

Existence of Negative Gravity Material Identification of Dark Energy

James G. Gilson j.g.gilson@qmul.ac.uk*

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Abstract

A solution to Einstein's field equations via the Friedman equations is shown to produce a cosmological model that is in exact agreement with the measurements made by the dark energy astronomers. All the essential physical parameters are obtained as epoch dependent functions all in closed form. The equations of state are obtained for a non-dark energy density, a dark energy density and a total gravitationally weighted mass density. An interpretation of the structure involving a dark energy mass distribution that is twice the usual value is shown to clarify greatly the physical significance of the mathematics. It is asserted that the astronomer's measurements together with the mathematical model proves that the universe is permeated uniformly with a positive mass density that carries a negative gravitational constant, $-G$, characteristic. This mass component is identified with the dark energy content of the universe that has been postulated to explain the observed acceleration. Another result implied by the model is that there is twice the amount of dark energy that is usually considered to be present.

Keywords: Dust Universe, Dark Energy, Friedman Equations

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1 Introduction

The work to be described in this paper is a detailed analysis of a conclusion hinted at in the paper *A Dust Universe Solution to the Dark Energy Problem* [5]. The conclusion arrived at there was that the dark energy *substance* is physical material with a positive density, as is usual, but with a negative gravity, $-G$, characteristic. References to equations in that paper will be prefaced with the letter *A*. The work in *A* and its extension here has its origins in the studies of Einstein's general relativity in the Friedman equations context to be found in references ([17],[23],[22],[21],[20],[19],[4],[5]) and similarly motivated work in references ([11],[10],[9],[8],[6]) and ([13],[14],[15],[16],[8]). Other useful sources of information have been references ([18],[3]) with the measurement essentials coming from references ([1],[2],[12]).

After the astronomical observations some eight years ago implying that the expansion of the universe is accelerating there has been overwhelming research into trying to find an explanation of what is happening and how it all does or does not fit into general relativity theory. The acceleration phenomenon is widely described as very mysterious. This is particularly in connection with

*School of Mathematical Sciences, Queen Mary University of London, Mile End Road, London E1 4NS, United Kingdom.

the material called *dark energy* which is thought to produce a *negative pressure* that causes the acceleration. As a positive pressure in the general relativity generated Friedman equations is thought to be due to gravitational attraction, the measured acceleration is seen *vaguely* as being caused by dark energy being gravitationally repulsive. However, it has seemed to be difficult to fit the observations into general relativity theory. I think the basis of all the theoretical difficulties lies with what Einstein called his greatest mistake, the introduction of his cosmological constant Λ to stabilise his model which without Λ would itself suffer negative acceleration. I think the mistake was in fact *not* the introduction of Λ which actually brilliantly achieved his objective but rather the association of Λ with the mass density function ρ_Λ which was assumed to represent the mystery repulsive dark energy material. As this concept was obviously effective in stabilising Einstein's model it has for the last hundred years been regarded as a physically correct assumption and so it always appears in Λ orientated discussions of the Friedman equations. In *A*, I showed that if the density function for dark energy is rather defined as $\rho_\Lambda^\dagger = \Lambda c^2/(4\pi G) = 2\rho_\Lambda$ then, dark energy becomes a distribution of positive mass that carries a negatively valued gravitational constant, $-G$. The dark energy model for the universe introduced in *A*, then accurately describes the dynamics and kinematics of the actual measured universe parameters and most of the mystery vanishes. It all turns on recognising that the universe contains a uniform and minutely small density distribution of negative gravity characterised material. I give a strong case for these ideas in the following pages by showing that the model from *A* has a *complete* solution giving for example the equations of state for the total *gravitationally positive* mass distribution density, $\rho(t)$, the dark energy material distribution, ρ_Λ , and the density a distribution, $\rho_G(t)$, which expresses the positive gravitational mass density relative to the dark energy mass density.

2 Summary of Mathematical Structure of Model

The main theoretical basis for the work to be discussed here are the two Friedman equations that derive from general relativity,

$$8\pi G\rho(t)r^2/3 = \dot{r}^2 + (k - \Lambda r^2/3)c^2 \quad (2.1)$$

$$-8\pi GP(t)r/c^2 = 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2 \quad (2.2)$$

in the case $k = 0$ and a positively valued Λ we have,

$$8\pi G\rho(t)r^2/3 = \dot{r}^2 - |\Lambda|r^2c^2/3 \quad (2.3)$$

$$-8\pi GP(t)r/c^2 = 2\ddot{r} + \dot{r}^2/r - |\Lambda|rc^2. \quad (2.4)$$

The main results obtained from the dark energy model for ease of reference are listed next,

$$r(t) = (R_\Lambda/c)^{2/3}C^{1/3}\sinh^{2/3}(\pm 3ct/(2R_\Lambda)) \quad (2.5)$$

