A Dust Universe Solution to the Dark Energy Problem

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Abstract

Astronomical measurements of the Omegas for mass density, cosmological constant lambda and curvature k are shown to be sufficient to produce a unique and detailed cosmological model describing dark energy influences based on the Friedman equations. The equation of state Pressure turns out to be identically zero at all epochs as a result of the theory. The *partial* omega, ω_Λ for dark energy, has the exact value, minus unity, as a result of the theory and is in exact agreement with the astronomer's measured value. Thus this measurement is redundant as it does not contribute to the construction of the theory for this model. Rather, the value of ω_Λ is predicted from the theory. The model has the characteristic of changing from deceleration to acceleration at exactly half the epoch time at which the input measurements are taken. This is a mysterious feature of the model for which no explanation has so far been found. An attractive feature of the model is that the acceleration change time occurs at a red shift of approximately 0.8 as predicted by the dark energy workers. Using a new definition of dark energy density it is shown that the contribution of this density to the acceleration process is via a negative value for the gravitational constant, -G, exactly on a par with gravitational mass which occurs via the usual positive value for G.

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1 Introduction

The work to be described in this paper is the generation and study of the theory that should go with the experimental work[1, 2] of the astronomers who claim that their measurements indicate that the universe expansion is accelerating. Assuming that this theory is based on the Friedman equations[15, 21], I have found a solution to those equations that seems to be inevitable if the ideas put forward by the dark energy workers are to be realized as a consequence of the model. Very briefly their main conclusion from observational astronomy is that a universe expansion process is taking place now that changed from deceleration to acceleration at some time t_c in the past. This change is identified roughly as occurring in association with events at a universe radius $r_c = r(t_c)$ with an observed red shift here and now in the range, $0.5 < z < 1$. I shall use the subscript c as indicating the time of *change* from deceleration to acceleration. They assume that the cosmological constant Λ is positive and deduce from experiment that $k = 0$.

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2 Accelerating Model

Using Friedman's equations,

$$8\pi G\rho r^2/3 = \dot{r}^2 + (k - \Lambda r^2/3)c^2 \tag{2.1}$$
$$-8\pi GPr/c^2 = 2\ddot{r} + \dot{r}^2/r + (k/r - \Lambda r)c^2 \tag{2.2}$$

in the case $k = 0$ and a positively valued Λ we have,

$$8\pi G\rho r^2/3 = \dot{r}^2 - |\Lambda| r^2 c^2/3 \tag{2.3}$$
$$-8\pi GPr/c^2 = 2\ddot{r} + \dot{r}^2/r - |\Lambda| rc^2. \tag{2.4}$$

I define a radial length R_Λ associated with Λ as

$$R_\Lambda = |3/\Lambda|^{1/2} \tag{2.5}$$

and use Rindler's constant C,

$$C = 8\pi G\rho r^3/3 \tag{2.6}$$

to write the first Friedman equation in the form,

$$\dot{r}^2 = C/r + (rc/R_\Lambda)^2. \tag{2.7}$$

Having the quantity, Rindler's[15] constant C, in equation (2.7) fixed at a constant value turns out to be an important asset when we come to integrate the Friedman equations. If as is usually taken to be the case, the mass density function ρ is taken to be of uniform value throughout the universe at any given epoch, t, though having a value that depends on epoch through $r(t)$ as $\rho(r(t))$ and the quantity $4\pi r^3/3 = V_U(r)$ is interpreted as the total 3-dimensional volume of the universe when its radius is r, then the mass of the universe will be given by

$$M_U(r) = \rho(r)V_U(r) = 4\pi r^3\rho(r)/3 \tag{2.8}$$

and then the constant C takes the form

$$C = 2M_U(r)G. \tag{2.9}$$

From equation (2.9), it follows while C can be kept constant for variations of the boundary radius r of the universe we can incorporate an r dependent gravitation *constant*, $G(r)$, provided an r dependent mass, $M_U(r)$, is suitably chosen to conform with equation (2.9). This point will be returned to when we discuss quantization of the integrated Friedman equations. In big bang models in which all the mass of the universe is created at the big bang event and thereafter keeps at a constant value M_U, G must also keep at a constant value if the equation (2.9) is to be retained for constant C. The equation (2.9) can also be retained for constant C in cases where the mass of the universe is generated over time provided a suitable time dependent G is incorporated to keep C constant over time. Most of the work to follow applies to either of these cases so that we do not need to specify which form of G is involved. However, we do need to distinguish between the two cases if we wish to examine the mass density function ρ of the universe explicitly.

