Thermodynamics of a Dust Universe Energy density, Temperature, Pressure and Entropy for Cosmic Microwave Background

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Abstract

This paper continues the building of the cosmological theory that was introduced in two earlier papers under the title *A Dust Universe Solution to the Dark Energy Problem*. The model introduced in this theory has existence before time zero so that it is not necessary to interpret it as of big-bang origin. The location of the Cosmic Microwave Background, within the theoretical structure gives a closing of the *fundamentals* of the model in terms of the definitions of Temperature, Entropy and other Thermodynamic aspects. Thus opening up a research tool in cosmology in exact agreement with experiment that can compete with the so-called *Standard Big Bang Model* as a mathematical-physical description of our universe based *rigorously* on Einstein's general relativity. It is suggested that the singularity at time zero involves a *population inversion* in the statistical mechanics sense and so justifies the use of negative temperature for the *CMB* at negative times. This also has the satisfactory consequence that the Universe's evolution involves entropy steadily increasing over all time from minus infinity through the singularity to plus infinity.

Keywords: Dust Universe, Dark Energy, Friedman Equations, Entropy, Population Inversion, Negative Temperature

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1 Introduction

The work to be described in this paper is an extension and general discussion of the significance and physical interpretations of the papers *A Dust Universe Solution to the Dark Energy Problem* [23] and *Existence of Negative Gravity Material. Identification of Dark Energy* [24]. The conclusions arrived at in those papers was that the dark energy *substance* is physical material with a positive density, as is usual, but with a negative gravity, -G, characteristic and which is twice as abundant as has usually been considered to be the case. References to equations in those papers will be prefaced with the letter A and B respectively. The work in A, B, the discussion here and the extensions here have origins in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([16],[22],[21],[20],[19],[18],[4],[23]) and similarly motivated work in references ([10],[9],[8],[7],[5]) and ([12],[13],[14],[15],[7]). Other

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useful sources of information have been references ([17],[3]) with the measurement essentials coming from references ([1],[2],[11]). Further references will be mentioned as necessary. After writing A and B, I found that Abbe Georges Lemaître [25] had produced much the same model in 1927 but had presented it with a greatly different emphasis and interpretation while also missing important aspects of the significance of the model that have emerged from the version and structure that I had found. He seems to have been only aware of the positive time solution and then only in a restricted form in his 1927 paper. See remarks on page 66 of *McVitties's* book [28]. Abbe Georges Lemaître is often referred to as the father of the big-bang but it seems to me that this reputation arose out of his omission of the negative time solution in his considerations of this early model. The explicit solution for $r(t)$, equation (3.8) here, is also not mentioned in the 1927 paper. Up to date, there are generally few references to the explicit solution, (3.8), and sparse developments of its form. In particular, I have found a substantial application of this formula by the authors *Ronald J. Adler et al* [26] in a paper called *Finite Cosmology and a CMB Cold Spot*. Their work has common ground with the present paper. However, they fail to realise the power of the scale factor (3.8) and use it in conjunction with a time patching scheme making use of the related looking formula (1.1),

$$a(t) \sim \sinh^{1/2}(t/(2R_d)) \tag{1.1}$$

where R_d is the de Sitter radius, to represent the radiation dominated era just as the formula (2.2) is used in the standard model to represent this era. This patching will be discussed in detail in the next section. I have not found who first wrote down the formula (3.8). The scale factor (3.8) and its universe although known about for many years, seems to have been almost totally eclipsed and overshadowed by the vast effort put into developing over the last eighty years of what is now called *The Standard Model*.

2 Defect in Standard model

The standard cosmological model is generally accepted as giving a good and plausible description of the evolution of the universe from the initially assumed big bang through to the present time and into predicting the future of the universe. At the same time, it can be used to give a convincing case for how the material structures of which the universe is composed now has evolved from simpler and more fundamental materials that were likely predominant at earlier times. This is seen as a building and complexifying process with time which is assumed to be driven by temperature change from the very high temperature in the vicinity of the big bang to the low temperate of the present day. The physical processes that are involved can be identified from general earthbound physical theory and technology that associates the need for specific ranges of temperature with the possibility of known processes occurring. Thus the dependence of the temperature $T(t)$ on time, t, is the main key to the structural evolution of the universe.

At this moment in time, we have only one convincing theory of gravity, Einstein's General Relativity and its consequence the *Friedman equations*, as a description for the process of cosmological evolution with epoch. It is generally assumed that the standard cosmological model is rigorously a solution to the Einstein field equations. I do not wish to be dogmatic but I believe the truth of this assumption is questionable. To examine this issue let us consider the mathematical structures that constitutes the standard model. Nowadays, the standard model is said to describe four main phases in the birth and evolution of the universe. They are (1) inflation and the big bang, (2) a radiation dominated phase, (3) a matter dominated phase and (4) accelerated expansion into the distant future. Essentially three distinct *rigorous* solutions to Einstein's field equations as represented by their *distinct* scale factors, $r_A(t), r_B(t), r_C(t)$ and *distinct* temperature time relations $T_A(t), T_B(t), T_C(t)$ can be found to accommodate the type of physical process assumed to be taking place within each epoch interval *A,B and C*. The three scale factors or radii involved are usually represented as,

$$r_A(t) \quad \sim \quad \exp(Ht). \tag{2.1}$$

This first A stage involves Guth's inflation determined by a very large value for Einstein's cosmological constant, Λ_A and the big-bang event.

