2(t) + (kc^2/r^2) \quad (2.12)\\
8\pi G\rho_E^T &= 3(H^2(t) + kc^2/r^2) \quad (2.13)\\
\rho_E^T &= \rho + \rho_\Lambda, \quad (2.14)
\end{aligned}
$$

where ρ_E^T is the total density for mass at points within the boundary of the universe as perceived by Einstein. Rearranging the first Friedman equation, we have

$$
\begin{aligned}
8\pi G(\rho + \rho_\Lambda) - 3(kc^2/r^2) &= 3H^2(t) \quad (2.15)\\
\frac{8\pi G\rho}{3H^2(t)} + \frac{8\pi G\rho_\Lambda}{3H^2(t)} - \frac{kc^2}{r^2H^2(t)} &= 1. \quad (2.16)
\end{aligned}
$$

The three Omegas which the astronomers use to display their measurements are defined using the three terms on the left hand side of (2.16) according to which they have to add up to *unity*,

$$\begin{aligned} \Omega_M(t) &= 8\pi G\rho/(3H^2(t)) && (2.17) \\ \Omega_\Lambda(t) &= 8\pi G\rho_\Lambda/(3H^2(t)) && (2.18) \\ \Omega_k(t) &= -kc^2/(r^2 H^2(t)) && (2.19) \\ \Omega_M(t) + \Omega_\Lambda(t) + \Omega_k(t) &= 1. && (2.20) \end{aligned}$$

There is a very strong case (A,B,C,D,E) for identifying the dark energy mass density that should account for Einstein's constant Λ term as given by twice the density introduced by Einstein,

$$\rho_\Lambda^\dagger = 2\rho_\Lambda \quad (2.21)$$

$$\rho^{T\dagger} = \rho + \rho_\Lambda^\dagger \quad (2.22)$$

and this implies the formula (2.22) for the total amount of physical mass density within the boundaries of the spherical universe in contrast with (2.14). Thus equation (2.15) should be replaced by

$$8\pi G(\rho + \rho_\Lambda^\dagger) - 3(kc^2/r^2) = 3H^2(t) + 8\pi G\rho_\Lambda \quad (2.23)$$

$$\frac{8\pi G\rho}{3H^2(t) + c^2\Lambda} + \frac{8\pi G\rho_\Lambda^\dagger}{3H^2(t) + c^2\Lambda} - \frac{3kc^2}{r^2(3H^2(t) + c^2\Lambda)} = 1. \quad (2.24)$$

Thus we now have three new Omegas

$$\begin{aligned} \Omega_M^\dagger(t) &= 8\pi G\rho/(3H^2(t) + c^2\Lambda) && (2.25) \\ \Omega_\Lambda^\dagger(t) &= 8\pi G\rho_\Lambda^\dagger/(3H^2(t) + c^2\Lambda) && (2.26) \\ \Omega_k^\dagger(t) &= -k3c^2/(r^2(3H^2(t) + c^2\Lambda)) && (2.27) \\ \Omega_M^\dagger(t) + \Omega_\Lambda^\dagger(t) + \Omega_k^\dagger(t) &= 1. && (2.28) \end{aligned}$$

Here I shall be mostly concerned with the flat space case $k = 0$ so that the two possible and equivalent sets of Omegas satisfy the relations

$$\begin{aligned} \Omega_M(t) + \Omega_\Lambda(t) &= 1 && (2.29) \\ \Omega_M^\dagger(t) + \Omega_\Lambda^\dagger(t) &= 1. && (2.30) \end{aligned}$$

Inspection of the formulae for $H(t)$, $\Omega_M(t)$ and $\Omega_\Lambda(t)$ shows that $\Omega_\Lambda(t)$ varies between 0 and 1 as t varies between 0 and ∞ and consequently from

(2.29), $\Omega_M(t)$ varies between 1 and 0. It follows that there will be a time when

$$\Omega_M(t_0) = 1/4 \tag{2.31}$$
$$\Omega_\Lambda(t_0) = 3/4 \tag{2.32}$$

and this event will happen regardless of any measurements. I have assumed that the epoch time of this event in the history of the universe is given by t_0. Thus the usual use of the subscript 0 to denote *time now* has been abandoned and *time now* will in future be denoted by $t^\dagger$. The corresponding and more realistic time t_0 relation between non-dark energy materials and dark energy will with a simple calculation be represented in terms of the dagger Omegas by

$$\Omega^\dagger_M(t_0) = 1/7 \tag{2.33}$$
$$\Omega^\dagger_\Lambda(t_0) = 6/7. \tag{2.34}$$