The accelerating universe astronomical observational workers[1] give measured values of the three Ωs, and w_Λ to be

$$\Omega_{M,0} = 8\pi G\rho_0/(3H_0^2) = 0.25 \tag{2.10}$$
$$\Omega_{\Lambda,0} = \Lambda c^2/(3H_0^2) = 0.75 \tag{2.11}$$
$$\Omega_{k,0} = -kc^2/(r_0^2H_0^2) = 0, \Rightarrow k = 0, \tag{2.12}$$
$$\omega_\Lambda = P_\Lambda/(c^2\rho_\Lambda) = -1\pm \approx 0.3. \tag{2.13}$$

Here the value of Hubble's constant will be taken to be

$$H_0 = 72\ Km\ s^{-1}\ Mpc^{-1} \tag{2.14}$$

or in inverse seconds[1]

$$H_0 = 2.333419756287 \times 10^{-18}\ s^{-1}. \tag{2.15}$$

From these values it follows that Rindler's constant C has the form and value

$$C = \Omega_{M,0} H_0^2 r_0^3 = 1.361211939757935 r_0^3 \times 10^{-36} \tag{2.16}$$

in terms of the radius now, r_0. Λ and R_Λ will have the forms and values

$$\Lambda = \Omega_{\Lambda,0} 3(H_0/c)^2 = 1.363097286965269 \times 10^{-52} \tag{2.17}$$
$$R_\Lambda = |3/\Lambda|^{1/2} = 1.483532963676604 \times 10^{26}. \tag{2.18}$$

It is convenient to rearrange the first Friedman equation (2.3) in the successive forms,

$$8\pi G\rho r^2/3 + \Lambda r^2 c^2/3 = \dot{r}^2 \tag{2.19}$$
$$8\pi G\rho r^2/3 + 8\pi G\rho_\Lambda r^2/3 = \dot{r}^2 \tag{2.20}$$

and so identify a mass density ρ_Λ which can be used to account for the cosmological constant contribution as an additional mass density along with the original ρ. Thus

$$8\pi G\rho_\Lambda = \Lambda c^2 \tag{2.21}$$

or

$$\rho_\Lambda = \Lambda c^2/(8\pi G). \tag{2.22}$$

Thus in these terms the first Friedman equation (2.3) becomes

$$8\pi G(\rho + \rho_\Lambda) = 3(\dot{r}/r)^2 = 3H(t)^2. \tag{2.23}$$

The second Friedman equation (2.4) can be written as

$$8\pi G r(-P/c^2 + \rho_\Lambda) = 2\ddot{r} + \dot{r}^2/r. \tag{2.24}$$

The introduction of the additional mass density ρ_Λ as in equation (2.22) to explain the mathematical appearance or existence of the cosmological constant Λ as a physical contributor to the theory is the usual approach. However, it is not necessarily the best way of physically accounting for the cosmological constant as the resulting equation (2.23) has the built in implication of putting ρ and ρ_Λ on a par with respect to the kinematic quantity $(\dot{r}/r)^2 = H(t)^2$ and this may not be physically very relevant. An alternative approach with an alternative density, $\rho_\Lambda^\dagger$, will be discussed later.

