Queen Mary
University of London

# B. Sc. Examination by course unit 2010

## MTH4102 Differential Equations

**Duration: 2 hours**

**Date and time: 17th May 2010, 1000-1200**

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**Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.**

**You should attempt all questions. Marks awarded are shown next to the questions.**

**Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.**

**Complete all rough workings in the answer book and cross through any work which is not to be assessed.**

**This is an OPEN BOOK exam**

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| **permitted:** | **any printed material, e.g. books** |
| | **any handwritten notes** |
| | **photocopies of any kind** |
| **prohibited:** | **using electronic devices, e.g. calculators or mobile phones** |
| | **sharing material with other students** |

**Exam papers must not be removed from the examination room.**

**Examiner(s): J. E. Lidsey**

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 **TURN OVER**

**Question 1**

(a) Show that $y = \sqrt{x^4 + 2x}$ is a particular solution of the differential equation

$$y' = \frac{2y}{x} - \frac{3}{y}.$$

(8 marks)

(b) Determine the solution of the initial value problem

$$\frac{dy}{dx} = \frac{1}{x^2 + 7x + 12}, \qquad y(0) = 0$$

by the method of direct integration. (8 marks)

(c) Given the initial condition $y(0) = -1$, find the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{x\tan(x^2)}{2y}.$$

(8 marks)

**Question 2**

(a) Use the integrating factor method to find the general solution to the linear, inhomogeneous differential equation

$$\frac{dy}{dx} = 2xy + xe^{2x^2}.$$

(10 marks)

(b) Show that the differential equation

$$(x - y^2) + (2xy + x^2y^3)\frac{dy}{dx} = 0$$

is not an exact differential equation. (7 marks)

(c) Use the integrating factor $\mu = 1/x^2$ to determine the general solution (in implicit form) of the differential equation given in part (b). (9 marks)

**Question 3**

(a) Find the solution to the linear, second-order differential equation

$$y'' + 2y' - 3y = 0$$

that satisfies the initial conditions $y(0) = -1$ and $y'(0) = 1$. (8 marks)

(b) Find the general solution to the inhomogeneous differential equation

$$y'' + 2y' - 3y = 5\sin(3x)\,.$$

(6 marks)

(c) Find a particular solution to the inhomogeneous differential equation

$$y'' - 3y' + 2y = \frac{e^{3x}}{1+e^x}\,.$$

(10 marks)

**Question 4** Consider the linear two-dimensional system of differential equations

$$y_1' = 2y_1 - 3y_2, \qquad y_2' = 3y_1 + 2y_2\,.$$

(a) Solve the eigenvalue problem of the corresponding two-dimensional matrix, i.e., compute the eigenvalues and eigenvectors. (10 marks)

(b) Determine the general solution of the system of differential equations. (2 marks)

(c) Find the values of the constants of integration such that the solution satisfies the initial conditions $y_1(0) = 2$ and $y_2(0) = 1$. (4 marks)

(d) Express the corresponding solutions $y_1(x)$ and $y_2(x)$ of the system of differential equations in terms of real-valued functions. (6 marks)

(e) Calculate the gradient (slope) of the solutions $y_1(x)$ and $y_2(x)$ of part (c) at $x = 0$. (4 marks)

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**End of Paper**