This implies that about 85.7% of the universe mass is dark energy rather than the usually assumed 75%, a *substantially* changed assessment. If this assessment of the percentage of dark energy to conserved mass is accepted, it will also have some effect on the amount of *visible mass* assumed to be present within the total mass of the universe. The ratio dark mass to visible mass is often taken to be 4 to 1. Thus the percentage of dark mass in the universe according to (2.33) and (2.34) would become reduced to $20 \times (4/7)\% \approx 11.44\%$. The total non-visible mass would then be $85.7\% + 11.44\% \approx 97.14\%$ leaving us with being able to see just about 2.86% of the total mass. If it is taken that we know nothing about the dark elements, as is often suggested, then our actual knowledge of the universe is mass wise abysmal. However, fortunately it is not true that we have *no* knowledge of the dark elements. We do have indirect knowledge of these aspects. The theory associated with this model give a definite relation between dark energy and dark mass this relation can be read off from the gravitation polarisation equations (2.1, 2.2) repeated next

$$G\rho_\Lambda = G_-\Gamma_B(t) + G_+\Delta_B(t) \tag{2.35}$$
$$0 = G_-\Gamma_Z(t) + G_+\Delta_Z(t) \tag{2.36}$$
$$\rho(t) = \rho_{\Delta,\nu_c} + \rho_{\Gamma,\nu_c}. \tag{2.37}$$

The third equation above expresses the total time conserved density $\rho(t)$ in terms of the CMB mass density, ρ_{Γ,ν_c} , and the rest of the universe mass density ρ_{Δ,ν_c}. The ν_c subscript indicates that zero point energies are included in these terms. The second equation above defines the zero-point energy of the dark energy as being zero, effectively defining energy zero for this cosmology theory. The total energy density for this model equation (2.22) can thus be written as (2.41)

$$
\begin{aligned}
\rho_\Lambda^\dagger &= 2\rho_\Lambda &(2.38)\\
\rho^{T\dagger}(t) &= \rho(t) + 2\rho_\Lambda &(2.39)\\
\rho^{T\dagger}(t) &= \rho_{\Delta,\nu_c} + \rho_{\Gamma,\nu_c} + 2(\Delta_B(t) - \Gamma_B(t)) &(2.40)\\
\rho^{T\dagger}(t) &= \rho_{\Delta,\nu_c} + 2\Delta_B(t) + \rho_{\Gamma,\nu_c} - 2\Gamma_B(t) &(2.41)\\
\rho^{T\dagger}(t) &= \tilde{\rho}_{\Delta,\nu_c} + \tilde{\rho}_{\Gamma,\nu_c} &(2.42)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{\rho}_{\Delta,\nu_c} &= \rho_{\Delta,\nu_c} + 2\Delta_B(t) &(2.43)\\
\tilde{\rho}_{\Gamma,\nu_c} &= \rho_{\Gamma,\nu_c} - 2\Gamma_B(t). &(2.44)
\end{aligned}
$$

The tilde versions of the basic two densities are the resultants of a gravitational vacuum polarisation process in which the basic Γ and Δ densities induce, via their pressures and coexistence, the two polarisation densities $2\Gamma_B(t)$ and $2\Delta_B(t)$ which together represent the dark energy density ρ_Λ, equation (2.35). This process takes place through the equations of motion of the two components. Thus from this point of view dark energy within the universe boundary is a vacuum polarisation consequence of the of the existence of the basic Γ and Δ fields in interaction under general relativity. The dark energy density also exists outside the universe boundary but in an un-polarised condition. Thus the polarisation within the universe is constrained by the constant value that exists everywhere. To examine the weight of this gravitational vacuum polarisation on the none polarised fields separately at time $t^\dagger$ using the numerical results from (A,B,C)

$$
\begin{aligned}
2\omega_\Delta(t^\dagger) &\approx 6 &(2.45)\\
2\omega_\Gamma(t^\dagger) &= 2/3 &(2.46)
\end{aligned}
$$