$$r_B(t) \quad \sim \quad t^{1/2}. \tag{2.2}$$

This second stage B is the radiation dominated epoch, with some appropriate value for Einstein's cosmological constant, Λ_B.

$$r_C(t) \quad \sim \quad t^{2/3}. \tag{2.3}$$

This third stage C which includes the present time, $t_0 = t^\dagger$, is the mass dominated epoch with accelerating expansion into the future determined by the definite *measured* value of Einstein's cosmological constant, $\Lambda_C = \Lambda$. I have emphasised that the three solutions *A,B,C* above are rigorously each a solution of Einstein's field equations. Further, full versions can be found with detailed coefficients so that the $\sim$ sign can be replaced with the $=$ sign. However, these solution are called the *Standard Model* when they are patched together in time sequence. Mathematically, this patching process can be carried through for both the scale factors and the time temperature relations to find a single form covering the time range $0 \to +\infty$. It seems to me that this final form is not strictly speaking a rigorous solution to Einstein's field equations and for the following reasons. Einstein's field equations are *causal* in the sense that paths calculated from theory are determined for all time by initial conditions and this applies to the points on the boundary of the expanding universe given by the radial variable $r(t)$. Thus for rigorous solutions $r(t)$ cannot change its form in time passage without some external intervention *outside* the guiding influence of general relativity. There is no statistical aspect of relativity in its accepted unmodified form that allows for choice between random selection of options at any stage of an evolution process. Statistics is involved in the thermal aspects of modern cosmology but this process the *CMB* is effectively locked away in a background subspace and can only influence paths through a *constant* thermal mass contribution in spite of its density changing with temperature and epoch. If such a change of motion had occurred in actual history, it could be regarded as due to the actual randomness of natural events or as an act of God but this later explanation is certainly outside relativity. Such a change would be on a par with the big-bang concept itself which is generally admitted not to be understood or likely to be explainable by present theory. From the theoretical point of view of the standard model, such changes have been allowed by modern cosmologists in order to conform to human intuitions and preconceptions of how things should be. They may, of course, be right with their intuitive pictures of event sequences but still this is not science and not necessarily proof that the time patched solutions are rigorous solutions to the field equations. This argument against the standard model is strongly reinforced by the fact that solutions of Einstein's field equation involving the cosmological constant requires that this quantity really remains constant over all the time for which the theory or solution is being used. We have seen that the standard model involves at least two definite changes to its parameter values for $r(t)$ and $T(t)$ and Λ. There is another important aspect of this issue and that is that the model I am proposing is rigorously a solution in isolation. It does not need patching up or joining to other solution to give a full time solution. Further it can supply all the facilities and options that are offered by the standard model for material synthesis resulting from temperature and pressure conditions that change with epoch and more. Showing that this is the case is what this paper is all about.

3 Summary of Mathematical Structure of Model

The main theoretical basis for the work to be discussed here are the two Fried-man equations that derive from general relativity with the curvature parameter $k = 0$ and a positively valued Λ. For ease of reference these equations and the main results obtained so far from the dark energy model are listed next,

$$\begin{aligned}
8\pi G\rho(t)r^2/3 &= \dot{r}^2 - |\Lambda|r^2c^2/3 && (3.1)\\
-8\pi GP(t)r/c^2 &= 2\ddot{r} + \dot{r}^2/r - |\Lambda|rc^2. && (3.2)\\
r(t) &= (R_\Lambda/c)^{2/3}C^{1/3}\sinh^{2/3}(\pm 3ct/(2R_\Lambda)) && (3.3)\\
b &= (R_\Lambda/c)^{2/3}C^{1/3} && (3.4)\\
C &= 8\pi G\rho(t)r^3/3 && (3.5)\\
R_\Lambda &= |3/\Lambda|^{1/2} && (3.6)\\
\theta_\pm(t) &= \pm 3ct/(2R_\Lambda) && (3.7)\\
r(t) &= b\sinh^{2/3}(\theta_\pm(t)) && (3.8)\\
v(t) &= \pm(bc/R_\Lambda)\sinh^{-1/3}(\theta_\pm(t))\cosh(\theta_\pm(t)) && (3.9)\\
a(t) &= b(c/(R_\Lambda))^2\sinh^{2/3}(\theta_\pm(t))(3-\coth^2(\theta_\pm(t)))/2 && (3.10)\\
H(t) &= (c/R_\Lambda)\coth(\pm 3ct/(2R_\Lambda)) && (3.11)\\
P(t) &= (-c^2/(8\pi G))(2\ddot{r}(t)/r(t) + H^2(t) - 3(c/R_\Lambda)^2) && (3.12)\\
P(t) &\equiv 0\ \forall\ t && (3.13)\\
P_\Lambda &= (-3c^2/(8\pi G))(c/R_\Lambda)^2 && (3.14)\\
P_G &= (3c^2/(8\pi G))(c/R_\Lambda)^2 && (3.15)\\
\rho_\Lambda &= (3/(8\pi G))(c/R_\Lambda)^2 && (3.16)\\
\rho_\Lambda^\dagger &= (3/(4\pi G))(c/R_\Lambda)^2 \ = \ 2\rho_\Lambda. && (3.17)\\
\rho(t) &= (3/(8\pi G))(c/R_\Lambda)^2(\sinh^{-2}(3ct/(2R_\Lambda)) && (3.18)\\
&= 3M_U/(4\pi r^3(t)) = M_U/V_U(t). && (3.19)\\
\rho_G(t) &= (G_+\rho(t) + G_-\rho_\Lambda^\dagger)/G && (3.20)\\
G_+ &= +G && (3.21)\\
G_- &= -G, && (3.22)
\end{aligned}$$

