Queen Mary
University of London

B. Sc. Examination by Course Unit 2008

MAE 113 Discrete Techniques for Computing

Duration: 2 hours

Date and time: 7th May 2008, 10.00 am

This paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE INVIGILATOR.

SECTION A *Each question carries 11 marks. You should attempt ALL questions.*

Question 1 Let A be the set $\{1,3,5,7,9\}$ and B the set $\{1,2,3,4,5,6\}$. What are the following?

(a) [2 marks] The set $A \cap B$.

(b) [4 marks] The numbers $|A \cap B|$ and $|A \cup B|$.

(c) [2 marks] $\{x : x \in A \text{ and } x \notin B\}$.

(d) [3 marks] The number of subsets of A.

Question 2 Answer the following for the boolean formula $(p' \vee q)' \vee q$ in the two variables p and q.

(Note the alternative notation for this formula is: $\neg(\neg p \vee q) \vee q$.)

(a) [4 marks] Draw the logic circuit for this boolean formula.

(b) [5 marks] Write out the truth table for this boolean formula.

(c) [2 marks] Is this boolean formula equivalent to $p \vee q$? Explain your answer.

Question 3 (a) [3 marks] Multiply the binary numbers : 111×11001.

(b) [3 marks] Convert the binary number 1011011 into the corresponding decimal number.

(c) [5 marks] Convert the decimal number 505 into the corresponding binary number.

Question 4 (a) [3 marks] Simplify the following expression in $\mathbb{Z}_7$, the integers modulo 7. (Your answer should be one of $[0],[1],[2],\ldots,[6]$.)

$$([3]-[5])+([2]+[3])([5]-[2]]).$$

(b) [2 marks] Calculating in $\mathbb{Z}_7$ find a number a such that $[3] \times [a] = [1]$.

(c) [6 marks] Draw a digraph with vertices u, v, w, x and adjacency matrix

	u	v	w	x
u	1	0	0	1
v	0	1	0	1
w	0	1	0	1
x	0	1	0	0

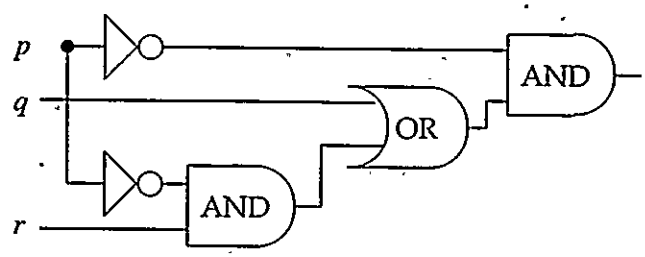
Question 5 (a) [3 marks] Calculate the number $\frac{8!}{5!}$.

(b) [3 marks] Calculate the number $C(7,3)$.

(c) [5 marks] How many rearrangements are there for the letters of the word REGREET ?

SECTION B *Each question carries 15 marks. You may attempt all questions. Except for the award of a bare pass, only marks for the best THREE questions will be counted.*

Question 6 (a) [5 marks] Write out the boolean formula for the logic circuit



(b) [6 marks] Write out the truth table for the boolean formula you have found in (a).

(c) [4 marks] Express the boolean formula in (a) as a disjunction of one or more minterms.

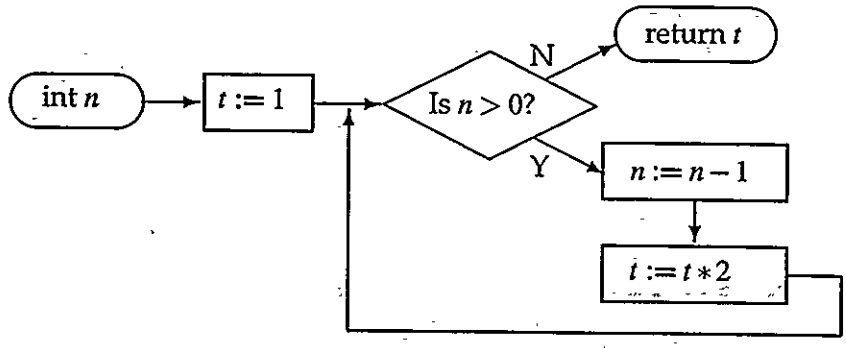
Question 7 (a) [5 marks] Draw an algorithm for calculating the function $x(x+4)$.

