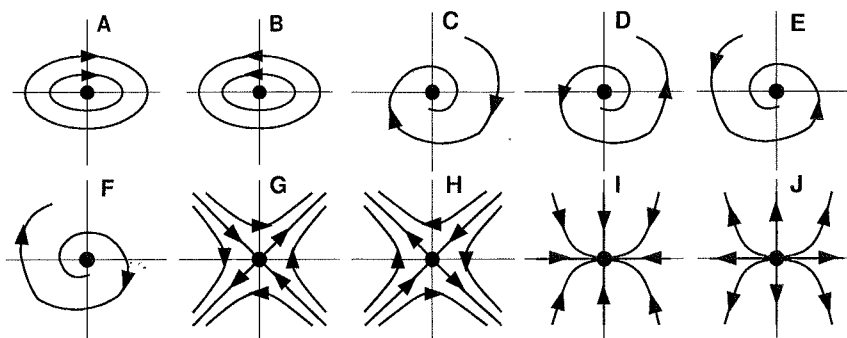


Differential Equations (MTH4102) Problem Sheet 10

Problem 28

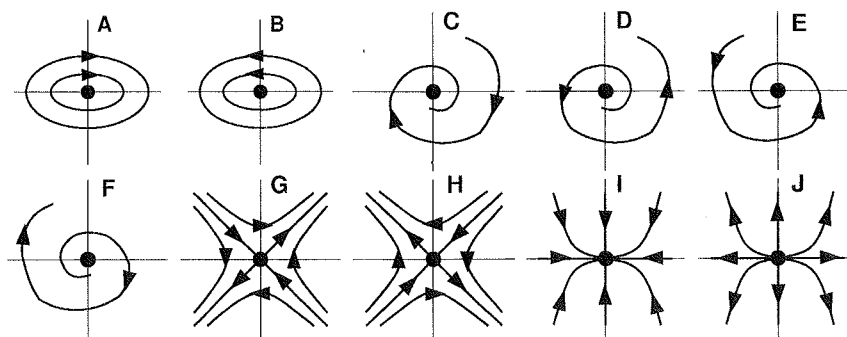
Assign to the following two dimensional differential equations the (qualitatively) correct phase portrait (as usual the y_1 -axis is the horizontal axis, the y_2 -axis the vertical axis).

- | | | | |
|--------------------------|----------------------|--------------------------|----------------------|
| a) $y_1' = 4y_1 + y_2,$ | $y_2' = 4y_2 + y_1$ | b) $y_1' = y_1 - 4y_2,$ | $y_2' = y_2 + 4y_1$ |
| c) $y_1' = y_1 + 4y_2,$ | $y_2' = y_2 - 4y_1$ | d) $y_1' = -y_1 + 4y_2,$ | $y_2' = -y_2 - 4y_1$ |
| e) $y_1' = -y_1 - 4y_2,$ | $y_2' = -y_2 + 4y_1$ | f) $y_1' = -4y_1 + y_2,$ | $y_2' = -4y_2 + y_1$ |
| g) $y_1' = 4y_2,$ | $y_2' = y_1$ | h) $y_1' = 4y_2,$ | $y_2' = -y_1$ |
| i) $y_1' = -4y_2,$ | $y_2' = y_1$ | j) $y_1' = -4y_2,$ | $y_2' = -y_1$ |



Problem 29

For initial condition $y_1(0) = 0, y_2(0) = 1$, sketch for each of the following phase portraits the functions $y_1(x)$ and $y_2(x)$ in a diagram (as usual the y_1 -axis is the horizontal axis, the y_2 -axis the vertical axis).



Problem 30

$$y_1(x) = e^x \cos(x), \quad y_2(x) = e^x \sin(x)$$

is a particular solution of a system of two linear differential equations with constant coefficients.

- a) Write down the system of differential equations.
- b) Sketch the phase portrait of the system of differential equations.

Problem J

Consider the system of differential equations

$$y_1' = (\sqrt{2} - 1)y_1 + (2/\sqrt{3} + 1)y_2, \quad y_2' = -\sqrt{3}y_1 + (\sqrt{2} + 1)y_2.$$

- a) Sketch (qualitatively) the phase portrait. (30 marks)
- b) For initial condition $y_1(0) = 1$, $y_2(0) = -1$, compute $y_1'(0)$ and $y_2'(0)$. (30 marks)
- c) Sketch (qualitatively) the solution for the initial condition of part b) in a x - y_1 and x - y_2 diagram. (40 marks)

END