$$\begin{aligned}
\ddot{r}(t) &= -4\pi r(t)G\rho_G(t)/3 && (3.23)\\
\ddot{r}(t) &= 4\pi rG(\rho^{\dagger}_{\Lambda} - \rho(t))/3 \;=\; a(t) && (3.24)\\
&= 4\pi r^3 G(\rho^{\dagger}_{\Lambda} - \rho(t))/(3r^2) && (3.25)\\
&= M^{\dagger}{}_{\Lambda}G/r^2 - M_U G/r^2 && (3.26)\\
M^{\dagger}_{\Lambda} &= 4\pi r^3 \rho^{\dagger}_{\Lambda}/3 && (3.27)\\
M_U &= 4\pi r^3 \rho(t)/3 && (3.28)\\
P_G(t)/(c^2\rho_G(t)) &= 1/(\coth^2(3ct/(2R_{\Lambda})) - 3) \;=\; \omega_G(t) && (3.29)\\
\omega_{\Lambda} &= P_{\Lambda}/(c^2\rho_{\Lambda}) = -1. && (3.30)
\end{aligned}$$

where $M^{\dagger}_{\Lambda}$ is the total dark energy mass within the universe and M_U is the total non-dark energy mass within the universe. Equation (3.23) or equation (3.24) is exactly the classical Newtonian result for the acceleration at the boundary of a spherical distribution positive G mass density. The function $r(t)$ (3.8) is the radius or scale factor[1] for this model([3]). After writing A and B, I found the connection with Lemaître's work.

An interesting and significant result that follows easily from this theory is the identification of the Newtonian gravitational Coulomb like potential that is equivalent to Einstein's general relativity with his positive cosmological constant $\Lambda > 0$ or expressed otherwise, the *Newtonian* Coulomb potential limit of relativity with Einstein's dark energy. All that is needed is to replace the Newtonian equivalent density for a point source potential at epoch-time, t, with the general relativity gravity weighted density, $\rho_G(t)$, equation (3.20),

$$\rho_G(t) = \rho(t) - \rho^{\dagger}_{\Lambda}.$$

This procedure gives the result,

$$\ddot{r} = -MG/r^2 + r\Lambda c^2/3$$

in place of the Newtonian result

$$\ddot{r} = -MG/r^2.$$

The inverse square is simply modified with the addition of a term linear in r and proportional to Einstein's Λ.

4 Thermodynamics of the CMB

The thermodynamics of dust with its characteristically zero pressure as described here by the quantity $P(t)$ equation (3.12) may seem to present this theory with some conceptual difficulties, if it is to include the cosmical microwave background as part of its structure. However, this turns out not to be the case as will be explained. The model is geometrically and dynamically completely defined, it does have pressure as part of its structure but the thermodynamical significance of this pressure which is identically zero over the whole life span of this model and any relation it may have with a temperature is not obvious. The zero dust pressure does decompose into positive and negative signed components and as usual the dark energy material contributes the negative pressure. The dark energy component is identified firmly with truly negatively characterised gravitating material but takes the form of a positive mass density, $\rho^{\dagger}_{\Lambda}$ which is twice the dark energy density, ρ_{Λ}, identified by Einstein. There is no reason to