(b) [10 marks] Walk through the following algorithm

(i) when $n = 5$, and

(ii) when $n = -2$

If an integer n is input, what number will the algorithm return?



Question 8 (a) [4 marks] Describe how one can tell from the lookup table for a function from a finite set X to a finite set Y whether it is (i) onto and (ii) one-to-one.

(b) [11 marks] Let X be the set $\{1,2,3,4,5,6,7,8,9\}$ and let f and g be functions from X to X with the following lookup tables:

f:

X	X
1	3
2	5
3	1
4	2
5	7
6	6
7	9
8	4
9	8

g:

X	X
1	2
2	2
3	4
4	2
5	8
6	8
7	6
8	3
9	3

(i) Write out the lookup table for $g \circ f$.

(ii) One of the functions f and g has an inverse from X to X. Which one is it? Write out a lookup table for this inverse.

(iii) Let R be the relation

$$aRb \Leftrightarrow g(a) = g(b).$$

This relation R is an equivalence relation on the set X; write down its equivalence classes.

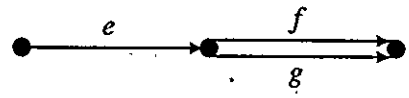
Question 9 (a) [5 marks] A coin is tossed four times. The coin is biased and the probability of heads is 1/4. Calculate the probabilities of each of the following events:

(i) The outcome is HTTH (i.e. heads then tails then tails then heads).

(ii) There is at least one T.

Question 9 continues on next page

(b) [10 marks] We are given the following communication network



A message will get through this network as long as e doesn't fail and at least one of f and g doesn't fail. We write A_e, A_f, A_g for the events that edge e fails, that f fails and that g fails. The probabilities of A_e, A_f and A_g are respectively 0.1, 0.2 and 0.3, and these three events are independent.

(i) Suppose we call a row of the following table *good* if it allows a message to get through the communication network. Which are the good rows?

	A_e	A_f	A_g	probability
1.	T	T	T	
2.	T	T	F	
3.	T	F	T	
4.	T	F	F	
5.	F	T	T	
6.	F	T	F	
7.	F	F	T	
8.	F	F	F	

(ii) Fill in the probabilities in this table ONLY for the good rows.

(iii) What is the probability that a message will get through the network?

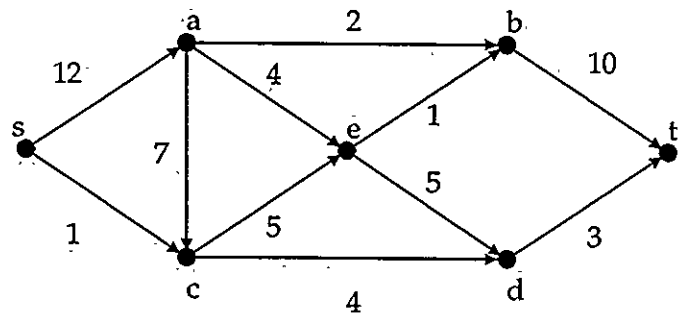
Question 10 (a) [2 marks] Explain what is meant by saying that a list of edges is a *cut* for a network.

(b) [2 marks] If f is a flow in a network, and C is a cut for the network, state what information can be obtained about the value of f from the value of C.

(c) [5 marks] Which of the following lists of edges are CUTS for the network below? For each that is a cut, what is its value?

(i) *ab*, *ae*, *cd*, *ed* (ii) *ab*, *eb*, *dt*.

(d) [6 marks] Find a maximum flow in the network below. Give the value of the flow, and explain how you know that it is maximal.



 END OF EXAMINATION