

Killing Spinor Initial Data

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Question:

how to characterise exact solutions of the Einstein field equations (Schwarzschild, Kerr) in terms of initial data?

One possible strategy:

- Find a **invariant characterisation** of the spacetime.
- Obtain a $3 + 1$ decomposition of the characterisation. This will give **necessary** conditions.
- In a second (more complicated) stage, show that the conditions are **sufficient** by constructing evolution equations for them. Add further conditions (if required) to reconstruct the original spacetime decomposition.

Which invariant characterisation?

A crucial feature of Schwarzschild and Kerr is that they are of Petrov type D.

The Petrov classification:

- An **algebraic characterisation** of the Weyl tensor of the spacetime, independent of any special coordinate system.

The Petrov classification:

- An **algebraic characterisation** of the Weyl tensor of the spacetime, independent of any special coordinate system.
- It is based in the **eigenvalue problem**:

$$\frac{1}{2}C_{\mu\nu}{}^{\lambda\rho}x_{\lambda\rho} = \lambda x_{\lambda\rho},$$

where the **self dual** of the Weyl tensor $C_{\mu\nu\lambda\rho}$ is given by

$$C_{\mu\nu\lambda\rho} \equiv C_{\mu\nu\lambda\rho} + \frac{i}{2}\epsilon_{\mu\nu\sigma\tau}C^{\sigma\tau}{}_{\lambda\rho},$$

and

$$x_{\mu\nu} \equiv X_{\mu\nu} + \frac{i}{2}\epsilon_{\mu\nu\sigma\tau}X^{\sigma\tau},$$

where $X_{\mu\nu} = X_{[\mu\nu]}$ is a **bivector**.

Principal null directions:

- **Petrov type I:** exists k^μ such that

$$k_{[\sigma} C_{\mu]\nu\lambda[\rho} k_{\tau]} k^\nu k^\lambda = 0.$$

- **Petrov type III:** exists k^μ such that

$$C_{\mu\nu\lambda[\rho} k_{\sigma]} k^\lambda = 0.$$

- **Petrov type N:** exists k^μ such that

$$C_{\mu\nu\lambda\rho} k^\lambda = 0.$$

- **Petrov type II:** exists k^μ such that

$$C_{\mu\nu\lambda[\rho} k_{\tau]} k^\nu k^\lambda = 0.$$

- **Petrov type D:** exist k^μ and l^μ such that

$$k_{[\sigma} C_{\mu]\nu\lambda[\rho} k_{\tau]} k^\nu k^\lambda = 0,$$

$$l_{[\sigma} C_{\mu]\nu\lambda[\rho} l_{\tau]} l^\nu l^\lambda = 0.$$

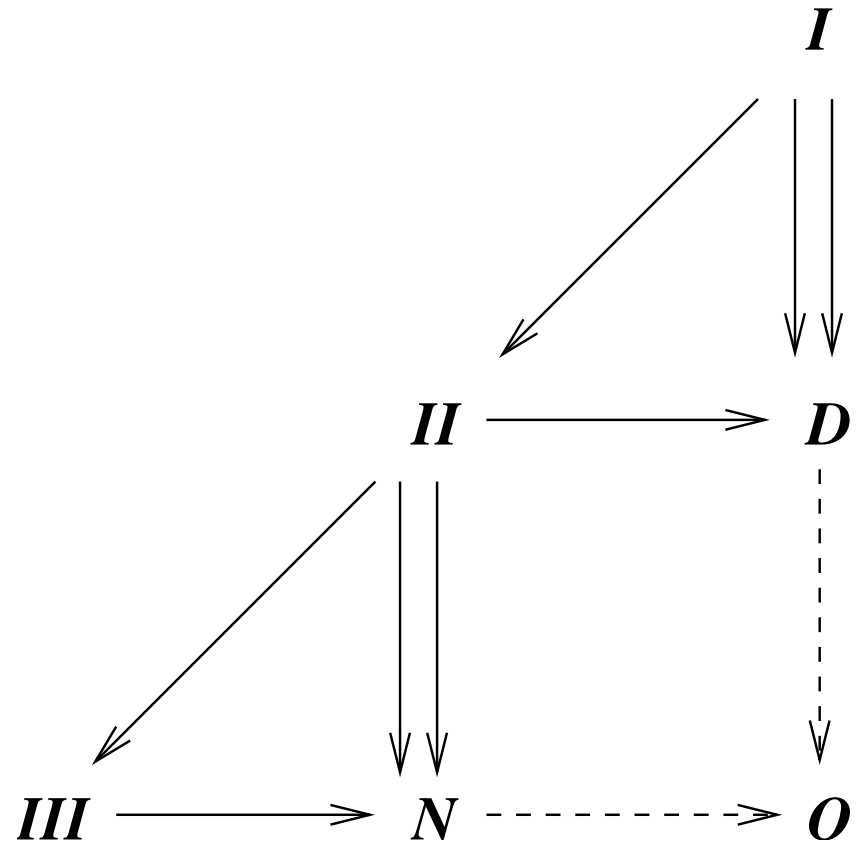
- **Petrov type O:** conformally flat,

$$C_{\mu\nu\lambda\rho} = 0.$$

Invariant characterisations using explicitly the **algebraic type** of the Weyl tensor contain involved algebraic conditions.

- Construction of **necessary conditions** on the initial data is more or less direct.
- However, the discussion of the propagation of the conditions —to discuss **sufficiency**— is much more complicated!

Penrose diagram:



As an example of the **explicit use of the Petrov type** consider the characterisation of Schwarzschild data given in ^a

- Necessary conditions given in terms of conditions on concomitants of E_{ij} and B_{ij} on the hypersurface.
- In order to ensure sufficiency, use an explicit formula for a candidate of the **stationary Killing vector** in terms of E_{ij} and B_{ij} .

^aA. García Parrado & JA Valiente Kroon. *Initial data sets for the Schwarzschild space-time*. Phys. Rev. D **75**, 024027 (2007).

Proposition 1. *Let*

$$\begin{aligned}\rho &= \left(\frac{1}{2} B_i^j B^{il} E_{jl} - \frac{1}{6} E_{ij} E_l^j E^{il} \right)^{1/3}, \\ P &= -\frac{1}{2} E^{ij} K_{ij} - \rho K - \frac{1}{6\rho} \epsilon^{jk}{}_i \left(E^{il} D_k B_{lj} + B^{il} D_k E_{lj} \right), \\ P_i &= \frac{1}{6\rho} (-B^{kl} D_i B_{kl} + E^{kl} D_i E_{kl}).\end{aligned}$$

Necessary conditions for an initial data set (h_{ij}, K_{ij}) , satisfying the Einstein vacuum constraints, on a manifold S to be a Schwarzschild initial data set are:

$$\begin{aligned}B_{ij} &= -\frac{1}{\rho} (B_i^k E_{kj} + B_j^k E_{ki}) & E_{ij} &= \frac{1}{\rho} (B_i^k B_{kj} - E_i^k E_{kj}) + 2\rho h_{ij}, \\ B_{ij} P^i P^j &= 0, & E_{jk} \epsilon^k{}_{li} P^j P^l - P B_{ij} P^j &= 0, \\ (P^2 + P^k P_k) B_{ij} + 2P E_{k(i} \epsilon^k{}_{j)l} P^l - 2P_{(i} B_{j)l} P^l &= 0, \\ \frac{1}{3} P_k P^k - P^2 + \frac{2}{3} E_{kl} P^k P^l &> 0, & \frac{1}{9\rho^2} (P^k P_k - P^2) + 2\rho &> 0.\end{aligned}$$