[1] After writing A, I found mention of this scale factor in Michael Berry's Book, p. 129

believe that the *CMB* should be directly associated with the dark energy vacuum constituent and also be negative G characterised and so to add the *CMB* into the structure the obvious choice is to make it part of the conserved mass of the universe which has been denoted earlier by M_U, equation (3.28). This placement for the *CMB* is reinforced by the fact that blackbody radiation is not an absolute constant as is the dark energy density but rather depends on temperature which itself is usually assumed to vary with epoch time. I therefore make the strong assumption,

$$M_U = M_\Delta + M_\Gamma, \tag{4.1}$$

where M_U, the total conserved *non-dark* energy mass of the universe, M_Δ is the conserved mass that is neither *CMB* nor *dark* energy mass and M_Γ, is the conserved *CMB* mass of the universe and all are taken to be absolute constants. In this paper, I shall restrict the discussion to the assumption (4.1). I refer to this assumption as strong as there are other option possibilities all of which can keep C constant over all time because the model depends *essentially* on the assumption that Rindler's constant $C = 2M_U G$ is an absolute constant and this is what makes the integration of the Friedman equations yield the model. Clearly, C can be kept constant if, M_U varies with time with whatever side effects that may have or if, indeed, M_U is kept constant with time but M_Δ and M_Γ are allowed to vary with time. The main consequence of the assumption (4.1) is that total non-dark energy density, M_U and both Δ and Γ component densities acquire an epoch time dependence as a result of the equations,

$$\begin{aligned} M_U &= \rho(t)V_U(t), && (4.2)\\ M_\Delta &= \rho_\Delta(t)V_U(t), && (4.3)\\ M_\Gamma &= \rho_\Gamma(t)V_U(t), && (4.4) \end{aligned}$$

where the volume of the universe at time t is given by $V_U(t) = 4\pi r^3(t)/3$. Taking the cosmic microwave background radiation to conform to the usual blackbody radiation description, the *mass* density function for the *CMB* will have the form

$$\begin{aligned} \rho_\Gamma(t) &= aT^4(t)/c^2, && (4.5)\\ a &= \pi^2 k^4/(15\hbar^3 c^3), && (4.6)\\ &= 4\sigma/c, && (4.7) \end{aligned}$$

where σ is the Stephan-Boltzmann constant,

$$\begin{aligned} \sigma &= \pi^2 k^4/(60\hbar^3 c^2), && (4.8)\\ &= 5.670400 \times 10^{-8} \quad W\ m^{-2}\ K^{-4}. && (4.9) \end{aligned}$$

The total mass associated with the *CMB* is given by (4.4). That is

$$\begin{aligned} M_\Gamma &= \rho_\Gamma(t)V_U(t), && (4.10)\\ &= (aT^4(t)/c^2)4\pi r^3(t)/3 && (4.11)\\ &= (aT^4(t)/(3c^2))4\pi b^3 \sinh^2(\theta_\pm(t)) && (4.12)\\ &= (aT^4(t)/(3c^2))4\pi (R_\Lambda/c)^2 2M_U G \sinh^2(\theta_\pm(t)) && (4.13)\\ &= (8\pi aT^4(t)/(3c^4))(R_\Lambda)^2 M_U G \sinh^2(\theta_\pm(t)) && (4.14) \end{aligned}$$

It follows from (4.14) that the temperature, T(t) as a function of epoch can be expressed as,

$$\begin{aligned} (8\pi aT^4(t)/(3c^4)) &= M_\Gamma/((R_\Lambda)^2 M_U G \sinh^2(\theta_\pm(t))) && (4.15)\\ T^4(t) &= M_\Gamma(3c^4)/(8\pi a(R_\Lambda)^2 M_U G \sinh^2(\theta_\pm(t))) && (4.16)\\ T(t) &= \pm(M_\Gamma 3c^4/(8\pi a(R_\Lambda)^2 M_U G \sinh^2(\theta_\pm(t))))^{1/4} && (4.17)\\ T(t) &= \pm((M_\Gamma 3c^2/(4\pi a))(r(t)^3))^{1/4}. && (4.18) \end{aligned}$$

The fourth power equation in T has four solutions two of which are real and two of which are complex and so the latter two can be neglected. The positive solution is obviously important but the negative solution also turns out to play an important role in this theory. I shall return to this issue.

The well established fact that at the present time, $t^\dagger$, the temperature of the CMB is

$$T(t^\dagger) = T^\dagger = 2.728\ K \tag{4.19}$$

can be used to simplify the formula (4.18) because with (4.19) it implies that

$$0 < T(t^\dagger) \quad = \quad +(M_\Gamma 3c^4/(8\pi a(R_\Lambda)^2 M_U G \sinh^2(\theta_\pm(t^\dagger))))^{1/4} \tag{4.20}$$

or

$$\begin{aligned} T(t)/T(t^\dagger) &= \pm(\sinh^2(\theta_\pm(t^\dagger)))^{1/4}/(\sinh^2(\theta_\pm(t)))^{1/4} && (4.21)\\ T(t) &= \pm T^\dagger \left(\left(\frac{\sinh(\theta_\pm(t^\dagger))}{\sinh(\theta_\pm(t))}\right)^2\right)^{1/4} && (4.22)\end{aligned}$$

where it is understood that the fourth root is taken after the square. The formula relating temperature and time (4.22) is different from those which are used in the standard model. The formula here arises in this model in a very natural way. The other thermodynamics quantities for the *CMB*, free energy F(T,V) as a function of temperature and volume, entropy S_Γ, pressure P_Γ and energy E_Γ also arise naturally in their usual forms and as functions of time,

$$\begin{aligned} F(T,V) &= -(a/3)V_U(t)T^4(t) && (4.23)\\ S_\Gamma(t) &= \left(-\frac{\partial F}{\partial T}\right)_V = (4a/3)V_U(t)T^3(t), && (4.24)\\ P_\Gamma(t) &= \left(-\frac{\partial F}{\partial V}\right)_T = (a/3)T^4(t), && (4.25)\\ E_\Gamma &= aV_U(t)T^4(t). && (4.26)\end{aligned}$$

A very clear and accurate description of the thermodynamics of blackbody radiation can be found in *F. Mandl's* book [27] page 260.

