

MAS/118 Differential Equations

2:30 pm, Wednesday 14th May, 2008

Duration: 2 hours

You should attempt all questions. Marks awarded are shown next to the question. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

This is an OPEN BOOK exam

- permitted: any printed material, e.g. books
any handwritten notes
photocopies of any kind
- prohibited: using electronic devices, e.g. calculators or mobile phones
sharing material with other students

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Problem 1

- a) Determine the general solution of the differential equation

$$y' = \frac{3x}{2 - x - x^2}$$

by integration.

(10 points)

- b) Determine the solution of the initial value problem

$$y' = e^{x+y}, \quad y(0) = 0$$

by separation of variables.

(8 points)

- c) Show that $y = x/\sinh(x)$ solves the initial value problem

$$y' = \frac{y}{x} (1 - \sqrt{x^2 + y^2}), \quad y(0) = 1.$$

(12 points)

Problem 2

Consider the first order differential equation

$$\frac{2(1+x)^2 - y^2}{(1+x)} + 2(1+x+y)y' = 0.$$

- a) Show that the differential equation is not an exact differential equation.
(4 points)

- b) Use the integrating factor

$$\mu = \frac{1}{1+x}$$

to compute the general solution (in implicit form) of the differential equation.
(8 points)

- c) Fix the constant of integration according to the initial condition $y(0) = -2$ and state the solution of the initial value problem (in explicit form).
(8 points)

Problem 3

- a) Compute the general solution of the linear second order differential equation

$$y'' + 3y' + 2y = 0.$$

(4 points)

- b) Compute a particular solution of the inhomogeneous differential equation

$$y'' + 3y' + 2y = \frac{1}{(1 + e^x)^2}$$

and hence write down the general solution of this differential equation.

(10 points)

- c) Fix the constants of integration of the solution of part b) according to the initial condition $y(0) = 1$, $y'(0) = -3$ and sketch this solution in a diagram.

(12 points)

Problem 4

Consider the linear two-dimensional system of differential equations

$$y_1' = y_1 + 2y_2/3, \quad y_2' = -4y_1/3 - y_2/3.$$

- a) Solve the eigenvalue problem of the corresponding two-dimensional matrix, i.e. compute the eigenvalues and the eigenvectors. (8 points)

- b) Determine the general solution of the system of differential equations. (2 points)

- c) Fix the constants of integration in such a way that the solution fulfils the initial condition $y_1(0) = 1$ and $y_2(0) = -1$. Write down the corresponding solution $y_1(x)$ and $y_2(x)$ of the system of differential equations using real-valued functions. (8 points)

- d) State the phase portrait which shows the dynamics of the system of differential equations (horizontal axis: y_1 -coordinate, vertical axis: y_2 -coordinate). (6 points)