It follows from the definition (2.22) of ρ_Λ that the mass density associated with Λ has the same sign as does Λ itself which for some theory constructs is taken as positive and in other theoretical constructs is taken as negative. The negative mass density case does present conceptual difficulties. Using an equation of state involving pressure is also conceptually difficult in this context because an equation of state of the form

$$P_\Lambda = \omega_\Lambda c^2 \rho_\Lambda \tag{2.25}$$

[1] Many decimal places will be used to keep track of minutely different values that can arise from different calculation routes

for negative Λ and positive ω_Λ implies that the pressure P_Λ is also negative. For positive Λ and negative ω_Λ it also implies that the pressure P_Λ is negative and is the case that is used by the astronomers. This negative pressure is usually accommodated with some intellectual gymnastics. However, in the case of a partial pressure as in equation (2.25) it is more easily acceptable[2]. I show here that such conceptual difficulties are avoided in the cosmological model to be examined here. In the next section, I derive a solution to the Friedman equations that can accommodate all four of the dark energy researchers experimental values.

3 Dust Model Solution

From equation (2.7),

$$(dt/dr)^2 = \frac{1}{C/r + (cr/R_\Lambda)^2} \tag{3.1}$$

$$(dt/dr) = \pm\frac{1}{(C/r + (cr/R_\Lambda)^2)^{1/2}} \tag{3.2}$$

$$t = \pm\int \frac{dr}{(C/r + (cr/R_\Lambda)^2)^{1/2}}. \tag{3.3}$$

$$= \pm\int \frac{dr}{(C/r)^{1/2}(1 + (c/R_\Lambda)^2 r^3/C)^{1/2}} \tag{3.4}$$

$$= \pm\int \frac{r^{1/2}dr}{(C)^{1/2}(1 + (c/R_\Lambda)^2 r^3/C)^{1/2}}. \tag{3.5}$$

The change of variable $r \to y$ with the standard integration form at equation (3.9) gives equation (3.10) as t in terms of the transformed variable, y.

$$r^3 = y^2 \tag{3.6}$$

$$(2/3)d(r^{3/2})/dr = r^{1/2} \tag{3.7}$$

$$r^{1/2}dr = (2/3)dy \tag{3.8}$$

$$\int \frac{dy}{(1+ay^2)^{1/2}} = a^{-1/2}\ln(a^{1/2}y + (ay^2+1)^{1/2}), \tag{3.9}$$

$$t = \pm(C)^{-1/2}(2/3)\int \frac{dy}{(1 + (c/R_\Lambda)^2 y^2/C)^{1/2}} \tag{3.10}$$

$$a = (c/R_\Lambda)^2/C. \tag{3.11}$$

Thus the following sequence of steps gives the integral evaluated in terms of the transformed variable y, an inversion back to the original variable r at (3.20) and the introduction of the simplifying function $\theta_\pm(t)$ and the constant b to arrive finally at the solution(3.24).

[2] A confusion as between the total pressure P and the partial pressure P_Λ in the first version of this paper has been rectified in this present version.

$$\begin{aligned}
t &= \pm(C)^{-1/2}(2/3)a^{-1/2}\ln(a^{1/2}y + (ay^2+1)^{1/2}) && (3.12)\\
&= \pm(C)^{-1/2}(2/3)((c/R_\Lambda)^2/C)^{-1/2}\\
&\times \ln(((c/R_\Lambda)^2/C)^{1/2}y + (((c/R_\Lambda)^2/C)y^2+1)^{1/2}) && (3.13)\\
\pm 3ct/(2R_\Lambda) &= \ln(cy/R_\Lambda)C^{-1/2} + ((cy/R_\Lambda)^2/C+1)^{1/2}). && (3.14)\\
((cy/R_\Lambda)^2/C+1)^{1/2} &= \exp(\pm 3ct/(2R_\Lambda)) - (cy/R_\Lambda)C^{-1/2} && (3.15)\\
(cy/R_\Lambda)^2/C+1 &= \exp(\pm 3ct/(R_\Lambda)) - 2\exp(\pm 3ct/(2R_\Lambda))(cy/R_\Lambda)C^{-1/2}\\
&+ (cy/R_\Lambda)^2C^{-1}. && (3.16)\\
1 &= \exp(\pm 3ct/(R_\Lambda))\\
&- 2\exp(\pm 3ct/(2R_\Lambda))(cy/R_\Lambda)C^{-1/2}. && (3.17)\\
cy/(R_\Lambda C^{1/2}) &= \frac{\exp(\pm 3ct/(2R_\Lambda)) - \exp(\mp 3ct/(2R_\Lambda))}{2}\\
&= \sinh(\pm 3ct/(2R_\Lambda)). && (3.18)\\
cy &= R_\Lambda C^{1/2}\sinh(\pm 3ct/(2R_\Lambda)) && (3.19)\\
cr^{3/2} &= R_\Lambda C^{1/2}\sinh(\pm 3ct/(2R_\Lambda)) && (3.20)\\
r(t) &= (R_\Lambda/c)^{2/3}C^{1/3}\sinh^{2/3}(\pm 3ct/(2R_\Lambda)). && (3.21)\\
b &= (R_\Lambda/c)^{2/3}C^{1/3} && (3.22)\\
\theta_\pm(t) &= \pm 3ct/(2R_\Lambda) && (3.23)\\
r(t) &= b\sinh^{2/3}(\theta_\pm(t)). && (3.24)
\end{aligned}$$