The temperatures and pressures associated with various epoch times associated with this model and labelled alphabetically are given in the following list. The times have been selected to be only the frequently mentioned standard model special times associated with various important physical processes ([30],[29]).

$Time, secs/yrs$		$Temp, Kelvin$	$Pressure,\ Nm^{-2}$	
$t_A = 10^{-43}\ s$	$\rightarrow$	$6.521 \times 10^{30}\ K,$	4.56×10^{107}	(4.27)
$t_B = 10^{-32}\ s$	$\rightarrow$	$2.062 \times 10^{25}\ K,$	4.56×10^{85}	(4.28)
$t_C = 10^{-6}\ s$	$\rightarrow$	$2.062 \times 10^{12}\ K,$	4.56×10^{33}	(4.29)
$t_D = 180\ s$	$\rightarrow$	$1.537 \times 10^{8}\ K,$	1.41×10^{17}	(4.30)
$t_E = 3 \times 10^5\ yr$	$\rightarrow$	$670.4\ K,$	5.09501×10^{-5}	(4.31)
$t_F = 10^9\ yr$	$\rightarrow$	$11.60\ K,$	4.57156×10^{-12}	(4.32)
$t_G = t^\dagger \approx 14 \times 10^9\ yr$	$\rightarrow$	$2.728\ K,$	1.32978×10^{-14}	(4.33)
$t_H = 15 \times 10^9\ yr$	$\rightarrow$	$2.552\ K,$	1.07066×10^{-14}	(4.34)

These time temperature values are much the same as time temperature values that can be found in the *composite* standard model and imply temperature ranges that suit various quantum particle physical structure generation processes. They can be found in this theory with the single temperature *all-time* formula, $T(t)$, given by equation (4.22).

Approximate list of physical processes associated with temperature ranges implied by $T(t)$:

Preliminary remark: Processes are associated with times in this list using times often quoted in the standard model [29]. The association of process with temperature in the standard model is of very low-level accuracy so I have made no attempt to get accurate connection of time with process. The list is just to show qualitatively that the processes described in the standard model can also be described in the model that I am proposing with at least equal detail.

The zero and positive valued times listed above from the standard model, $t_A \to t_H$, interspersed with theoretical values $(0, t_1, t_c, t^\dagger, t_2)$ from the present theory, in numerical time order, are given in the following list. The present theory contains time symmetrical event $t \to -t$ that are not listed.

Main Events in History of the Universe

$t = 0$ The *singularity* when the radial speed is ambiguously, $\pm\infty$

t_A Planck epoch range of super-fast inflation

t_B Post inflation when there is a hot soup of electrons and quarks

t_C Rapid cooling when quarks convert into protons and neutrons

t_D Super hot fog of charged electrons and protons impede passage of photons

t_E Electrons protons and neutrons form atoms and photons have free passage

t_1 The time when the radial boundary speed of the universe has descended through finite values from ∞ to the value c

t_F Hydrogen and helium coalesce under gravity to eventually become galaxies

t_c The time when the universes motion changes from deceleration to acceleration

$t_G = t^\dagger$ The present time, $t^\dagger$, conditions as they are now

t_H Conditions 10^9 years into the future

t_2 The time when the radial boundary speed of the universe ascends from below to attain the value c again

$t_\infty = \infty$

5 Ratio of Radiation Mass to Delta Mass

The accelerating universe astronomical observational workers [1] give measured values of the three Ωs, and w_Λ to be

$$\Omega_{M,0} = 8\pi G\rho_0/(3H_0^2) = 0.25^{+0.07}_{-0.06} \tag{5.1}$$
$$\Omega_{\Lambda,0} = \Lambda c^2/(3H_0^2) = 0.75^{+0.06}_{-0.07} \tag{5.2}$$
$$\Omega_{k,0} = -kc^2/(r_0^2 H_0^2) = 0, \ \Rightarrow k = 0, \tag{5.3}$$
$$\omega_\Lambda = P_\Lambda/(c^2\rho_\Lambda) = -1 \pm \approx 0.3. \tag{5.4}$$

