MAE113 DISCRETE TECHNIQUES FOR COMPUTING

Coursework 5–to be handed in by noon, Wednesday 03/11/2010.
Write your name and student number at the top of your assignment before handing it in. You should attempt all questions because only one question will be marked.

1. Which of the following lookup tables represent a function $f : X \to Y$ for $X = \{1, 2, 3, 4, 5\}$, $Y = \{1, 2, 3, 4, 5\}$?

(a)

X	Y
1	3
2	2
3	3
4	1
5	4

(b)

X	Y
3	1
2	2
1	3
1	4
4	5

(c)

X	Y
1	1
2	1
4	1
3	1
5	1

(d)

X	Y
1	1
2	2
3	3
4	4

2. For each of the following functions, state whether they are one-to-one, onto, both or neither.

(a) $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$, where f given by the lookup table

X	Y
1	2
2	2
3	3
4	4
5	5

(b) $g : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$, where g is given by the lookup table

X	Y
1	5
2	4
3	3
4	2
5	1

(c) $h : \mathbb{R} \to \mathbb{R}$, $h(x) = x^3$,

(d) $k : \{x \in \mathbb{R} : x \geq 3\} \to \{y \in \mathbb{R} : y \geq 2\}$, $k(x) = \sqrt{x + 1}$,

(e) $j : \mathbb{R} \setminus \{1\} \to \mathbb{R} \setminus \{-1\}$, $f(x) = \frac{1+x}{1-x}$.

3. Which of the functions in question 2 are invertible? For those that are, give the inverse function.

4. Consider the functions f, g, h, j from question 2. Calculate the following functions, or else explain why they cannot exist:

   (a) $f \circ g$,

   (b) $g \circ f$,

   (c) $f^{-1} \circ g^{-1}$,

   (d) $h \circ j$,

   (e) $j \circ h$.

5. Consider the functions $f : \mathbb{R} \to \{x \in \mathbb{R} : x \geq 1\}$, $f(x) = \sqrt{x^2 + 1}$ and $g : \{x \in \mathbb{R} : x \geq 1\} \to \mathbb{R}$, $g(x) = \sqrt{x^2 - 1}$. Calculate the following functions, or else explain why they cannot exist:

   (a) $f \circ g$,

   (b) $g \circ f$.