Equation (3.21) or (3.24) is a formula for the radius of an expanding *accelerating* universe conforming to all four measurements of the dark energy investigators.

4 Characteristics of Model

The velocity, $v(t) = \dot{r}(t) = dr(t)/dt$, of expansion as a function of t is given by

$$\begin{aligned}
v(t) &= (R_\Lambda/c)^{2/3}C^{1/3}(2/3)\sinh^{-1/3}(\theta_\pm(t))\\
&\quad \times\cosh(\theta_\pm(t))(\pm 3c/(2R_\Lambda)) && (4.1)\\
&= \pm(Cc/R_\Lambda)^{1/3}\sinh^{-1/3}(\theta_\pm(t))\cosh(\theta_\pm(t)) && (4.2)\\
&= \pm(bc/R_\Lambda)\sinh^{-1/3}(\theta_\pm(t))\cosh(\theta_\pm(t)). && (4.3)
\end{aligned}$$

The acceleration $a(t) = \dot{v}(t) = \ddot{r}(t)$ is found to be

$$\begin{aligned}
a(t) &= ((c/R_\Lambda)^{4/3}C^{1/3}/2)(3\sinh^{2/3}(\theta_\pm)\\
&\quad -\cosh^2(\theta_\pm(t))\sinh^{-4/3}(\theta_\pm(t))) && (4.4)\\
&= b(c/(R_\Lambda))^2\sinh^{2/3}(\theta_\pm(t))(3-\coth^2(\theta_\pm(t)))/2. && (4.5)
\end{aligned}$$

From equation (3.21) and equation (4.2), Hubble's constant is given as a function of t as

$$\begin{aligned}
H(t) &= \dot{r}/r = \frac{\pm(Cc/R_\Lambda)^{1/3}\sinh^{-1/3}(\theta_\pm(t))\cosh(\theta_\pm(t))}{(R_\Lambda/c)^{2/3}C^{1/3}\sinh^{2/3}(\theta_\pm(t))} && (4.6)\\
&= (c/R_\Lambda)\coth(\pm 3ct/(2R_\Lambda)). && (4.7)
\end{aligned}$$

The measured value of $H(t)$ is given by equation (2.15). Thus a consistent value for time now, t_0, is the time solution of (4.8) at (4.9) with a numerical value in seconds at (4.10)

$$
\begin{aligned}
& (c/R_\Lambda)\coth(\pm 3ct/(2R_\Lambda)) = 2.333419756287\times 10^{-18} && (4.8)\\
t_0 &= (\pm 2R_\Lambda)/3c)\coth^{-1}((R_\Lambda/c)\times 2.333419756287\times 10^{-18}) && (4.9)\\
&= 4.34467334479058\times 10^{17} \quad s, && (4.10)
\end{aligned}
$$

using the value of R_Λ from equation (2.18).