they must be expressed in terms off the none polarised fields as in (2.47) and (2.48)

$$
\begin{aligned}
2\Delta_B(t^\dagger) &\approx 6\rho_{\Delta B,\nu_c}(t^\dagger) && (2.47)\\
2\Gamma_B(t^\dagger) &= (2/3)\rho_{\Gamma B,\nu_c}(t^\dagger) && (2.48)\\
\rho_{\Delta B,\nu_c}(t^\dagger) &\approx (10^4/1.9)\rho_{\Gamma B,\nu_c}(t^\dagger) && (2.49)\\
\rho_{\Gamma B,\nu_c}(t^\dagger) &\approx 1.9\times 10^{-4}\rho_{\Delta B,\nu_c}(t^\dagger) && (2.50)\\
\rho_\Lambda &= \Lambda c^2/(8\pi G) \approx 7.3\times 10^{-27} && (2.51)\\
\rho_{\Gamma B,\nu_c}(t^\dagger) &= aT^4(t^\dagger) \approx 4.66\times 10^{-31}. && (2.52)
\end{aligned}
$$

The relation (2.49) also comes from (A,B,C). Thus we can express the positively weighted Δ_B and negative weighted Γ_B vacuum polarisation density poles as

$$
\begin{aligned}
2\Delta_B(t^\dagger) &\approx (6\times 10^4/1.9)\rho_{\Gamma B,\nu_c}(t^\dagger) && (2.53)\\
2\Gamma_B(t^\dagger) &\approx (2/3)1.9\times 10^{-4}\rho_{\Delta B,\nu_c}(t^\dagger) && (2.54)\\
2\Delta_B(t^\dagger) &\approx 3\times 10^4\rho_{\Gamma B,\nu_c}(t^\dagger) && (2.55)\\
2\Gamma_B(t^\dagger) &\approx 1.26\times 10^{-4}\rho_{\Delta B,\nu_c}(t^\dagger). && (2.56)
\end{aligned}
$$

Returning to the gravitational vacuum polarisation equation (2.1) repeated here for convenience,

$$
\begin{aligned}
G\rho_\Lambda &= G_-\Gamma_B(t) + G_+\Delta_B(t) && (2.57)\\
0 &= G_-\Gamma_Z(t) + G_+\Delta_Z(t), && (2.58)
\end{aligned}
$$

we can do a spot numerical check using the values above and without the G factor as follows

$$
\begin{aligned}
7.3\times 10^{-27} &\approx \rho_\Lambda = \Delta_B(t^\dagger) - \Gamma_B(t^\dagger) && (2.59)\\
&= \rho_\Delta\omega_\Delta - \rho_\Gamma\omega_\Gamma && (2.60)\\
&\approx (3\times 10^4 - (1/3))\rho_\Gamma && (2.61)\\
&\approx (3\times 10^4)\rho_\Gamma && (2.62)\\
&\approx (3\times 10^4)\times 4.66\times 10^{-31} && (2.63)\\
&\approx (13.98/1,9)\times 10^{-27} && (2.64)\\
&\approx 7.3\times 10^{-27}. && (2.65)
\end{aligned}
$$

This is just a rough check that does give a good though approximate result while showing that the induced Δ and induced Γ fields in the form of a difference are the *source* of the dark energy density within the universes boundaries. At step (2.61), the $-1/3$ term from the Γ field is abandoned because it contributes negligibly in relation to the 10^4 from the Δ term. However, at step (2.62) the Γ field only appears to be a main contributor because it occurs as multiplicatively weighted by the Δ factor, 10^4. As the Δ field is all the conserved universe field density less the CMB the induced delta field Δ is all the induced conserved density universe field less the induced CMB field. The Δ field includes the so called *dark matter* as its major contributor of about 80% with normal visible mass making a smaller percentage of about a 20% contribution. Thus the important conclusion is that *dark energy* value within the universe is a direct consequence of the induced mass from the Δ field which itself is largely *dark mass*. Briefly, dark energy within the universe is numerically very close in value to the *vacuum polarised* dark mass and if the Γ field is also classified as dark the closeness becomes coincidence. From the preceding discussion and equation (2.57) it should not be inferred that dark mass is a primary source of dark energy. I think the reverse is nearer to the truth and equation (2.57) is the direct result of a mechanical equilibrium between pressure equivalent induced density from the CMB and the sum of the pressure induced densities from the Δ and Λ field at the boundary and within the universe. Thus this mechanical equilibrium effectively transfers the dark energy pressure from outside the universe to its boundary and hence by homogeneity to inside the universe. The $PEID$ concept will be explained in the next section on pressure equivalent induced densities.

