

Research Interests

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My research interests lie in the area of Mathematical Classical General Relativity, that is, the qualitative study of the properties of solutions of the equations governing the gravitational field. In particular, my research focuses on the study of asymptotic and conformal properties of the Einstein field equations.

In recent years I have been analysing the structure of spatial infinity and null infinity for asymptotically flat spacetimes. This is a long-term project geared towards providing a mathematically rigorous framework to R. Penrose's proposal of characterising the gravitational field of isolated systems in General Relativity by means of the notion of asymptotic simplicity [4]. This project will provide an assessment of the physical relevance and limitation of Penrose's ideas. Extending H. Friedrich's ideas —see e.g. [3] on the conformal field equations I have been able to show that the standard smoothness assumptions on the behaviour of the gravitational field at the conformal boundary of spacetime —null infinity— are more restrictive than originally thought - see [LOP12, LOP15, LOP16, LOP17]. More precisely, I have gathered evidence for a rigidity conjecture for the notion of asymptotic simplicity. This conjecture states that if a given asymptotically flat spacetime happens to have a smooth extension to null infinity (both future and past), then any Cauchy hypersurface from which the spacetime could have arisen is necessarily stationary near infinity. The evidence for this conjecture shows that the developments of the initial data sets for spacetimes containing dynamical black holes —like the Brill-Lindquist, Misner and Bowen-York— do not have a smooth null infinity —see [LOP28].

Currently, I am particularly interested in the effects that the non-smoothness of the gravitational field at infinity has on objects of physical interest for which the notion of asymptotic simplicity provides a suitable framework. These include reductions of the asymptotic symmetry group (the BMS group), the Bondi mass, angular momentum and the Newman-Penrose constants. On a short term I will be working on a proof for the rigidity conjecture for asymptotically simple spacetimes mentioned in the previous paragraph —starting with the case of time symmetric, conformally flat initial data, and then move to more general classes of data. Once obtained, such a proof should exhibit a very delicate interplay between conformal structures of the spacetime, PDE-theory aspects of the propagation equations and group theoretical properties of the Einstein field equations.

Initial data sets of the type arising in the rigidity conjecture have been abstractly constructed by means of some gluing constructions by Corvino & Schoen and Chruściel & Delay —see e.g. [2, 1]. Recently, I have started a collaboration with M. Ansorg (Max-Planck Institute for Gravitational Physics, Golm, Germany) to implement these gluing constructions numerically. In this way one will obtain initial data sets for numerical simulations with a smooth null infinity. A natural progression of these calculations is the numerical evolution of such initial data sets.

My work on the asymptotics of the gravitational field has led to a working interest on the properties of the Einstein constraint equations and the construction of initial data sets. In particular, using some ideas from asymptotics I have been able to provide proofs for some conjectures concerning the existence of particular hypersurfaces for certain spacetimes —like conformally flat slices in the Kerr spacetime [LOP13, LOP14]. In the future I plan to work on some aspects of the construction and characterisation of initial data sets with a particular emphasis on finding ways of encoding physics in the solutions to the Einstein constraints —so that the resulting initial data sets are more realistic.

On a related issue, I have a collaboration with A. García-Parrado (University of Linköping, Sweden) in order to develop a 3+1 version of the equivalence problem for spacetime manifolds —see [LOP18, LOP20]. The aim of this project is to combine analytical techniques with ideas of the study of exact solutions to the Einstein field equations to obtain invariant characterisations of initial data sets of interesting and physically relevant solutions —like Schwarzschild and Kerr spacetimes. These characterisations have a natural application in numerical simulations.

Note: LOPX refers to publication number X in the attached list of publications.

References

- [1] P. T. Chruściel & E. Delay, *Existence of non-trivial, vacuum, asymptotically simple spacetimes*, Class. Quantum Grav. **19**, L71 (2002).
- [2] J. Corvino, *Scalar curvature deformations and a gluing construction for the Einstein constraint equations*, Comm. Math. Phys. **214**, 137 (2000).
- [3] H. Friedrich, *Gravitational fields near space-like and null infinity*, J. Geom. Phys. **24**, 83 (1998).
- [4] R. Penrose, *Asymptotic properties of fields and space-times*, Phys. Rev. Lett. **10**, 66 (1963).