From the formula for the acceleration (4.3), it can be seen that there is a time t_c at which the acceleration changes from negative to positive given by the time solution of (4.11) at (4.12)

$$
\begin{aligned}
\cosh^2(\theta(t))\sinh^{-4/3}(\theta(t)) &= 3\sinh^{2/3}(\theta(t)) && (4.11)\\
t_c = (2R_\Lambda/(3c))\coth^{-1}(3^{1/2}) &= 2.172336672394881\times 10^{17}. && (4.12)
\end{aligned}
$$

From equations (4.17) and (3.21) we can find the value of $r_0 = r(t_0)$, the radius of the universe formally at time-now as

$$r_0 = (R_\Lambda/c)^{2/3} C^{1/3}\sinh^{2/3}(\pm 3ct_0/(2R_\Lambda)). \tag{4.13}$$

The evaluation of which requires that we have a value for Rindler's constant C. However, an inspection of the definition for this constant equation (2.6) shows that to know this constant we require knowing H_0 for which I have assumed an input value at equation (2.15) but it also requires knowing the present radius of the universe r_0 for which we have *not* got a measured value. There is an alternative approach to finding the value for the present time, t_0, and radius, $r_0 = r(t_0)$, of the universe and thus getting the appropriate value for Rindler's constant. From equations (2.6) and (2.10), we note that Rindler's constant can be expressed in the form

$$C = \Omega_M H^{\dagger 2} r^{\dagger 3}, \tag{4.14}$$

where I am now using the dagger superscript to denote the present day radius of the universe and the present day value for Hubble's constant. The dagger notation is to emphasise the alternative route used to calculate quantities such as $r^\dagger$. From equation (3.21) for $r(t)$ it follows that

$$r^{\dagger 3} = (R_\Lambda/c)^2 C\sinh^2(\pm 3ct^\dagger/(2R_\Lambda)). \tag{4.15}$$

The value for Rindler's constant, C, from equation (4.14) can be substituted into equation (4.15) to give

$$1 = 0.25(R_\Lambda/c)^2 H^{\dagger 2}\sinh^2(3ct^\dagger/(2R_\Lambda)) \tag{4.16}$$

having cancelled the $r^{\dagger 3} = r^3(t^\dagger)$ that occurs on both sides of the equation. Thus

$$\pm 1 = 0.5(R_\Lambda/c)H^\dagger\sinh(3ct^\dagger/(2R_\Lambda)). \tag{4.17}$$

This can be solved for $t^\dagger$. Taking the plus sign with unity and using equation (4.7) at time now as in (4.18).

$$H^\dagger = H(t^\dagger) = (c/R_\Lambda)\coth(\pm 3ct^\dagger/(2R_\Lambda)). \tag{4.18}$$

It follows that the value of $t^\dagger$ is given by the time solution of (4.19) as $t^\dagger$ at (4.20).

$$2 = \cosh(\theta(t^\dagger)) \tag{4.19}$$
$$t^\dagger = (2R_\Lambda/3c)\cosh^{-1}(2) = 4.344673344789258 \times 10^{17}\ s. \tag{4.20}$$

Remarkably close to the value obtained for the original t_0 displayed again at equation (4.21).

$$t_0 = 4.34467334479058 \times 10^{17} \quad s. \tag{4.21}$$

However, the value of $t^\dagger$ was obtained without directly using the numerical value of H at equation (4.6) although it does indirectly depend on H through the value of R_Λ so that it can be used to check the theoretical numerical value of Hubble's constant $H^\dagger = H(t^\dagger)$ using the theoretical formula for $H(t)$. Thus the time $t^\dagger$ is greater than t_c, the acceleration change time, by an amount

$$t^\dagger - t_c = 2.172336672394881 \times 10^{17}\ s = 6.888434400035 \times 10^9\ y. \tag{4.22}$$

The $t^\dagger$ notation from now on will be used for time-now rather than the original t_0 but this implies that the value of Hubble's constant should be given by formula (4.7) as

$$H(t^\dagger) = (c/R_\Lambda)\coth(\pm 3ct^\dagger/(2R_\Lambda)) = 2.333419756287 \times 10^{-18}\ s^{-1}. \tag{4.23}$$