The KID candidate:

$$Y = \frac{\sqrt{|\rho P_i P^i - E_{ij} P^i P^j|}}{\rho^{11/6} \sqrt{3}},$$

$$Y^i = \frac{-\rho P P^i + E^{ij} P_j P - B^{kl} P_l P_m \epsilon^{im}_k}{\rho^{11/6} \sqrt{3} |\rho P_j P^j - E_{kl} P^k P^l|},$$

where

$$\rho = \left(\frac{1}{2} B_i^j B^{il} E_{jl} - \frac{1}{6} E_{ij} E_l^j E^{il} \right)^{1/3},$$

$$P = -\frac{1}{2} E^{ij} K_{ij} - \rho K - \frac{1}{6\rho} \epsilon^{jk}_i \left(E^{il} D_k B_{lj} + B^{il} D_k E_{lj} \right),$$

$$P_i = \frac{1}{6\rho} (-B^{kl} D_i B_{kl} + E^{kl} D_i E_{kl}).$$

An alternative?

Use the Petrov type in an indirect manner: *Killing spinors*.

- A Killing spinor is a totally symmetric spinor $\kappa_{(AB\dots P)} = \kappa_{AB\dots P}$ satisfying

$$\nabla_{Q'(Q} \kappa_{AB\dots P)} = 0.$$

- Consider the particular cases where the spinor has either one or two indices

$$\nabla_{A'(A} \kappa_{B)} = 0, \quad (\text{Twistor equation})$$

$$\nabla_{A'(A} \kappa_{BC)} = 0. \quad (\text{Valence-2 Killing spinor equation})$$

A brief tensor-spinor dictionary:

$$\xi^\mu \iff \xi^{AA'} = \sigma^{AA'}_{\mu} \xi^\mu,$$

$$\nabla_\mu \iff \nabla_{AA'} = \sigma_{AA'}^{\mu} \nabla_\mu,$$

$$g_{\mu\nu} \iff g_{AA'BB} = \sigma_{AA'}^{\mu} \sigma_{BB'}^{\nu} (\epsilon\epsilon)$$

- Note that κ_A and $\bar{\kappa}_{A'}$ do not have a tensorial correspondence!

$$\kappa_A \bar{\kappa}_{A'} \iff \text{null vector}$$

Gauge transformations: $t^C_A \in SL(2, C)$.

$$\epsilon_{AB} = t^C_A t^D_B \epsilon_{CD}.$$

- Spinorial representation of the Weyl tensor:

$$C_{\mu\nu\lambda\rho} = \epsilon_{A'B'}\epsilon_{C'D'}\Psi_{ABCD} + \epsilon_{AB}\epsilon_{CD}\bar{\Psi}_{A'B'C'D}.$$

- Petrov type in spinorial terms:

$$\Psi_{ABCD} = \alpha_{(A}\beta_B\gamma_C\delta_{D)}.$$

- Decomposition in irreducible spinors:

$$X_{A\dots P} = X_{(A\dots P)} + \epsilon's \times \text{symmetric contractions of } X.$$

These Killing spinor equations are **overdetermined** —i.e. do not admit a solution in a general spacetime $(\mathcal{M}, g_{\mu\nu})$.

Proposition 2. *A spacetime admits a solution to the **twistor equation** if and only if it is of **Petrov type N**.*

Proposition 3. *Any **Petrov type D** spacetime admits a **valence-2 Killing spinor**. Conversely, if a spacetime admits a **valence-2 Killing spinor** the the spacetime is of **Petrov type D** (and the spinor is algebraically special) or the spacetime is of **Petrov type N** (and the Killing spinor is degenerate).*

Further properties of (valence-2) Killing spinors:

Proposition 4. Any vacuum spacetime with a Killing spinor κ_{AB} has a (complex) Killing vector given by

$$\xi_{AA'} \equiv \nabla^Q_{A'} \kappa_{AQ}.$$

Thus, one has that

$$\nabla_{AA'} \xi_{BB'} + \nabla_{BB'} \xi_{AA'} = 0.$$

In addition one has that

Proposition 5. For *Kerr* the Killing vector associated to κ_{AB} is *degenerate* (i.e. its real and imaginary parts are proportional).

- This is almost a characterisation of the Kerr. The converse may require *asymptotic flatness*.

