Queen Mary, University of London
MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Mid-Term Test Solutions.

Time allowed: 45 minutes

1. (a) $A \cap B = \{2, 4\}$ [4 marks], $A \cup B = \{1, 2, 3, 4, 6, 8\}$ [4 marks, total 8]

   (b) The inclusion-exclusion formulae are $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |C \cap A| + |A \cap B \cap C|$ [2 marks] and $|B \cup C| = |B| + |C| - |B \cap C|$ [2 marks]. Substituting the formula for $|B \cup C|$ into the formula for $|A \cup B \cup C|$ we see that $|A \cup B \cup C| = |A| - |A \cap B| + |B \cup C| - |A \cap C| + |A \cap B \cap C|$ [4 marks]. Plugging in the numbers we get $|A \cup B \cup C| = 45 - 21 + 65 - 20 + 9 = 78$ [4 marks, total 12].

2. (a) (Should be in columns, with carries shown) $10101 \times 1010 = 101010 + 10101000 = 11010010$ [6 marks]. In decimal notation 10101 is $16 + 4 + 1 = 21$ and 1010 is $8 + 2 = 10$ [2 marks] and 11010010 is $2 + 16 + 64 + 128 = 210$ [2 marks], and since $21 \times 10 = 210$ our calculation was correct [2 marks, total 12].

   (b) (Again in columns with borrowing shown) $10101 - 1010 = 1011$ [8 marks]. Alternatively can use method of complements: $10101 - 1010 = 10101 + 1010^c - 10000 + 1 = 10101 + 101 - 10000 + 1 = 11010 - 10000 + 1 = 1010 + 1 = 1011$. [8 marks, inc 2 for formula and 2 for getting complement right]. No extra marks for checking in decimal but well worth doing anyway as most of the converting was done in part (a).

3. $\mathbb{Z}_8$ consists of the equivalence classes $[0], [1], [2], [3], [4], [5], [6], [7]$.

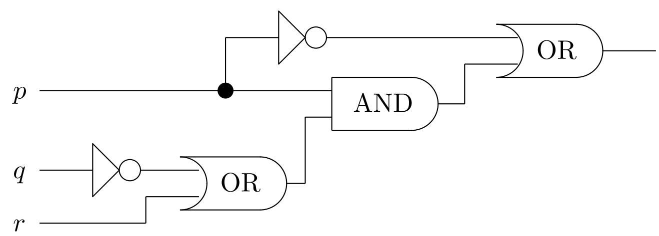
   (a) $[1], [3], [5], [7]$, [4 marks] since these are the only numbers not sharing a common factor with 8 [4 marks, total 8].

   (b) Calculate in $\mathbb{Z}_8$:

      (i) $([2] + [7]) \times ([1] - [6]) = [9] \times [-5] = [1] \times [3] = [3]$ [6 marks],

      (ii) $[3] \div [5] = [35] \div [5] = [7]$ [6 marks, total 12].

4. (a) The simplest logic circuit comes on observing $pq' \vee p' \vee pr \equiv p(q' \vee r) \vee p'$. There are other solutions - [5 marks for any correct circuit]



Truth table [5 marks for this, making 10 in total]:

p	q	r	pq'	p'	pr	$pq' \vee p' \vee pr$
1	1	1	0	0	1	1
1	1	0	0	0	0	0
1	0	1	1	0	1	1
1	0	0	1	0	0	1
0	1	1	0	1	0	1
0	1	0	0	1	0	1
0	0	1	0	1	0	1
0	0	0	0	1	0	1

(b) The smart way is to use Boolean algebra: $(p' \vee q) \to r \equiv r(p' \vee q) \vee (p' \vee q)' \equiv rp' \vee rq \vee pq'$ by De Morgan's law, for a very easy 10 marks. The long way to do it is to draw the truth table [4 marks for this]:

p	q	r	$p' \vee q$	$(p' \vee q) \to r$
1	1	1	1	1
1	1	0	1	0
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	1	0
0	0	1	1	1
0	0	0	1	0

From this we obtain the formula $pqr \vee pq'r \vee pq'r' \vee p'qr \vee p'q'r$ [2 marks]. We can simplify this to $pr \vee pq'r' \vee p'r$ to satisfy the requirements of the question, or even further to $r \vee pq'r'$ [4, total 10 marks].