3 Pressure Equivalent Induced Density, PEID

It turns out to be very useful to introduce the concept of *Pressure Equivalent Induced Density, PEID*, in relations to the equations of state associated with specific subsystems of the total system. For example, suppose one subsystem is called the Δ system with the equation of state,

$$P_\Delta(t) = c^2\rho_\Delta\omega_\Delta(t) \tag{3.1}$$

$$\Delta(t) = \rho_\Delta(t)\omega_\Delta(t) \tag{3.2}$$

$$= P_\Delta(t)/c^2, \tag{3.3}$$

then I take the definition for the $PEID$, $\Delta(t)$, to be given by equation (3.2). Thus $\Delta(t)$ has the same dimensions as density because in common with all the omegas, $\omega_\Delta(t)$, is dimensionless and it is derived from $\rho_\Delta(t)$ through the multiplicative action of the inducing function, $\omega_\Delta(t)$. From (3.3) it is clearly essentially a pressure with the dimensions of density. It represents this pressure in the form of the *mass density*, $\Delta(t)$. I am not aware that the $PEID$ slant on equations of state has any important part elsewhere in physics but it seems that it does play an essential role in cosmology in relation to the understanding of dark energy and its connection to other key densities. This is clear from inspection of equation (2.1) again with and without the G weightings,

$$G\rho_\Lambda = G_-\Gamma_B(t) + G_+\Delta_B(t) \tag{3.4}$$

$$\rho_\Lambda = \Delta_B(t) - \Gamma_B(t). \tag{3.5}$$

Thus from equation (3.5) the source of dark energy density within the universe is just the difference of the $PEID$s for the Δ and Γ fields which together constitute all the conserved mass of the universe. Thus the mystery of the origin of the dark energy density, $\rho_\Lambda = \Lambda c^2/(8\pi G)$ in Einstein's form or in my revised form $\rho_\Lambda^\dagger = 2\rho_\Lambda$, within the universe is completely resolved by this theory. Possibly this is the reason that dark energy is not visible. It could be because *pressures* are not usually visible and the *pressure status* of the dark energy density is its dominant characteristic. However, it seems to me that dark energy with approximately an equivalent density of 5 hydrogen atoms per cubic meter would not be visible anyway. The formula (3.5) can also be used to show a simple relation between *dark mass* and *dark energy* but before discussing that aspect it is useful to consider in the next paragraph the way this theory structure has developed and can continue developing. In the first two papers, *A and B* of the four *A,B,C,D*, I found the dust universe model from scratch by just integrating the Friedman equations. The result subsequently turned out to be a reincarnation of the first model introduced by *Lemaître* [25] but with substantially different interpretations and additional details. The version of the model in *A and B*, like most cosmological models, involved the assumption that the mass density of the universe only depended on time and so was space-wise homogeneous. However, the structure unearthed in that version of the model was completely adequate to describe cosmological expansion and its change from deceleration to acceleration at some time t_c in the past and various other new understandings of the cosmological process, all in complete agreement with up to date measurement. Thus this basic structure did not depend on differentiating the mass density into separate components to represent various contributory fields such as the electromagnetic or heavy particle contributions. The dark energy contribution was involved in that version of the theory but not included as part of the conserved mass of the universe, it was rather treated as a permanent constant density resident of the hyperspace into which the universe expands. I shall here denote that model by $U_0 = U_\Lambda(DM)$, meaning that it can be assumed to *only* contain an *energy conserved over all time* quantity of *dark mass*, M_U, while, as we have seen, it swims in and is permeated with the dark energy content of an enveloping 3D-hyperspace. The conserved mass density, $\rho(t) \sim \Omega_M(t)$, in this model *must* represent all the dark mass if we assume that none of this dark mass has converted into visible mass and further because it satisfies the equation (2.29) which has to add up to unity to ensure that fact. Thus the model $U_\Lambda(DM)$, can be conceived as not containing any visible hadronic matter, which as we know can only be present in a very small proportion anyway and it would also likely be none uniformly distributed. It follows that the model $U_0 = U_\Lambda(DM)$ can be regarded as a very bland, over all time, approximation to the actual universe and which can be built up in stages to represent the universe with increasing accuracy. I emphasise the usual cosmological basic assumption that the model's density function is space-wise homogeneous means that if the model contains any dark mass within its boundaries then it contains *only uniform* dark mass and together with the uniformly distributed dark energy background. The next stage in the build up process in which the cosmic microwave back ground was added was published in C and will be denoted by $U_1 = U_\Lambda(DM = \Delta(t) \cup \Gamma(t))$. This means that the fixed amount of dark mass in the first version is now able to transform into time dependent components $\Delta(t)$ for one part and $\Gamma(t)$ for the complementary part, the CMB, with the same total mass quantity as the original dark mass. The next stage of complexity is the introduction of the possibility that part of the Δ mass, M_U can transform into visible mass, often called hadronic mass. This universe can be represented by $U_2 = U_\Lambda(DM = (\Delta(t) = \Delta_D(t) \cup \Delta_V(t)) \cup \Gamma(t))$ with now the quantity of Δ mass being shared between the dark and visible versions as denoted by the D and V subscripts. Clearly the increasing complexity procedure can continue to produce universes with lower homogeneity described by U_3 and so on. Let us now return to discussing the relation between dark mass and dark energy.