$$b = (R_\Lambda/c)^{2/3}C^{1/3} \quad (2.6)$$

$$C = 8\pi G\rho(t)r^3/3 \quad (2.7)$$

$$R_\Lambda = |3/\Lambda|^{1/2} \quad (2.8)$$

$$\theta_\pm(t) = \pm 3ct/(2R_\Lambda) \quad (2.9)$$

$$r(t) = b\sinh^{2/3}(\theta_\pm(t)) \quad (2.10)$$

$$v(t) = \pm(bc/R_\Lambda)\sinh^{-1/3}(\theta_\pm(t))\cosh(\theta_\pm(t)) \quad (2.11)$$

$$a(t) = b(c/(R_\Lambda))^2\sinh^{2/3}(\theta_\pm(t))(3 - \coth^2(\theta_\pm(t)))/2 \quad (2.12)$$

$$H(t) = (c/R_\Lambda)\coth(\pm 3ct/(2R_\Lambda)) \quad (2.13)$$

$$P(t) = (-c^2/(8\pi G))(2\ddot{r}(t)/r(t) + H^2(t) - 3(c/R_\Lambda)^2) \quad (2.14)$$

$$P(t) \equiv 0 \quad \forall t \quad (2.15)$$

$$P_\Lambda = (-3c^2/(8\pi G))(c/R_\Lambda)^2 \quad (2.16)$$

$$\rho_\Lambda = (3/(8\pi G))(c/R_\Lambda)^2 \quad (2.17)$$

$$\rho_\Lambda^\dagger = (3/(4\pi G))(c/R_\Lambda)^2 = 2\rho_\Lambda. \quad (2.18)$$

The function $r(t)$ is the radius or scale factor¹ for this model([3]). The acceleration $a(t)$ can also be expressed as

$$\ddot{r}(t) = 4\pi r G(\rho_\Lambda^\dagger - \rho(t))/3 = a(t) \quad (2.19)$$

$$= 4\pi r^3 G(\rho_\Lambda^\dagger - \rho(t))/(3r^2) \quad (2.20)$$

$$= M_\Lambda^\dagger G/r^2 - M_U G/r^2 \quad (2.21)$$

$$M_\Lambda^\dagger = 4\pi r^3 \rho_\Lambda^\dagger/3 \quad (2.22)$$

$$M_U = 4\pi r^3 \rho(t)/3 \quad (2.23)$$

where M_Λ^\dagger is the total dark energy mass within the universe and M_U is the total non-dark energy mass within the universe. From equation(A2.19), we have

$$8\pi G\rho(t)r^2/3 + \Lambda r^2 c^2/3 = \dot{r}^2 \quad (2.24)$$

$$8\pi G\rho(t) + \Lambda c^2 = 3H^2(t) \quad (2.25)$$

$$8\pi G\rho(t) = 3H^2(t) - \Lambda c^2 \quad (2.26)$$

$$\rho(t) = (3/(8\pi G))(H^2(t) - (c/R_\Lambda)^2) \quad (2.27)$$

$$= (3/(8\pi G))(c/R_\Lambda)^2(\coth^2(\theta_+(t)) - 1) \quad (2.28)$$

$$= (3/(8\pi G))(c/R_\Lambda)^2(\sinh^{-2}(3ct/(2R_\Lambda))) \quad (2.29)$$

$$= (3/(8\pi G))C/r^3(t) \quad (2.30)$$

$$= 3M_U/(4\pi r^3(t)) = M_U/V_U(t). \quad (2.31)$$

Equation (2.31) confirms the mass density of the universe character of $\rho(t)$.

3 The Pseudo Mass Density

The mass densities introduced in this theoretical construction, so far, are ρ_Λ , ρ_Λ^\dagger and $\rho(t)$. The first of these is what Einstein would have regarded as the repulsive material density within the universe to be associated with his Λ term added to overcome the intrinsic expansion process and give his static universe. The second one, is numerically twice the first which I have introduced for reasons about to be explained. The third density, $\rho(t)$, is the total *gravitational positive* mass density for the model being discussed. These are three legitimate mass densities in particular conforming to the usually accepted requirement that mass should always be *positive*. I now introduce another density of great importance which will be denoted by $\rho_G(t)$ and defined by

$$\rho_G(t) = \rho(t) - \rho_\Lambda^\dagger. \quad (3.1)$$

This is clearly *not* a legitimate mass density as it can become negative if $\rho(t) < \rho_\Lambda^\dagger$. However, if we rewrite it as

$$\rho_G(t) = (G_+\rho(t) + G_-\rho_\Lambda^\dagger)/G \quad (3.2)$$

$$G_+ = +G \quad (3.3)$$

$$G_- = -G, \quad (3.4)$$

¹After writing A, I found mention of this scale factor in Michael Berry's Book, p. 129.

the quantity $\rho_G(t)$ can then be regarded as a legitimate gravitationally *weighted mass density* and the equation (2.19) becomes

$$\ddot{r}(t) = -4\pi r(t)G\rho_G(t)/3 \quad (3.5)$$

$$2\ddot{r}(t)/r(t) = -8\pi G\rho_G(t)/3. \quad (3.6)$$

There is another way in which the quantity $\rho_G(t)$ can be legitimised. This is by calling it a *relative mass density*. Then possible negative values can be explained by arguing that it falls below the reference density ρ_Λ^\dagger for negative values. In that situation the uniform constant dark energy value is simply acting as a standard reference value throughout all space. Equation (3.5) or equation (3.6) is exactly the classical Newtonian result for the acceleration at the boundary of a spherical distribution positive G mass density.