Killing spinors are related to other (tensorial) objects —Killing-Yano tensors:

$$K_{\mu\nu} = i(\kappa_{AB}\epsilon_{a'B'} - \epsilon_{AB}\bar{\kappa}_{A'B'}).$$

Then

$$\nabla_{(\mu}K_{\nu)\lambda} = 0.$$

Further

$$H_{\mu\nu} = K_{\mu}{}^{\lambda}K_{\lambda\nu},$$

with

$$\nabla_{(\mu}H_{\nu)\lambda} = 0.$$

That is, $H_{\mu\nu}$ is a Killing tensor —related to Carter's constant of motion in the case of Kerr.

- Thus, most of this talk, could be reformulated in tensorial terms rather spinorial.

*Can one construct for Killings spinors an analogue
of the theory of KIDs?*

KIDs

When do initial data (S, h_{ij}, K_{ij}) for the vacuum EFE give rise to a development with a Killing vector?

- If and only if (S, h_{ij}, K_{ij}) admits a Killing Initial Data (KID), i.e. (N, Y^i) satisfying

$$2NK_{ij} + 2D_{(i}Y_{j)} = 0,$$

$$\mathcal{L}_Y K_{ij} + D_i D_j N = N(r_{ij} + K K_{ij} - 2K_{ik} K_j^k).$$

Proposition 6. *KIDs in (S, h_{ij}, K_{ij}) are in one to one correspondence with Killing vectors in $(\mathcal{M}, g_{\mu\nu})$.*

In order to obtain a “3+1” decomposition of the Killing spinor equations, consider space spinors —or SU(2) spinors.

- The space spinor formalism is used, for example, in the so-called Ashtekar variables.
- Given a foliation of spacetime with normal given by τ_μ ($\tau_\mu \tau^\mu = 2$), contract suitably $\tau_{AA'}$ to eliminate all the primed indices in the expressions. Decompose vectors using

$$\xi_{AA'} = \frac{1}{2} \tau_{AA'} \xi - \tau_{A'}^C \xi_{AC}$$

with

$$\begin{aligned} \xi &\equiv \xi_{AA'} \xi^{AA'}, \\ \xi_{AB} &\equiv \tau^{A'}_{(A} \xi_{B)A'}. \end{aligned}$$

A small dictionary of space spinors:

Gauge transformations: $t^C_A \in SU(2)$.

$$\tau_{CC'} t^C_A \bar{t}^{C'}_{A'}$$

Hermitian conjugation

$$\hat{\mu}_A = \tau_A^{A'} \bar{\mu}_{A'}, \quad \hat{\hat{\mu}}_A = -\mu_A.$$

- $X_{A_1 B_1 \dots A_p B_p}$ associated to a real spatial tensor iff

$$X_{A_1 B_1 \dots A_p B_p} = (-1)^p \hat{X}_{A_1 B_1 \dots A_p B_p}.$$

- In the latter case

$$X_{i_1 \dots i_p} = \sigma^{A_1 B_1}_{i_1} \dots \sigma^{A_p B_p}_{i_p} \hat{X}_{A_1 B_1 \dots A_p B_p}.$$

- Derivatives:

$$\nabla_{AB} = \tau_B^{A'} \nabla_{AA'} \quad (\text{Sen connection})$$

- Derivative intrinsic to the leaves of the foliation:

$$D_{AB}\kappa_C = \nabla_{AB}\kappa_C - \frac{1}{2}K_{ABCQ}\kappa^Q,$$

where K_{ABCD} is the second fundamental form.

- Decomposition:

If $X_{A_1 B_1 \dots A_p B_p} = X_{(A_1 B_1 \dots A_p B_p)}$, then $X_{i_1 \dots i_p}$ is symmetric and tracefree!