Comparing this with the experimental value $2.333419756287 \times 10^{-18}\ s^{-1}$ at equation (2.15), we see that the value coming from this theory via the indirect route through the value of R_Λ is exactly the experimental value given at (2.15). However, although we have found $t^\dagger$ at formula (4.22), we still have not found the value of $r^\dagger = r(t^\dagger)$ that is necessary to find Rindler's constant C and the constant b. It seems that mathematically C is to be taken as arbitrary or alternatively $r^\dagger$ is to be taken as arbitrary or either C or $r^\dagger$ is to be obtained from experiment or further some other theoretical consideration needs to be used to obtain one or other of these two key constants. There are values ascribed to $r^\dagger$ from astronomical observation based on a various speculative extrapolations and which are therefore not greatly reliable see Gravitation[16] page 738, box(27.4). I have here decided to use a value for $r^\dagger$ that comes from an assumption about the dependence of the gravitation constant on $t^\dagger$ or on $ct^\dagger$. This assumption comes from a suggestion about a formula for G in terms of other physical constants of great numerical accuracy noticed by *Ross McPherson*, see references [14, 13]. His original suggestion had dimensions different from G. A generalised version of McPherson's suggestion but which has the usual dimensions associated with the gravitation constant G is as follows

$$G = \hbar^2/(m_p^2 m_e c t_0), \tag{4.24}$$

where m_p and m_e are the rest masses of the proton and electron respectively. A formula similar to (4.24) for the gravitation constant was suggested by P. A. Dirac[12] in the thirties. There are a number of ways in which this formula can be interpreted. Taking t_0 to have one of values associated with the present age of the universe gives a very accurate value for G[10, 17, 19]. Otherwise ct can be given a value associated with the present radius of the universe $r^\dagger$ multiplied by $\cos(\chi_G)$, the numerical constant, $\cos(\chi_G) \approx 1$ coming from gravitational coupling, which will also give a value for G. I have used this last option to develop a quantum theory for gravity[20, 11, 12] that successfully gives accurate formulae conformable with Dirac's large number hypothesis. This gives me some confidence in now inverting the formula (4.24) and using the measured value for G to supply the missing numerical value for $r^\dagger$ as

$$r^\dagger = \hbar^2/(m_p^2 m_e G) = 6.539532681821 \times 10^{25}, \tag{4.25}$$

where the numerical constant, $\cos(\chi_G) \approx 1$, has been omitted as its values is very close to unity at nearly all times. The situation when it could make some significant difference, at the start of expansion of the universe when $t \approx +0$, will be discussed in a future paper. The issues of reformulating quantum theory so that it is consistent with the theory of stochastic processes, the quantization of gravity theory and the consequent production of a quantized cosmology is discussed in references [3, 4, 5, 6, 7, 8, 9]. The use of the formula (4.24) for G in terms of quantum quantities in this work represents a weak partial quantization of the structure. A more complete quantization will be discussed in a later paper.

This can now be used to complete the numerical computation for the present theory by using the value for $r^\dagger$ in equation (4.25) to give us Rindler's constant C through equation (2.16) which is repeated below with $r^\dagger$ replacing r_0 at (4.26) and its numerical value is given at (4.27). The value of the constant b is then deduced at (4.28).

$$\begin{aligned} C &= \Omega_{M,0} H_0^2 r^{\dagger 3} = 1.361211939757935 r^{\dagger 3} \times 10^{-36} && (4.26)\\ &= 1.361211939757935 r^{\dagger 3} \times 10^{-36} = 3.806871984611 \times 10^{41} && (4.27)\\ b &= (R_\Lambda/c)^{2/3} C^{1/3} = 4.534258713925 \times 10^{25}. && (4.28) \end{aligned}$$