4 Dark Mass, Dark Energy Ratio

Consider firstly the basic universe type Friedman dust universe, U_0. The model in this basic case is an excellent representation of the modern astronomical measurements. However the basic density function is assumed to be rigorously homogeneous and contains only conserved with time dark mass and the hyperspace permeating dark energy. The density functions for the dark mass, dark energy and the ratio, $r_{\Lambda,DM}(t)$, of dark energy to dark mass as functions of time are respectively represented by

$$\begin{aligned} \rho(t) &= (3/(8\pi G))(c/R_\Lambda)^2 \sinh^{-2}(3ct/(2R_\Lambda)) && (4.1) \\ \rho_\Lambda^\dagger &= (3/(4\pi G))(c/R_\Lambda)^2 && (4.2) \\ r_{\Lambda,DM}(t) &= \rho_\Lambda^\dagger/\rho(t) = 2\sinh^2(3ct/(2R_\Lambda)) && (4.3) \\ r_{\Lambda,DM}(\pm t_c) &= 2\sinh^2(\pm 3ct_c/(2R_\Lambda)) = 1. && (4.4) \end{aligned}$$

Equation (4.3) is a general result but in the case of a U_0 universe it can be expressed differently by using equation (3.5) with the Γ term taken zero as

$$\begin{aligned} \rho_\Lambda &= \Delta_{B,0}(t) && (4.5) \\ &= \rho(t)\omega_{\Delta,0}(t), && (4.6) \end{aligned}$$

the zero subscripts having been added to differentiate the functions concerned from those in the U_1 version. From paper C, we know that

$$\omega_\Delta(t) = \left(\frac{M_\Gamma}{3M_U} + \frac{3(c/R_\Lambda)^2\rho^{-1}(t)}{8\pi G}\right)/(1 - M_\Gamma/M_U). \qquad (4.7)$$

Thus the zero Γ version for U_0 is given by

$$\omega_{\Delta,0}(t) = \left(\frac{3(c/R_\Lambda)^2\rho^{-1}(t)}{8\pi G}\right). \qquad (4.8)$$

Substituting this into equation (4.6) confirms the validity of (4.6). Thus the rather trivial equation (4.6) gives the *all* time dependent relation between dark energy and dark mass for the nontrivial model U_0. However, trivial or not, the dark energy and dark mass densities are *strongly* numerically related through the function $\omega_{\Delta,0}(t)$ and this applies for all time, $(-\infty < t < +\infty)$. Let us now consider the ratio, $r_{\Lambda,DM}(t)$, of dark energy to dark mass in the case of a universe in which the homogeneity has been broken by the addition of the cosmic microwave background, replacing some by the CMB. From (4.3), we have generally,

$$r_{\Lambda,DM}(t) = \rho_\Lambda^\dagger/\rho(t) = 2\sinh^2(3ct/(2R_\Lambda)). \qquad (4.9)$$