Propagation of the “twistor equation”

Proposition 7. *Let $\mathring{\kappa}_A$ be a spinorial field on \mathcal{M} defined in a neighbourhood of S_0 such that for $p \in S_0$,*

$$\nabla_{A'(A}\mathring{\kappa}_{B)}(p) = 0, \quad \nabla_{EE'}\nabla_{A'(A}\mathring{\kappa}_{B)}(p) = 0.$$

Then there exists a spinor field κ_A on \mathcal{M} satisfying

$$\begin{aligned} \nabla_{A'(A}\kappa_{B)} &= 0, \\ p \in S_0, \quad \kappa_A(p) &= \mathring{\kappa}_A(p), \quad \nabla_{AA'}\kappa_B(p) = \nabla_{AA'}\mathring{\kappa}_B(p). \end{aligned}$$

Proof:

Let

$$H_{A'AB} \equiv \nabla_{A'(A} \kappa_{B)} \implies \square H_{A'AB} = \nabla_{A'(A} \square \kappa_{B)} + 2\Psi_{AB}{}^{PQ} H_{A'PQ}$$

A necessary condition for κ_{AB} to be Killing spinor is that

$$\square \kappa_A = 0.$$

So, consider a *Killing spinor candidate* satisfying the above with initial data given by

$$p \in S_0, \quad \kappa_A(p) = \dot{\kappa}_A(p), \quad \nabla_{AA'} \kappa_B(p) = \nabla_{AA'} \dot{\kappa}_B(p).$$

Thus, one has

$$\square H_{A'AB} = 2\Psi_{AB}{}^{PQ} H_{A'PQ},$$

which together with the initial data

$$\nabla_{A'(A} \dot{\kappa}_{B)}(p) = 0, \quad \nabla_{EE'} \nabla_{A'(A} \dot{\kappa}_{B)}(p) = 0,$$

imply $H_{A'AB} = 0$, and hence κ_{AB} is a Killing spinor!

Proposition 8 (Twistor initial data). *If on \mathcal{S} there exists a $\tilde{\kappa}_A$ such that*

$$\begin{aligned}
D_{(AB}\tilde{\kappa}_{C)} + \frac{1}{2}K^Q{}_{(ABC)}\tilde{\kappa}_Q &= 0, \\
2D_{(AB}D_{F)C}\tilde{\kappa}^C + \frac{1}{2}\tilde{\kappa}_{(F}D_{AB)}K^{CD}{}_{CD} - \frac{1}{2}K^{CD}{}_{CD}D_{(BF}\tilde{\kappa}_{A)} \\
&\quad - 3\Omega_{CD(BF}D^{CD}\tilde{\kappa}_{A)} + \frac{3}{2}\Omega_{(AF}{}^{DH}\Omega_{B)CDH}\tilde{\kappa}^C - 3(iB_{ABFC} + E_{ABFC})\tilde{\kappa}^C \\
&\quad + \frac{3}{4}K^{DH}{}_{DH}\Omega_{ABFC}\tilde{\kappa}^C + \Omega_{ABF}{}^D D_{CD}\tilde{\kappa}^C = 0.
\end{aligned}$$

Then there exists a spinorial field $\mathring{\kappa}_A$ on \mathcal{M} satisfying

$$\nabla_{A'(A}\mathring{\kappa}_{B)}(p) = 0, \quad \nabla_{EE'}\nabla_{A'(A}\mathring{\kappa}_{B)}(p) = 0.$$

Valence-1 Killing spinor data equations in the time symmetric case:

$$D_{(AB}\kappa_{C)} = 0,$$
$$2D_{(AB}D_{F)C}\kappa^C - 3E_{ABFC}\kappa^C = 0.$$

How to extend this discussion to **valence-2 Killing spinors**?

Define

$$H_{A'ABC} \equiv \nabla_{A'(A} \kappa_{BC)},$$

$$S_{AA'BB'} \equiv \nabla_{AA'} \xi_{BB'} + \nabla_{BB'} \xi_{AA'} = -\frac{1}{4} \nabla^Q_{A'} H_{B'ABQ},$$

where $\xi_{AA'} = \nabla^Q_{A'} \kappa_{AQ}$.

A lengthy calculation renders

$$\square H_{A'ABC} = 4\Psi_{(AB}{}^{PQ} H_{C)PQA'} + 8\nabla_{(A}{}^{Q'} S_{BC)Q'A'}.$$

- Thus, one has to consider the propagation of the valence-2 Killing spinor equation in parallel to the propagation of the associated Killing equation!

