CW4 · SOLUTIONS

(1) (a) $\{1,2,3,4\} \cup \{1,3,5,7\} = \{1,2,3,4,5,7\}$

(b) $\{1,8,6,4\} \cap \{10,4,6,8\} = \{4,6,8\}$

(c) $\{1,4,9\} \cup \emptyset = \{1,4,9\}$

(d) $\{5,7,1,4\} \cap \{0,2,6\} = \emptyset$

(e) $\{5005, \text{apple}\}$

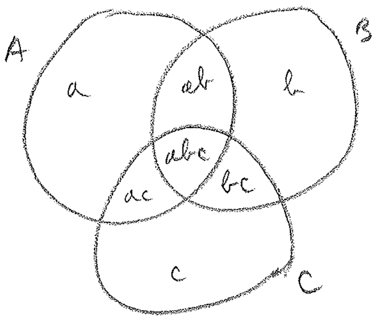
(f) if $n \in \{x \in \mathbb{Z} : x \text{ is divisible by } 2\} \cap \{x \in \mathbb{Z} : x \text{ is divisible by } 3\}$ then n is divisible both by 2 and 3 so we conclude that n is divisible by 6.

Thus: $\{x \in \mathbb{Z} : x \text{ divisible by } 2\} \cap \{x \in \mathbb{Z} : x \text{ divisible by } 3\} =$

$$= \{x \in \mathbb{Z} : x \text{ divisible by } 6\}.$$

(g) $\{1,2,3\} \times \{a,b,c\} = \{ (1,a), (1,b), (1,c),$
$(2,a), (2,b), (2,c),$
$(3,a), (3,b), (3,c) \}$

(2)



$B \cap C = bc \cup abc$ so

$A \setminus (B \cap C) = a \cup ac \cup ab$

EQUAL

$A \setminus B = a \cup ac$

$A \setminus C = a \cup ab$ so

$(A \setminus B) \cup (A \setminus C) = (a \cup ac) \cup (a \cup ab) = a \cup ac \cup ab$

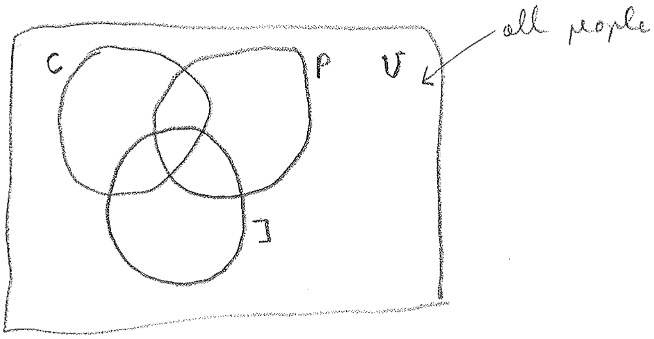
So we conclude $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

(3) Let C = set of people enjoying classical music

J = — — — — — jazz — — —

P = — — — — — pop — — —

③ (CONTINUED). The picture is :



Data :

$|C| = 18$ $|C \cap P| = 5$

$|P| = 11$ $|J \cap P| = 7$

$|J| = 18$ $|C \cap J| = 9$

$|U| = 30$ $|C \cap J \cap P| = 2$

(a) Question asks for $|C \cup J|$, and we can calculate it as

$$|C \cup J| = |C| + |J| - |C \cap J| = 18 + 18 - 9 = 27$$

(b) Question asks for $|U \setminus (C \cup P \cup J)| = |U| - |C \cup P \cup J|$

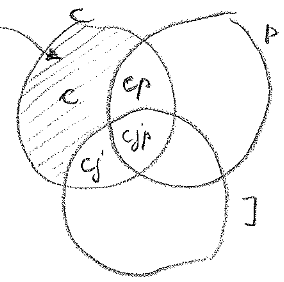
Using INCLUSION-EXCLUSION formula,

$$|C \cup P \cup J| = |C| + |P| + |J| - |C \cap P| - |C \cap J| - |J \cap P| + |C \cap J \cap P| =$$

$$= 18 + 11 + 18 - 5 - 9 - 7 + 2 = 28$$

So $|U \setminus (C \cup P \cup J)| = 30 - 28 = 2$.