I finish this discussion with a few remarks about the parametric values that arise in this theory. Firstly we can calculate the red shift, z, that would apply to light emitted at the acceleration change time t_c from the universe boundary $r(t_c)$ to arrive at a present day observer as

$$z = r(t^\dagger)/r(t_c) - 1 \approx 0.81712. \tag{4.29}$$

This certainly strengthens the theory-observation cohesion in that it is a theory result within the limits suggested by the dark energy workers. An interesting and *curious* direct result from this structure is that the ratio of time now to the time of acceleration change as calculated from their respective values is exactly 2,

$$t^\dagger/t_c = 2. \tag{4.30}$$

One might have noticed the value of this ratio from equations (4.12) and (4.20), if one had at some time earlier come across the *exact* numerical relation (4.36),

$$\begin{aligned}
\coth^2(\theta(t_c)) &= 3, &(4.31)\\
\cosh(\theta(t^\dagger)) &= 2, &(4.32)\\
\sinh(\theta(t^\dagger)) &= 3^{1/2}, &(4.33)\\
H(t_c) &= 3^{1/2}c/R_\Lambda &(4.34)\\
H(t^\dagger) &= c\coth(\theta(t^\dagger))/R_\Lambda &(4.35)\\
\cosh^{-1}(2) &= 2\coth^{-1}(3^{1/2}). &(4.36)\\
H(t) &= H(t_c)\coth(\theta(t))/3^{1/2} &(4.37)\\
H(t^\dagger) &= 2H(t_c)/3. &(4.38)
\end{aligned}$$

The first equation above (4.31) essentially defines the time t_c of change of acceleration from negative to positive and comes from equation (4.12). The second equation above comes from equation (4.17) or (4.19) and defines the *time-now*, $t^\dagger$, when the input measurements were made. The fourth equation (4.34) comes from equations (4.18) and (4.31). I have not been able to decide whether or not the result equation (4.30) is some remarkable coincidence in values of the present day $t^\dagger$ and the past t_c or some deep indication of a need to have more than just the past epoch value t_c to be able to talk about a definite location in the time history of an event such as a universally distributed change from deceleration to acceleration. However, equation (4.37) does clearly show up the background *mathematical* theory structure of the relation between t_c and t at equation (4.30) in relation to the time evolution of the system and gives equation (4.38) the Hubble equivalent of (4.30) when $t = t^\dagger$ but, I have to admit, that I have not yet deciphered the physical significance of this mathematics as expressed by (4.30) or (4.38). It would not seem so remarkable if they, (4.30) and (4.38), were just approximate results but in the construction of this model they are numerically *exact* and unexpected.

The formula for the pressure from the Friedman equation (2.4) is, using the definitions for H and R_Λ at equations (4.6) and (2.18) in (4.41),

$$\begin{aligned}
P &= -(c^2/(8\pi G r))(2\ddot{r} + \dot{r}^2/r - |\Lambda| r c^2) &(4.39)\\
&= -(c^2/(8\pi G))(2\ddot{r}/r + \dot{r}^2/r^2 - |\Lambda| c^2) &(4.40)\\
&= (-c^2/(8\pi G))(2\ddot{r}/r + H(t)^2 - 3(c/R_\Lambda)^2). &(4.41)
\end{aligned}$$

From equations (3.24), (4.3) and (4.5) we have

$$\begin{aligned}
2\ddot{r}(t)/r(t) &= (c/R_\Lambda)^2(3 - \coth^2(\theta_+(t))) &(4.42)\\
H(t)^2 &= (c/R_\Lambda)^2\coth^2(\theta_+(t)) &(4.43)\\
2\ddot{r}(t)/r(t) + H(t)^2 &= 3(c/R_\Lambda)^2. &(4.44)
\end{aligned}$$

Substuting these formulae into the form for pressure equation (4.41), we find complete cancellation on the right hand side to give an identically equivalent to zero total pressure at all times,

$$\begin{aligned}
P(t) &= (3c^2/(8\pi G))(-(c/R_\Lambda)^2 + (c/R_\Lambda)^2) &(4.45)\\
P(t) &\equiv 0 \Rightarrow \omega \equiv 0. &(4.46)
\end{aligned}$$