4 Component Masses

Using equations (2.14) and (3.6) the identically zero total pressure $P(t)$ can be expressed in the form

$$P(t) = c^2\rho_G(t)/3 + (c^2/(8\pi G))(3(c/R_\Lambda)^2 - H^2(t)) \equiv 0. \quad (4.1)$$

In A , $P_M(t)$, the positive component of the pressure associated with ordinary non-dark energy mass, was defined as $P(t) - P_\Lambda$. Here the notation has been changed so that the M subscript is replaced with the G subscript, $P_G(t) = P_M(t)$, an improved designation for the pressure produced by the non-dark energy material component of the universe. It follows from equation (4.1) with the total pressure $P(t) = P_G(t) + P_\Lambda$ that

$$\rho_G(t) = (3/(8\pi G))(H^2(t) - 3(c/R_\Lambda)^2) \quad (4.2)$$

$$P_G(t) = P(t) - P_\Lambda \quad (4.3)$$

$$= c^2\rho_G(t)/3 + (-c^2/(8\pi G))H^2(t) + (6c^2/(8\pi G))(c/R_\Lambda)^2 \quad (4.4)$$

using equation (2.16). Then as we have both $P_G(t)$ and $\rho_G(t)$ for this component, we can write down the equation of state for this component as

$$\begin{aligned} P_G(t)/(c^2\rho_G(t)) &= 1/3 - (c^2/(8\pi G))/(3c^2/(8\pi G)) \\ &+ (c/R_\Lambda)^2/(H^2(t) - 3(c/R_\Lambda)^2) \end{aligned} \quad (4.5)$$

$$= 1/3 - 1/3 + 1/(\coth^2(3ct/(2R_\Lambda)) - 3) \quad (4.6)$$

$$= 1/(\coth^2(3ct/(2R_\Lambda)) - 3) = \omega_G(t). \quad (4.7)$$

The equation of state for the density, $\rho(t)$, is given by equation (4.1) in either of the forms,

$$P(t) = 0 \quad (4.8)$$

$$\rho_G(t) = (3/(8\pi G))(H^2(t) - 3(c/R_\Lambda)^2) \quad (4.9)$$

and because of the identically zero value of $P(t)$ an ω value is not defined. The equation of state for the dark energy component is given by equation A(4.50) as

$$\omega_\Lambda = P_\Lambda/(c^2\rho_\Lambda) = -(3c^2/(8\pi G))(c/R_\Lambda)^2/(c^2\rho_\Lambda) \quad (4.10)$$

$$= -1. \quad (4.11)$$

5 Conclusions

The cosmological model developed in *A* and further amplified here is *rigorously* a solution to the full set of Einstein's field equations of general relativity via the Friedman equations. The model also satisfies *exactly* the recent measurements by the astronomical dark energy workers. The model is mathematically complete as all the essential physical functions for, scale, velocity, acceleration, and Hubble's *not so* constant are given in closed form. The equations of state are obtained for the total *positive* G or non-dark energy density, $\rho(t)$; the dark energy density, ρ_Λ , used originally by Einstein and a total gravitationally weighted mass density, $\rho_G(t)$, are given in closed form. One difference from the usual physical interpretation is that the dark energy material is associated with a density, ρ_Λ^\dagger , twice the usual density that Einstein used. Thus if this density is used to replace the Einstein density in the equation of state (4.11) for dark energy, the -1 would be replaced with $-1/2$. This difference has radical consequences for the understanding of the whole physical structure of the theory. In particular, it implies that there is twice as much dark energy material present as hitherto has been thought to be present. Another major interpretational consequence is that dark energy material is *positive* mass that carries a negative valued, $-G$, characteristic property. With regard to the often suggested *mystery* that the enormous amount of dark energy present cannot be seen, a calculation of the density value of ρ_Λ^\dagger shows it to be about the equivalent of less than nine protons or hydrogen atoms per cubic meter. No wonder it cannot be seen. It is also the case that if all the observable mass were un-clumped from being galaxies and spread uniformly throughout the universe it also would be unobservable. The flatness of the model means that the expanding universe can be regarded simply as an expanding 3-sphere enveloping more dark energy as it increases in radius. Thus as this density is an absolute constant this increase must come from the regions outside the universe's spherical boundary. Thus dark energy is more than just a characteristic of the interior of the universe, it extends uniformly out into the three dimensional enveloping space into which the universe is expanding. It is hyper-universal.

It is my contention that the astronomical measurements[1] together with the inevitably following general relativity theoretical model derived in *A*[5] and further developed in this paper constitute *proof* that there exists throughout the universe and beyond extending into all hyperspace *positive* mass with a *negative* gravitational characteristic, $-G$, additional to the rest of the universe of swarming galaxies largely controlled *locally* by positive G .

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