(c) Question asks for $|C \setminus (J \cup P)| = |c|$



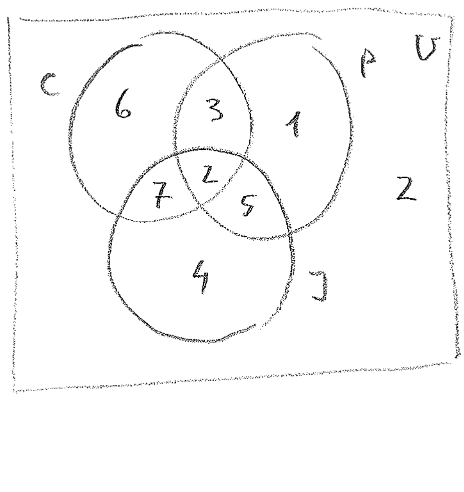
Since $|cjp| = |C \cap J \cap P| = 2$

and $5 = |C \cap P| = |cp| + |cjp| \Rightarrow |cp| = 3$

Similarly $9 = |C \cap J| = |cj| + |cjp| \Rightarrow |cj| = 7$.

Now $|c| + |cp| + |cj| + |cjp| = |C| \Rightarrow |c| = 18 - (3 + 7 + 2) = \underline{\underline{6}}$.

Arguing in a similar way, we can see that the situation is as follows:



④ $|A| = 28$ $\quad |A\cup B| = 45$ $\quad |A\cup B\cup C| = 64$

$|B| = 29$ $\quad |B\cup C| = 51$

$|C| = 33$ $\quad |A\cup C| = 53$

(a) $|B\cap C| = |B| + |C| - |B\cup C| = 29 + 33 - 51 = 11$

(b) IE-formula:

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$

First solution: since we already calculated $|B\cap C|$ needed for the IE-formula, we can calculate $|A\cap B|$ and $|A\cap C|$ in the same way:

$$|A\cap B| = |A| + |B| - |A\cup B| = 28 + 29 - 45 = 12$$
$$|A\cap C| = |A| + |C| - |A\cup C| = 28 + 33 - 53 = 8$$

Thus $|A\cap B\cap C| = |A\cup B\cup C| - |A| - |B| - |C| + |A\cap B| + |A\cap C| + |B\cap C|$

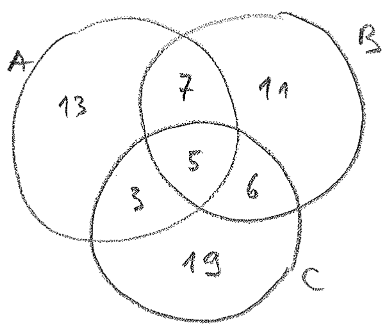
$$= 64 - 28 - 29 - 33 + 12 + 8 + 11 = \underline{\underline{5}}$$

Second solution: Substitute $|A\cap B| = |A| + |B| - |A\cup B|$ into
$|A\cap C| = |A| + |C| - |A\cup C|$
$|B\cap C| = |B| + |C| - |B\cup C|$

$$\Rightarrow |A\cap B\cap C| = |A\cup B\cup C| - \cancel{|A|} - \cancel{|B|} - \cancel{|C|} + \cancel{|A|} + \cancel{|B|} - |A\cup B| + |A| + \cancel{|C|} - |A\cup C| + |B| + |C| - |B\cup C| =$$

$$= |A| + |B| + |C| - |A\cup B| - |A\cup C| - |B\cup C| + |A\cup B\cup C| =$$
$$= 28 + 29 + 33 - 45 - 53 - 51 + 64 = \underline{\underline{5}}$$

In fact, a bit more work allows us to describe the situation entirely as:



(5) SUPPOSE $A \subseteq B$.

(a) TRUE: no matter what C may be,

$$A \cap C \subseteq A \subseteq B \text{ so } A \cap C \subseteq B$$

(b) FALSE: Take $A = \{1\}$, $B = \{1,2\}$, $C = \{3\}$.

$$A \subseteq B \text{ but } A \cup C \not\subseteq B.$$

(c) TRUE: no matter which C we take

$$A \setminus C \subseteq A \subseteq B \text{ so } A \setminus C \subseteq B.$$

(d) FALSE: Take $A = \{1\}$, $B = \{1,2\}$, $C = \{1,2\}$

$$A \subseteq B \text{ but } A \not\subseteq B \setminus C = \emptyset$$

(e) FALSE: Take $A = \{1\}$, $B = C = \{1\}$.

$$A \not\subseteq B \times C = \{(1,1)\}.$$

(f) TRUE:

$$A \times C = \{(a,c) : a \in A, c \in C\}$$

$$B \times C = \{(b,c) : b \in B, c \in C\}$$

Take an arbitrary $x \in A \times C$. Then $x = (a,c)$ for some $a \in A, c \in C$.

Given that $A \subseteq B$, if $a \in A$, then also $a \in B$ so

$x = (a,c)$ for $a \in B$, $c \in C$ so $x \in B \times C$.

Thus every element of $A \times C$ is also an element of $B \times C$ and we conclude $A \times C \subseteq B \times C$.