Proposition 9. Let $\mathring{\kappa}_{AB}$ be a spinor defined on a neighbourhood of \mathcal{S}_0 such that for $p \in \mathcal{S}_0$ one has

$$\nabla_{A'(A} \mathring{\kappa}_{BC)}(p) = 0,$$

$$\nabla_{EE'} \nabla_{A'(A} \mathring{\kappa}_{BC)}(p) = 0,$$

$$(\nabla_{AA'} \nabla_{B'}^P \mathring{\kappa}_{BP} + \nabla_{BB'} \nabla_{A'}^P \mathring{\kappa}_{AP})(p) = 0,$$

$$\nabla_{EE'} (\nabla_{AA'} \nabla_{B'}^P \mathring{\kappa}_{BP} + \nabla_{BB'} \nabla_{A'}^P \mathring{\kappa}_{AP})(p) = 0.$$

Then there is a spinor κ_{AB} on \mathcal{M} satisfying

$$\nabla_{A'(A} \kappa_{BC)} = 0,$$

$$p \in \mathcal{S}_0, \quad \kappa_{AB}(p) = \mathring{\kappa}_{AB}(p), \quad \nabla_{EE'} \kappa_{AB}(p) = \nabla_{EE'} \mathring{\kappa}_{AB}(p).$$

Proof:

From the Killing spinor equation one deduces:

$$\square \kappa_{AB} = -\Psi_{ABPQ} \kappa^{PQ}.$$

From it and the initial conditions

$$\kappa_{AB}(p) = \dot{\kappa}_{AB}(p), \quad \nabla_{CC'} \kappa_{AB}(p) = \nabla_{CC'} \dot{\kappa}_{AB}(p), \quad p \in S_0,$$

one constructs a *Killing spinor candidate*. Another lengthy calculation renders

$$\begin{aligned} \square S_{AA'BB'} &= -3\nabla_{AA'}(\Psi_B^{PQR} H_{B'PQR}) - 3\nabla_{BB'}(\Psi_A^{PQR} H_{A'PQR}) \\ &\quad + 2\Psi_{AB}^{PQ} S_{PA'QB'} + 2\bar{\Psi}_{A'B'}^{P'Q'} S_{AP'BQ'}. \end{aligned}$$

The latter, together with

$$\square H_{A'ABC} = 4\Psi_{(AB}^{PQ} H_{C)PQA'} + 8\nabla_{(A}^{Q'} S_{BC)Q'A'}.$$

and the given initial data imply

$$H_{A'ABC} = 0, \quad S_{AA'BB'} = 0.$$

Proposition 10. *If on \mathcal{S} there is a $\tilde{\kappa}_{AB}$ such that*

$$D_{(AB}\tilde{\kappa}_{CD)} - K_{E(ABC}\tilde{\kappa}_{D)}{}^E = 0,$$

$$\begin{aligned} D_{(AC}D_B{}^D\tilde{\kappa}_{F)D} + \frac{1}{2}D_{(AB}(\tilde{\kappa}^{DH}\Omega_{CF)DH}) + \Omega_{DH(BF}D^{DH}\tilde{\kappa}_{AC)} \\ - \frac{1}{2}\Omega_{H(ABC}D_F{}^D\tilde{\kappa}_D{}^H - \frac{1}{2}\Omega_{H(ACF}D^{DH}\tilde{\kappa}_{B)D} + 2(iB_{D(BCF} + E_{D(BCF)}\tilde{\kappa}_A)^D \\ - \left(\frac{1}{3}K^{HL}{}_{HL}\Omega_{D(ABC} + \Omega_{DHL(A}\Omega_{BC}{}^{HL})\right)\tilde{\kappa}_F{}^D + \frac{1}{2}\tilde{\kappa}^{DH}\Omega_{DHL(A}\Omega_{BCF)}{}^L \\ - \frac{1}{3}\tilde{\kappa}_{(AB}D_{CF)}K^{DH}{}_{DH} = 0, \end{aligned}$$