However, with the addition of the Γ field

$$\rho(t) = \rho_{\Delta}(t) + \rho_{\Gamma}(t) \tag{4.10}$$

so that the dark energy dark mass ratio of U_0 at (4.9) becomes in U_1

$$r_{\Lambda,DM,1}(t) \;\; = \;\; \frac{\rho_{\Lambda}^{\dagger}}{\rho_{\Delta}(t) + \rho_{\Gamma}(t)} = 2\sinh^2(3ct/(2R_{\Lambda})). \tag{4.11}$$

The denominator of the ratio remains unchanged as also does the second equality because the numerical values are unchanged. It might be thought that the left and right sides of the first equality do not now agree because only the Δ part contains dark mass, that which is left from the U_0 universe case after some has converted to CMB. Numerically there is no problem as the quantity of dark mass is presumably shared between the Δ and Γ fields. However, the terminology might be questioned. *Arguably*, the CMB is composed of photons which are not visible and therefore the CMB can be classified as *dark mass* equivalent material. Of course photons convey information about *other visible* materials to the eye but photons themselves are not *seen* in the usual meaning of the word. I have added the extra subscript 1 in the U_1 ratio so that no confusion can arise if the case I have just made is not accepted. The dark energy dark mass ratio in either form above represents a *fundamental* time conditioned relation between dark mass and dark energy. This result and the formula (3.5) both of which hold inside and on the boundary of the universe show how totally interdependent are the two *dark* facets. The ratio $r_{\Lambda,DM}(t)$ is of great generality and could play an important part in helping to understand cosmological *voids*, a recent astronomical discovery. This ratio has come out of general relativity but it can be shown that it is independent of general relativity and its existence only depends on some simple assumptions added to Newtonian gravitational theory. The very basic and major significance of this ratio will be discussed and demonstrated in the next section by showing that it is directly derivable from Newtonian gravitational theory. It will be indicated how this implies a context for its significance within smaller regions of space within the universe's boundary.

5 Newtonian Dark Mass and Dark Energy

Consider an infinitely extended 3-dimensional Euclidean space such as that in which Newtonian gravity is usually considered to act between objects having the physical characteristic called mass. I shall make the usual assumption that Newtonian gravity acts between enclosed regions of space of spherical shape that enclose a *uniform* density distribution of mass that can change with time but retaining an overall *fixed* quantity with respect to time of the usual positive gravitational mass within it boundary, an amount M, say. Usually there will be some moving gravitational centroid at which the gravitation force between objects will be thought to be acting. I also only use configurations in which this centroid is the centre of a sphere. The difference from Newtonian theory that I am about to introduce is the assumption that this Euclidean space is filled uniformly throughout all its extent by a positively mass density field of negatively characterised gravitational material such as the dark energy found to exist in the cosmos. This negative gravity material will be denoted by the constant density, $\rho^{\dagger}_{\Lambda} = c^2\Lambda/(4\pi G)$ just as in my double version of the Einstein theory quantity, $\rho_{\Lambda} = c^2\Lambda/(8\pi G)$. Consider now a spherical region of this space of radius r about the origin of this space as centre. Suppose this sphere contains a total amount of *dark mass*, M, with its positive gravitation characteristic, G. The sphere will also contain an amount of negative gravity, $-G$, dark energy given by

$$M_{\Lambda} = \rho^{\dagger}_{\Lambda} V(t) \quad (5.1)$$

$$V(t) = 4\pi r^3(t)/3. \quad (5.2)$$

Thus the total gravitational acceleration caused by the sphere's contents at its surface will be given by the Newtonian gravitational formula,

$$\ddot{r}(t) = M^{\dagger}_{\Lambda} G/r^2(t) - MG/r^2(t) \quad (5.3)$$

$$= 4\pi r^3 \rho^{\dagger}_{\Lambda} G/(3r^2) - C/(2r^2) \quad (5.4)$$

$$= 4\pi r \rho^{\dagger}_{\Lambda} G/3 - C/(2r^2) \quad (5.5)$$

$$= rc^2\Lambda/3 - C/(2r^2). \quad (5.6)$$

If we multiply equation (5.5) through by $\dot{r}$, we obtain

$$\ddot{r}\dot{r} = 4\pi r\dot{r}\rho^{\dagger}_{\Lambda} G/3 - C\dot{r}/(2r^2) \quad (5.7)$$

