Queen Mary
University of London

B. Sc. Examination by course unit 2010

MAE 113 Discrete Techniques for Computing

Duration: 2 hours

Date and time: Wednesday 19 May, 10.00 – 12.00

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

The paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J. Elmer

Section A: Each question carries 11 marks. You should attempt ALL questions.

Question 1 Let A be the set $\{0, 1, 2, 3, 4, 5\}$ and let B be the set $\{1, 3, 6\}$. What are the following?

(a) The sets $A \cup B$ and $A \setminus B$?

(b) The number $|A \cap B|$?

(c) The set of all subsets of B.

Question 2 (a) Convert the binary number 1100110 into the corresponding number in the decimal system.

(b) Convert the number 349 in the decimal system into the corresponding number in the binary system.

(c) Use Russian Peasant multiplication to multiply together 401 and 73.

Question 3 (a) What are the following?

$$\text{(i)} \quad \frac{11!}{8!}; \quad \text{(ii)} \quad C(8,5); \quad \text{(iii)} \quad \frac{(m+n-4)!}{(m+n-6)!}$$

(b) What is the coefficient of $x^2y^2z^4$ in the polynomial $(w + x + y + z)^8$?

Question 4 Draw the undirected graph G with the set of vertices $\{1, 2, 3, 4, 5\}$ and the following adjacency matrix:

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$$

Consider the relation $\sim$ on $\{1, 2, 3, 4, 5\}$ given by

$$x \sim y \;\Leftrightarrow\; x \text{ and } y \text{ are joined by an edge in } G.$$

Is this relation (a) reflexive, (b) symmetric, (c) transitive? Give reasons.

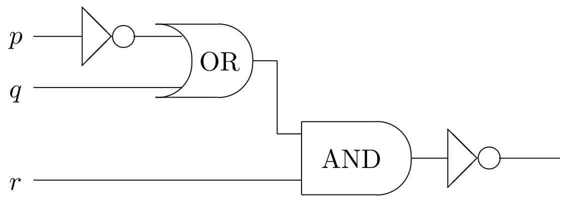
The next question is over the page.

Question 5 (a) Suppose two fair six-sided dice are thrown. Let A be the event that exactly one of the dice shows a 5, and let B be the event the total shown on the two dice is 9. Calculate $P(A)$ and $P(B)$.

(b) Are the two events above independent? Explain your answer.

Section B: Each question carries 15 marks. Except for the award of a bare pass, only the marks for the best THREE questions will be counted.

Question 6 Find the output of the following logic circuit in the following way: First work out the boolean formula of the circuit and then calculate the truth table of this formula. Express the formula as a disjunction of one or more minterms, simplifying as far as possible.



Question 7 Let X be the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let f and g be functions from X to X with the following lookup tables:

$f:$	x	$f(x)$	$g:$	x	$g(x)$
	1	3		1	2
	2	5		2	2
	3	1		3	4
	4	2		4	2
	5	7		5	8
	6	10		6	10
	7	9		7	10
	8	4		8	3
	9	8		9	2
	10	6		10	8

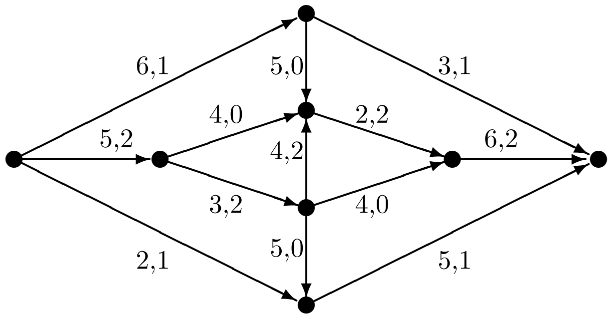
(a) State whether the functions f and g are onto, one-one, both, or neither.

(b) Which of the functions f and g is invertible? Write out the lookup table for its inverse. The first column should contain the numbers 1-10 in numerical order.

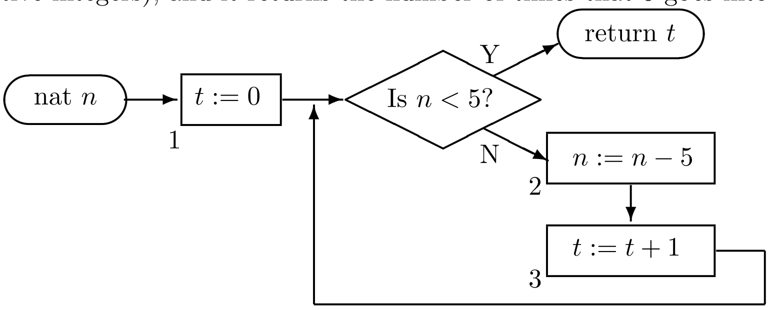
(c) Write out the lookup table for the functions $g \circ f$ and $g \circ g$. Are either of these functions invertible?

The next question is over the page

Question 8 In the following network, the source is on the left and the sink on the right. The first number against each edge is the capacity of the edge. Find a maximum flow in the network, starting from the flow given by the second numbers against the edges. State the value of your flow. To show that your flow is maximum, find a cut in the network that has the same value.



Question 9 The following algorithm accepts inputs n that are natural numbers (i.e. non-negative integers), and it returns the number of times that 5 goes into n.



(a) Walk through the algorithm when $n = 17$. The first two lines are done for you below:

$$n_1 = 17; \quad t_1 = 0$$
$$n_{2,1} = 12; \quad t_{2,1} = 0$$

(b) Draw an algorithm that calculates the remainder when n is divided by 5. (Hint: It needs only a slight change to the algorithm given above.)

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Question 10 In a group of 200 children, 141 have been inoculated against measles, mumps and rubella; 165 have been inoculated against measles; 156 have been inoculated against mumps; 157 have been inoculated against both measles and rubella; 174 have been inoculated against at least one of mumps and rubella; 144 have been inoculated against both measles and mumps; 25 have not been inoculated against either measles or rubella. Answer the following questions, and in each case state clearly the application that you are making of the principle of inclusion-exclusion.

(a) How many children in the group have not been inoculated against rubella? [Apply inclusion-exclusion to measles and rubella.]

(b) How many children in the group have been inoculated against both mumps and rubella?

(c) How many children in the group have not been inoculated against any of the three diseases?

End of Paper