According to the strong assumption (4.1) M_U, the total conserved mass of the universe, M_Δ the conserved Δ mass of the universe and M_Γ, the conserved *CMB* mass of the universe are all absolute constants. Thus the ratio of *CMB* mass to the Δ mass, $r_{\Gamma,\Delta} = M_\Gamma/M_\Delta$, will be an absolute constant with epoch change together with the same ratio in terms of the corresponding time dependent densities, $\rho_\Gamma(t)$ and $\rho_\Delta(t)$. The value of this ratio is,

$$r_{\Gamma,\Delta} = \rho_\Gamma(t)/\rho_\Delta(t) \approx 0.00019151 \approx 2 \times 10^{-4} \quad (5.5)$$
$$\Omega_M(t) = \Omega_\Delta(t) + \Omega_\Gamma(t) \quad (5.6)$$
$$r_{\Gamma,\Delta} = \Omega_\Gamma(t)/\Omega_\Delta(t) \approx 2 \times 10^{-4} \quad (5.7)$$
$$\Omega_M(t) \approx \Omega_\Delta(t)(1 + 2 \times 10^{-4}) \quad (5.8)$$
$$\Omega_\Delta(t) \approx \Omega_M(t)(1 - 2 \times 10^{-4}) \quad (5.9)$$
$$\Omega_\Delta(t) \approx \Omega_M(t)_{-0.00005}, \quad \Omega_{M,0} = \Omega_M(t^\dagger) = 0.25 \quad (5.10)$$
$$\Omega_\Gamma(t) \approx \Omega_M(t) \times 2 \times 10^{-4}, \quad (5.11)$$

where (5.6) is the Ω equivalent of the strong assumption, (4.1). Hence (5.7) through to (5.11).

If we compare (5.10) with (5.1), it is clear that the mass weight, M_Γ, of *CMB* contribution, $\Omega_\Gamma(t)$, to the conserved mass contribution M_U is way inside the error limits for $\Omega_{M,0}$. That is to say that as far as mass is concerned, the *CMB* could be neglected in relation to the Ω_Δ contribution as it makes no significant *numerical* contribution to the *basic* structure of the theory or the integration process used to derive it. However, it does introduce detail *internally* to the theory and it is clearly vital to the quantum synthesis of structures story attached to the theory. It reduces homogeneity in an important and useful way and, as will be shown, it induces a partitioning of the pressure $P(t)$ in the forms for the *CMB* pressure P_Γ and the pressure P_Δ from the Δ mass component. This will be addressed next.

The *CMB* pressure from equation (4.25) is

$$P_\Gamma(t) = (a/3)T^4(t), \quad (5.12)$$
$$= E_\Gamma/(3V_U(t)), \quad (5.13)$$
$$= M_\Gamma c^2/(3V_U(t)) = \rho_\Gamma(t)c^2/3. \quad (5.14)$$

From (3.13)and following equations the pressure of the dust universe is given by

$$P(t) = P_G + P_\Lambda \equiv 0 \ \forall \ t. \quad (5.15)$$

where both P_G and P_Λ are oppositely signed but numerically equal absolute constants. In the previous section, the *CMB* mass was incorporating into the theory by partitioning the total mass M_U which has been given a fixed numerical value from experiment into the two parts M_Δ and M_Γ while keeping the same constant value for M_U. This defines the Δ mass, M_Δ, but otherwise makes no difference to the structure of the theory or its correctness as rigorous solution to Einstein's field equations. However, as the value of the pressure, $P(t)$, is determined by the value within the theory of M_U, the non-dark energy part of the universe mass, the partitioning procedure implies a partitioning of the non-dark energy part of the pressure $P(t)$ which is P_G (5.15) corresponding to the partitioning of mass, (4.1).

There is an unfortunate anomaly in Friedman cosmology in the perception of pressure and in the way that it is defined. The pressure term in the Friedman equations is assumed to arise basically from the gravitational attraction of the material within the universe. Thus at the universe boundary massive objects will be attracted to within the universe by gravitation and this is regarded as the source of the positive pressure term, P, in the Friedman equations. Thus, effectively, P, is identified with the pressure that an inflated balloon in equilibrium would produce in reaction to the air pressure, $-P$, within it at equilibrium. It follows that, for consistency, if there is radiation pressure present within the universe it should be placed as a contribution to the $P(t)$ term, within the theory, with a negative sign. That is we should write for the induced partitioning of $P(t)$

$$P(t) = P_G + P_\Lambda \equiv 0 \ \forall \ t \tag{5.16}$$
$$P_G = P_\Delta(t) - P_\Gamma(t) \tag{5.17}$$

as the dark energy pressure, P_Λ, is not involved being not part of an effect from the conserved mass of the universe. The pressure, P_G, associated with the gravitational attraction of all the mass within the universe at any time t, as has been shown earlier, is an absolute constant while $P_\Gamma(t)$ depends on time. Thus the *non-CMB* part of the pressure, $P_\Delta(t)$ must also depend on time. Equation, (5.17) can be written as

$$1 = P_\Delta(t)/P_G - P_\Gamma(t)/P_G \tag{5.18}$$
$$r_{P\Gamma G}(t) = P_\Gamma(t)/P_G \tag{5.19}$$
$$r_{P\Delta G}(t) = r_{P\Gamma G}(t) + 1. \tag{5.20}$$

The ratios $r_{P\Gamma G}(t)$ and $r_{P\Delta G}(t)$ above give the weight of *CMB* pressure and the weight of *non-CMB* to the total non-dark pressure respectively

