MID-TERM TEST 2010

1. $$y = \int x e^{4x^2}\,dx = \frac{1}{8}\int e^{u}\,du = \frac{1}{8}e^{4x^2} + C$$

$u = \cancel{x}4x^2$ $\boxed{1}$ $\boxed{1}$

$du = 8x\,dx$

2. $$dy \cdot \frac{y}{1+y^2} = x\,dx \Rightarrow \frac{1}{2}\ln|1+y^2) = \frac{1}{2}x^2 + C \quad \boxed{1}$$

$$\Rightarrow 1+y^2 = e^{2C}e^{x^2}$$

$$\Rightarrow y = \pm\left[e^{2C}e^{x^2} - 1\right]^{1/2}$$

$$y(0) = -1 \Rightarrow e^{2C} - 1 = 1 \Rightarrow e^{2C} = 2$$

and take negative root $\quad y = -\left(2e^{x^2} - 1\right)^{1/2}$ $\boxed{1}$

3. I. F. $$u = e^{-\int \frac{2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2} \quad \boxed{1}$$

$$\Rightarrow \frac{d}{dx}\left(y/x^2\right) = x \Rightarrow y/x^2 = \int x\,dx = \frac{1}{2}x^2 + C$$

$$\Rightarrow y = \frac{1}{2}x^4 + Cx^2 \quad \boxed{1}$$

4. $P(x,y) = 2xy^3 + 2 \Rightarrow \dfrac{\partial P}{\partial y} = 6xy^2$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ exact [1]

$Q(x,y) = 3x^2y^2 + e^{2y} \Rightarrow \dfrac{\partial Q}{\partial x} = 6xy^2$

$$F = \int P(x,y)\,dx = \int (2xy^3+2)\,dx = x^2y^3 + 2x + C_1(y)$$

$$\frac{\partial F}{\partial y} = 3x^2y^2 + \frac{dC_1}{dy} = Q(x,y) = 3x^2y^2 + e^{2y}$$

$$\Rightarrow \frac{dC_1}{dy} = e^{2y} \Rightarrow C_1(y) = \frac{1}{2}e^{2y} + C$$

$\Rightarrow F(x,y) = \text{constant} \Rightarrow x^2y^3 + 2x + \frac{1}{2}e^{2y} = CD$ [1]

5. $y'' + y' - 2y = 0 \Rightarrow \lambda^2 + \lambda - 2 = 0$

$\Rightarrow (\lambda+2)(\lambda-1) = 0 \Rightarrow \lambda_1 = -2, \ \lambda_2 = 1$

$\Rightarrow y(x) = C_1 e^{-2x} + C_2 e^{x}$ [1]

$y' = -2C_1 e^{-2x} + C_2 e^{x}$

$y(0) = 1 = C_1 + C_2$

$y'(0) = 2 = -2C_1 + C_2$

$\Rightarrow C_1 = -\frac{1}{3}, \ C_2 = \frac{4}{3}$

$\rightarrow y(x) = -\frac{1}{3}e^{-2x} + \frac{4}{3}e^{x}$ [1]