$$\frac{d}{dt}\dot{r}^2/2 = \frac{d}{dt}r^2\Lambda c^2/6 - C\frac{d}{dt}r^{-1}/2 \quad (5.8)$$

$$\dot{r}^2 = (rc)^2\Lambda/3 + Cr^{-1} \quad (5.9)$$

$$C = 2MG. \quad (5.10)$$

The constant of integration that could occur in integrating (5.8) can be taken to be zero under the conditions that $\dot{r}(t)$ is taken to be infinite with $r(t) = 0$ at $t = 0$. Thus the spherical region expands with high speed from the origin, $r = 0$ at time $t = 0$. The solution to equation (5.9) was obtained in paper A in the form

$$\begin{aligned}
r(t) &= b \sinh^{2/3}(3ct/(2R_\Lambda)) && (5.11)\\
R_\Lambda &= (3/\Lambda)^{1/2} && (5.12)\\
b &= (R_\Lambda/c)^{2/3} C^{1/3} && (5.13)\\
C &= 2MG && (5.14)
\end{aligned}$$

where M here is any *dark mass* value. It follows that the *dark* mass density of the spherical region containing total dark mass, M, is as in (4.1) given by

$$\begin{aligned}
\rho(t) &= M/(4\pi r^3(t)/3) = M \sinh^{-2}(3ct/(2R_\Lambda))/b^3 && (5.15)\\
&= (3/(8\pi G))(c/R_\Lambda)^2 \sinh^{-2}(3ct/(2R_\Lambda)). && (5.16)
\end{aligned}$$

Thus the ratio of dark energy density to dark mass density within this region over time is

$$r_{\Lambda,DM}(t) = \rho_\Lambda^\dagger/\rho(t) = 2\sinh^2(3ct/(2R_\Lambda)) \quad (5.17)$$

which again is the same as (4.3). The formula for the ratio of dark energy to dark mass, $r_{\Lambda,DM}(t)$, depends only on the dark mass density through t and R_Λ. The time variable origin $t = 0$ depends only on where the sphere expansion is assumed to have started from with radius zero, an arbitrarily chosen space origin $r(t) = 0$ at time $t = 0$, in Euclidean three space. Thus it seems that this is a *fundamental* formula governing a time evolutionary process relating dark energy and dark mass. The consequence of this situation is that we can visualise, quite independently of relativity, such mixed mass region expansions. They can take place over time from anywhere in astro-space and *apparently* originate from a point quantity of *dark mass*, M, with infinite density. Further, the formula is time reversible so that it suggests that spherical contractions of spherical dark mass regions can also be visualised as a possible cosmological sequence of events resulting in the appearance of a point dark mass, M, with infinite density locally. As such an expansion proceeds the spherical region picks up dark energy mass from the enveloping Newtonian space, the expansion continuing with the expanding region having then a mixture of the two gravitational types of mass, $\pm G$. An important event in the history of such an expansion is when there are equal quantities of the two mass types within the sphere. At this event occurring, the sphere will be gravitationally neutral. The sphere will at that time exert no gravitational force on material outside its boundary, it will be gravitationally isolated from any material exterior to itself. If we denote the time when the sphere is so isolated by t_c this time can be found from the formula of dark mass and dark energy mass equivalent equality, either equation (5.18) or equation (5.19)