Let us now consider the relation between the *CMB* pressure, $P_\Gamma(t)$, and the Δ mass density,

$$P_\Gamma(t) = (a/3)T^4(t), \tag{5.21}$$
$$= E_\Gamma/(3V_U(t)), \tag{5.22}$$
$$= M_\Gamma c^2/(3V_U(t)) = \rho_\Gamma(t)c^2/3 \tag{5.23}$$
$$\approx \rho_\Delta(t)c^2 \times 2 \times 10^{-4}/3. \tag{5.24}$$

Equations (5.16) and (5.17) together, expressed as

$$P_\Gamma(t) \equiv P_\Delta(t) + P_\Lambda \ \ \forall \ t, \tag{5.25}$$

gives the important *interpretational* result that the *CMB* is in mechanical equilibrium with the Δ and Λ masses combined. This is an alternative characterisation of the dust universe property.

6 Entropy of the CMB

We found at equation (4.18) that there is a choice available for defining physical temperature as a positive or a negative quantity. As we all know, the usual choice in terrestrial physics is the positive one. This choice is made because S is a monotonic increasing function of E,

$$\frac{1}{T} = \frac{\partial S}{\partial E} > 0. \tag{6.1}$$

However, there are also physical theory situations where negative values of temperature are encountered and accommodated within conventional thermal physics. Negative temperatures occur for example in nuclear spin systems. Original work on this aspect is associated with the researchers *Purcell and Pound* ([31],[27]). The second of the previous two references gives much detail about such systems and explanations of their thermodynamics The condition under which negative temperature occurs is often considered to be due to the existence of a sub-system with finite maximum energy that is in weak interaction with the rest of the system so that its own temperature can be defined independently from the rest. The sub-system can then reach equilibrium without being in equilibrium with the whole system. In statistical mechanics, the process involved is sometimes referred to as *population inversion* with the temperature passing from positive infinity to negative infinity or conversely at the inversion time. This subsystem property is largely the situation in the cosmological model being presented here where the sub-system is the *CMB* which has a constant fixed total energy E_Γ and so its interaction with the whole universe system which changes markedly with time t must be judged as weak. The property is exactly fulfilled in the present model if, at the singularity time, population inversion actually takes place with a finite negative temperature just before time $t = 0$ and reaching $-\infty$ at the singularity then jumping to $+\infty$ immediately after the singularity to then descend through finite positive values to zero as epoch time advances. Such a singularity event depends theoretically on the temperature for negative time being chosen also to be negative and as we have seen we have this option from the theory (4.17). The infinite temperature jump at $t = 0$ can, as is well known, be avoided if one works with the parameter $1/T$ instead of T. This is equivalent to regarding inverse temperature as more physically significant than temperature itself. My feeling is that the idea of a singularity population inversion taking place at $t = 0$ is a likely scenario. However, I have to admit that the choice of negative temperature for $t < 0$ is speculative in spite of its apparent very good fit with the mathematics of the present model. In particular, this assumption does lead to the important conclusion that, if true, the entropy associated with the *CMB* steadily increases from $t = -\infty \to +\infty$ and, of course, that is what we would like to be the case. Firm decisions about what happens at the singularity at this time in theory development can only be speculative. Quantum Cosmology when it arrives may hopefully change the situation.

Starting at equation (6.2) is a list of the values of the ratio $r_{P\Gamma G}(t)$ of *CMB* pressure to the non-dark energy component component P_G of the pressure $P(t)$ that arises from the Δ mass, $r_{P\Gamma\Delta}(t)$, for fourteen important values of epoch time, t. This model is time symmetric about the time $t = 0$, the time of the singularity. Thus this model has existence before the time $t = 0$. This is evident from the form of the radius or scale factor equation, (3.8) provided it is understood that the square is taken before the cube root is taken in the index of the *sinh* function.

$$r_{P\Gamma G}(\pm 0) \approx \infty \tag{6.2}$$
$$r_{P\Gamma G}(\pm t_1/10) \approx 6.94625 \tag{6.3}$$
$$r_{P\Gamma G}(\pm t_1) \approx 0.06493 \tag{6.4}$$
$$r_{P\Gamma G}(\pm t_c) \approx 0.00012 \tag{6.5}$$
$$r_{P\Gamma G}(\pm t^\dagger) \approx 0.00002 \tag{6.6}$$
$$r_{P\Gamma G}(\pm t_2) \approx 2.03 \times 10^{-6} \tag{6.7}$$
$$r_{P\Gamma G}(\pm\infty) = 0. \tag{6.8}$$

The ratio $r_{P\Gamma G}(t)$ is a measure of how the *CMB* mass pressure relates to the P_G mass pressure. Towards the singularity, $t = 0$, it approaches ∞. The Δ mass pressure ratio to P_G at the times above is given by adding 1 to the corresponding values above. Thus towards $t = 0$ it also approaches ∞. Thus from the pressure point of view at the singularity there is *mechanical* equilibrium between *CMB* mass and the Δ mass and Λ mass combined with the contribution from the Λ mass being negligible (5.25). At the singularity temperature and pressure are both infinite. This is the equivalent of the idea from big-bang theory that near the singularity the universe is *radiation dominated*.