and

$$\tilde{\xi} \equiv D^{PQ}\tilde{\kappa}_{PQ}, \quad \tilde{\xi}_{BF} \equiv -\frac{1}{2}K^{DA}{}_{DA}\tilde{\kappa}_{BF} + \frac{3}{4}\tilde{\kappa}^{DA}\Omega_{BFDA} + \frac{3}{2}D_{(F}{}^D\tilde{\kappa}_{B)D},$$

satisfy

$$\begin{aligned} D_{AB}\tilde{\xi}_{CD} + D_{CD}\tilde{\xi}_{AB} + \tilde{\xi}K_{ABCD} = 0, \\ D_{DL}D_{CF}\tilde{\xi} + D_{CF}D_{DL}\tilde{\xi} + K_{DL}{}^{AB}D_{AB}\tilde{\xi}_{CF} + K_{CF}{}^{AB}D_{AB}\tilde{\xi}_{DL} \\ - \tilde{\xi}(2E_{CDL} - K_{CF}{}^{AB}K_{DLAB}) - D_{DL}(\tilde{\xi}^{AB}K_{CFAB}) \\ - D_{CF}(\tilde{\xi}^{AB}K_{DLAB}) + 4i\tilde{\xi}^A{}_{(C}B_{DFL)A} = 0. \end{aligned}$$

there exists a $\mathring{\kappa}_{AB}$ on \mathcal{M} such that

$$\nabla_{A'(A} \mathring{\kappa}_{BC)}(p) = 0,$$

$$\nabla_{EE'} \nabla_{A'(A} \mathring{\kappa}_{BC)}(p) = 0,$$

$$\left(\nabla_{AA'} \nabla^P_{B'} \mathring{\kappa}_{BP} + \nabla_{BB'} \nabla^P_{A'} \mathring{\kappa}_{AP} \right) (p) = 0,$$

$$\nabla_{EE'} \left(\nabla_{AA'} \nabla^P_{B'} \mathring{\kappa}_{BP} + \nabla_{BB'} \nabla^P_{A'} \mathring{\kappa}_{AP} \right) (p) = 0.$$

Valence-2 Killing spinor data equations in the **time symmetric case**:

$$D_{(AB}\kappa_{CD)} = 0,$$

$$D_{(AC}D_B^D\kappa_{F)D} + 2E_{D(BCF}\kappa_{A)}^D = 0,$$

$$D_{AB}D_{(C}^Q\kappa_{D)Q} + D_{CD}D_{(A}^Q\kappa_{B)Q} = 0,$$

$$D_{AB}D_{CD}D^{PQ}\kappa_{PQ} + D_{CD}D_{AB}D^{PQ}\kappa_{PQ} - 2D^{PQ}\kappa_{PQ}E_{ABCD} = 0.$$

Characterisation of “Kerr-like initial data”:

Proposition 11. *If (S, h_{ij}, K_{ij}) does not admit a solution to the valence-1 Killing spinor equations but admits a solution to the valence-2 Killing spinor equations, and in addition the associated Killing vector is degenerate (real), then the initial data is Kerr-like.*

- To obtain **Kerr data** one would probably require **asymptotic Euclideanity**.

So what?

- Can one construct geometric invariants from these equations (*à la Dain*)^a?
 - It has been shown that for time symmetric data one can construct a **generalised Killing vector**, solution of the composition of the KID operator and its adjoint.
 - This generalised Killing vector is a **critical point** of a certain (non-negative) functional. The functional vanishes if and only if the initial data is static.
- **Can one proceed in a similar fashion here?** Consider generalised Killing vectors which extremalise a certain (non-negative) functional. Critical points of the functional satisfy the generalised Killing spinor equation obtained by composing the Killing spinor operator with its formal adjoint.

^aS. Dain. *A New Geometric Invariant on Initial Data for the Einstein Equations*. Phys. Rev. Lett **93**, 231101 (2004)

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Also of interest:

- J. A. Valiente Kroon, *A characterisation of Schwarzschild initial data*, Phys. Rev. D **72**, 08003 (2005).
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