Thus this system rigorously describes a *dust* universe. From equation (4.41) or (4.45) we see that the total pressure P can be expressed as the sum of the two partial pressures P_Λ and P_M, the first of negative sign and the second of positive sign, as

$$
\begin{aligned}
P_\Lambda &= - (3c^2/(8\pi G))(c/R_\Lambda)^2 < 0 &(4.47)\\
P_M &= (3c^2/(8\pi G))(c/R_\Lambda)^2 > 0 &(4.48)\\
P &= P_M + P_\Lambda. &(4.49)
\end{aligned}
$$

The choice of which pressure is associated with which part of the total pressure being determined by the sign of that part so that normal gravitational mass gives negative acceleration or positive pressure, that is gravitational *attraction*. The ω_Λ that goes with the Λ equation of state involves negative pressure or positive acceleration is thus given by

$$
\begin{aligned}
\omega_\Lambda &= P_\Lambda/(c^2\rho_\Lambda) = -(3c^2/(8\pi G))(c/R_\Lambda)^2/(c^2\rho_\Lambda) &(4.50)\\
&= -1 &(4.51)
\end{aligned}
$$

an exact value from theory for ω_Λ at the centre of the measurement range at equation (2.13).

A simple interpretation of the force structure that is the cause of the acceleration (4.5) that is operative in this model is obtained if we introduce a mass density $\rho^\dagger$ to account for the cosmological constant Λ as

$$
{\rho^\dagger}_\Lambda = \Lambda c^2/(4\pi G) = 2\rho_\Lambda. \qquad (4.52)
$$

That is to say its values is twice the value of the usual density function at (2.22). The Friedman equation (2.4) repeated below at equation (4.53) with $P = 0$, can then be expressed as equation (4.54) through to equation (4.59).

$$
\begin{aligned}
0 &= 2\ddot{r} + \dot{r}^2/r - \Lambda r c^2. &(4.53)\\
\ddot{r} &= \Lambda r c^2/2 - \dot{r}^2/(2r). &(4.54)\\
&= \Lambda r c^2/2 - (4\pi G\rho r + \Lambda c^2 r/2)/3 &(4.55)\\
&= (\Lambda c^2 - 4\pi G\rho)r/3 &(4.56)\\
&= 4\pi r G(\rho^\dagger_\Lambda - \rho)/3 &(4.57)\\
&= 4\pi r^3 G(\rho^\dagger_\Lambda - \rho)/(3r^2) &(4.58)\\
&= {M^\dagger}_\Lambda G/r^2 - M_U G/r^2 &(4.59)\\
M^\dagger_\Lambda &= 4\pi r^3 \rho^\dagger_\Lambda/3 &(4.60)\\
M_U &= 4\pi r^3 \rho/3 &(4.61)
\end{aligned}
$$

Thus associating the density function $\rho^\dagger_\Lambda$ and consequent total mass $M^\dagger_\Lambda$ within universe with the cosmological constant Λ we get the very transparent formula for the dynamics under gravity, the *acceleration* that any particle would experience at the boundary of the universe being given by equation (4.59). It tells us clearly that the normal mass density ρ gravitationally causes the usual decelerating attraction of this particle to within the universe body whilst the dark energy mass density causes an acceleration that repels this particle towards the outside of the universe. The theoretical structure described here involves this as a naturally occurring effect which is built into this model. Consequently any need to introduce negative mass densities to describe the dark energy contribution together with the conceptually difficult concept of negative *total* pressure are removed from contention.

5 Conclusions

It has been shown that a subset of the measurements by the dark energy workers given at equations (2.10), (2.11) and (2.12) together with an input value for Hubble's constant now and the assumption that Rindler's constant, C, is an absolute constant lead to a unique solution of the Friedman equations. The fourth measurement from the dark energy observations at equation (2.13) is not necessary for finding this solution. In fact, we have found that this last measurement with the partial ω_Λ value, $\omega_\Lambda = -1$, is an exact result derivable from the unique model from the first three measurements. Thus the model derived from general relativity via Friedman's equations is conformable to all four of the dark energy measurements (2.10), (2.11), (2.12) and (2.13).

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