7 Conclusions

The cosmological model developed in *A* and *B* and further amplified here to show how the thermodynamics of the *CMB* is included in its structure is *rigorously* a solution to the full set of Einstein's field equations of general relativity via the Friedman equations. The model also satisfies *exactly* the recent measurements by the astronomical dark energy workers. The model need not necessarily be considered to be of the *big-bang* type because it has a history extending from $t = -\infty$ to $t = +\infty$. However, if the reader cannot accept or just disagrees with the existence of the solution before $t = 0$ being joined to the solution after $t = 0$, he or she can simply disregard the negative time phase and stick with the big-bang idea. The negative time phase can then be dismissed as an unphysical solution to Einstein's field equations. I shall now give a brief account of the time evolution of the model over the full time range $t = -\infty$ to $t = +\infty$ and mention a few of the new puzzles thrown up by it. The reader would find it helpful to look at the graphs for velocity and acceleration over this range as I *outline* the history of events as the model evolves with epoch. See the file darkenergy.pdf [32], presentation to the *2006 PIRT Conference* and the list *Main Events in History of the Universe* in section **4**.

The theory implies the following sequence of steps starting at $t = -\infty$. A definite quantity of positively gravitational mass, M_U, at density zero uniformly distributed over the whole of an infinitely extended spherical three dimensional Euclidean region of hyperspace is collapsing at a very high speed, $v \gg c$, towards a definite centre. This is the initial situation. Apart from the fact that this mentions the kinematics of a spherical boundary moving towards its centre and as the density of the contents of this collapsing sphere is essentially zero almost nothing *physical* is happening. Clearly this initial situation is somewhat like the big-bang. However, I think it can be given a plausible explanation with a slight extension of the theory which does not damage its correctness under relativity. I shall come back to this point at the end of this section. The hyperspace is itself filled with a uniform constant unchanging density of negatively characterised gravitating mass, *dark energy* from Einstein's positive Λ. Thus the repulsive gravitational effect of this material within the descending sphere will cause the incoming high velocity of the boundary to steadily be reduced until it reaches from above the value c at the negative time, $-t_2$. All this first phase does present some physical mystery because the boundary motion is superluminal. However, if this early stage motion can be seen as just kinematic that might reduce any detraction by this aspect. Similar problems occur in the standard model. We note that at and above the positive time, t_2, we have no information about what physical processes are taking place except that the mass density of the universe is very low indeed which is also obviously the case below and up to the negative time $-t_2$. The next main stage above $-t_2$ and below $-t_1$ is acceptable as within known physics and probably involves physical processes like those occurring above t_1 and below t_2. However, it is not necessarily true that the processes in this range are the time-reversed processes in the time symmetric range, displayed in the *Events in History* list. Lastly, in the negative time range from $-t_1$ to the singularity at 0 we are into a mystery superluminal range again but this will likely contain events like those in the positive time range 0 to t_1 but with the same caveat as for the previous range. The sequence of events for the positive time range can be read off from the *Main Events in History* list which takes us to the end of history at t_∞ with a very low density universe sphere with its boundary rushing to infinity with $v \gg c$.

Returning to the point raised earlier about *understanding* the initial state, we see that the final state at $+\infty$ is the same as the initial state at $-\infty$ with reversed radial speed of the boundary. This can be given a reasonable explanation by making a reinterpretation of the 3-dimension hyper-space into which the universe is contracting and then expanding after the singularity. Suppose that the original 3D-hyperspace is replaced with a fixed *3D-surface* of a 4D-sphere of very large radius, a well know geometrical trick. Thus the fixed hyper-space becomes a closed 3D-space. The expanding Universe can then be regarded as expanding from a point to cover the 4D-sphere surface to some maximum hyper-area and then to contract back over the adjacent hyper-surface to become a point sphere again. This is most easily pictured by considering the lower dimensional situation, a circle on an ordinary 3D-sphere surface expanding from a point on the surface to approaching a great circle at great *positive* speed away from its start point to when, after passing the great circle, descends initially at great negative speed to become an antipodal point. This is just like the kinematic process in the model is seen to be happening through $t = \pm\infty$. This hyper-spherical interpretation is not part of the model as it stands but it could be incorporated in the structure without damage to the rigorousness of the model as a solution of the Einstein field equations. We can in fact just use this idea to interpret what is happening at infinity and then let the radius of the 4D-hypersphere go to infinity. Its surface then becomes physically indistinguishable from the original hyperspace.

The main conclusions are that this model for the evolution of the universe has all the advantages that are found in the cosmological standard model together with new perspectives on the nature of dark energy and the amount of it that is present in the background together with a new and clearer representations for temperature, pressure and entropy. Importantly, it does not suffer from the serious defect of the standard model of *not* being *definitely* a solution of Einstein's field equations.

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