Queen Mary
University of London

B. Sc. Examination by course unit 2009

MAS118 Differential Equations

Duration: 2 hours

Date and time: 29th April 2009, 1000-1200

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

This is an OPEN BOOK exam

permitted:	**any printed material, e.g. books** **any handwritten notes** **photocopies of any kind**
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Exam papers must not be removed from the examination room.

Examiner(s): J. E. Lidsey

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Question 1

(a) Given the initial condition $y(0) = -1$, find the particular solution to the differential equation

$$\frac{dy}{dx} = \frac{x}{3 + 4x^2}$$

by the method of direct integration. (10 marks)

(b) Determine the solution of the initial value problem

$$\frac{dy}{dx} = \frac{(x-2)e^{x^2-4x}}{y}, \qquad y(0) = -1$$

by the method of separation of variables. (11 marks)

(c) Show that $y = \sqrt{1 + e^{2x}}$ is a particular solution to the differential equation

$$\frac{dy}{dx} = y - \frac{1}{y}\,.$$

(6 marks)

Question 2

(a) Find the general solution to the linear, inhomogeneous differential equation

$$\frac{dy}{dx} + \frac{2}{x}y = x^2 + x^3$$

by the integrating factor method. (8 marks)

(b) Show that the differential equation

$$\left(y^3 + \frac{1}{\cos^2(x)}\right)\sin(x) - 3y^2\left(1 + \cos(x)\right)\frac{dy}{dx} = 0$$

is an exact differential equation. (8 marks)

(c) Find the general solution to the differential equation given in part (b) (in implicit form). (8 marks)

Question 3

(a) Find the solution to the linear, second-order differential equation

$$3y'' - 4y' - 4y = 0$$

that satisfies the initial conditions $y(0) = 1$ and $y'(0) = -1$. (10 marks)

(b) Find the general solution to the inhomogeneous differential equation

$$3y'' - 4y' - 4y = 2e^{2x}\,.$$

(6 marks)

(c) Find a particular solution to the inhomogeneous differential equation

$$y'' + 2y' - 8y = 10\cos(2x)\,.$$

(6 marks)

Question 4 Consider the linear two-dimensional system of differential equations

$$y_1' = -y_1 - 2y_2, \qquad y_2' = 2y_1 - y_2\,.$$

(a) Solve the eigenvalue problem of the corresponding two-dimensional matrix, i.e., compute the eigenvalues and eigenvectors. (10 marks)

(b) Determine the general solution of the system of differential equations. (2 marks)

(c) Find the values of the constants of integration such that the solution satisfies the initial conditions $y_1(0) = 0$ and $y_2(0) = 1$. Write down the corresponding solutions $y_1(x)$ and $y_2(x)$ of the system of differential equations in terms of real-valued functions. (5 marks)

(d) Calculate the gradient (slope) of the solutions $y_1(x)$ and $y_2(x)$ of part (c) at $x = 0$. (4 marks)

(e) Sketch the solutions $y_1(x)$ and $y_2(x)$ of part (c) in $x - y$ diagrams. (6 marks)

End of Paper