$$r_{\Lambda,DM}(t_c) = \rho^{\dagger}_{\Lambda}/\rho(t_c) = 1 \quad (5.18)$$

$$\rho^{\dagger}_{\Lambda} = \rho(t_c) \quad (5.19)$$

$$\sinh^2(3ct_c/(2R_\Lambda)) = 1/2 \quad (5.20)$$

$$\Rightarrow t_c = \pm(2R_\Lambda/(3c))\sinh^{-1}(1/2^{1/2}) \quad (5.21)$$

and, curiously, the times $\pm t_c$ do not depend on the amount of dark mass within the expanding sphere but only depends on the value of the cosmological constant, Λ. It follows that the time t_c has exactly the same value as the relativistic epoch time when the universe changes from deceleration to acceleration. The time t_c is a *fundamental* universal time interval in the cosmological context. It is important to note that, as the process is time reversal invariant, the contraction sequence, in negative time, with mass M can be immediately followed by an expansion sequence with the same mass M, in positive time, so that conservation of mass is assured and mass is neither created from nothing nor is it destroyed at the singular event when $t = 0$. The non dependence of the process on the amount of dark mass within the boundary of the contracting or expanding sphere of dark mass has a surprising explanation. The process conforms exactly to the *principle of equivalence*. Just as the acceleration of a falling mass in a gravitational field does not depend on the value of the falling mass so the acceleration $\ddot{r}_{\Lambda,DM}(t)$ of the collapsing sphere process does not depend on its mass. The collapsing sphere in its own gravitational field conforms exactly too and is a manifestation of the principle of equivalence. It can occur locally and is a basic part of the description of the whole universe motion with epoch time. Recognition of this fundamental process in relation to other physical processes in cosmology will be discussed in the final section.

6 Conclusions

The cosmological model introduced in references A, B, C and applied to the finding of solutions to the *cosmological constant* problem in D has here been applied to unravelling the *dark mass* problem. Here it has been shown that a fundamental time moving relation holds between *dark energy* and *dark mass*. This relation was first shown to hold at the scale of the whole universe by using the Friedman equations from Einstein's general relativity and involving his positively valued cosmological constant Λ. Here it has been shown that the same relation can be derived from Newtonian gravitation theory with only the addition of a constant and universally distributed density of dark energy, $\rho_{\Lambda}^{\dagger} = 2\rho_{\Lambda}$, twice the Einstein value ρ_{Λ}, in Newtonian space and only subject to Newtonian gravity theory. This result implies that the formula relating dark mass and dark energy is independent of general relativity and the way it is derived also show that it can have applications at a much smaller scale than that of the entire universe. It can describe local space and time *small scale* movements of dark mass in relation to dark energy. Thus I suggest the formula could play an important role in explaining the way that dark mass, if taken to be primary positive gravity, $+|G|$, mass, can condense, precipitate or clump to become galaxies or just empty voids [35] in the cosmological fabric. As we have seen, there are five main events in the time sequences evolution of this dark energy dark mass process, $E_0, E_{\pm 1}, E_{\pm\infty}$, say. They involve E_0 when some definite random quantity of dark mass M is located at some definite point in three space at some definite time labelled as $t = 0$ for the process. At that time the dark mass is by itself because a point cannot contain any of the uniform and finite constant density of dark energy mass. Thus in space around the point mass it will own a Newtonian gravitational potential field $-MG/r$. At both the events $E_{\pm 1}$ at times $\pm t_c$ because of the time reversal invariance the contracting or expanding sphere will contain equal quantities of the dark mass and dark energy so that the sphere will be gravitationally neutral. It will thus be isolated gravitationally and so not own any gravitational potential. However the total mass density within the spheres boundaries will be $\rho(t_c) + \rho_{\Lambda}^{\dagger}$, a numerically very small value ≈ 9 proton masses per cubic meter. I think that such a sphere being gravitationally isolated and of such low density could qualify for the title *cosmological void*. At the events $E_{\pm\infty}$, the sphere will own a gravitation potential at points within its surface involving both the dark energy and dark mass within concentric spheres of radius $r < \infty$ but dominated by the repulsive dark mass for relatively large values of r. The contraction phase between $E_{-\infty}$ and E_0 might represent a moving platform for an original dark mass concentration to convert from pure dark mass to becoming dark mass contaminated with visible mass while its volume descends to occupying some relatively small region containing a group of visible galaxies or, a single galaxy or even a single particle. In other words, the descending spherical volume could represent a time dependant packaging process for cosmological clumping. A final remark about the relation of this theory structure to *aether theory* is appropriate. It is dark energy rather than dark mass that seems to play a role much like the all pervading aether which has been used to give a physical explanation for electromagnetic wave motion in so called *empty* space. The dark energy density is certainly an all-pervading effect in this cosmological theory as has been shown in this article and as it is also perceived in the present day arena of astronomical observations. It seems to be an everywhere present background reference level against which many astrophysical and quantum problems can be understood and measured. The dark mass or positive gravity element appears to represent a measure of a soliton like wave effect either universally or locally of a boundary motion at an interface between dark mass and dark energy described by the inverse of the ratio, $r_{\Lambda,DM}(t)$.

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