Topology
Exercise sheet 2

You do not need to do all the questions but you should think whether you could do them. Ideally, if I asked you in the tutorial how to do a question you would be able to answer it.

1. Suppose that A and B are disjoint subsets of $\mathbb{R}$. Must there be disjoint open sets U and V with $A \subset U$ and $B \subset V$. (Harder) What if A and B are closed?
2. Prove that any metric space is Hausdorff.
3. Suppose that X is a metric space. Prove that the definition of convergence given in this course is equivalent to the definition in terms of the metric.
4. Check that the property of being homeomorphic is an equivalence relation.
5. What functions are continuous from the topological space $\mathbb{R}$ with the indiscrete topology to $\mathbb{R}$ with the usual metric topology?
6. What functions are continuous from a topological space X into a space Y with the indiscrete topology?
7. Find a function $f\colon \mathbb{R} \to \mathbb{R}$ and an open set U with $f(U)$ not open.
8. Which of the following pairs are homeomorphic?
   (a) $(0, 1]$ and $[0, 1]$.
   (b) $[1, \infty)$ and $[0, 1]$. Why can we do this quickly?
   (c) $(-11, -10)$ and $\mathbb{R}$.
9. In a metric space we have the notion of completeness. Is this a topological property?
10. Consider the following property of a topological space: it can be written as a disjoint union of two open sets: that is $X = U \cup V$ with $U \cap V = \emptyset$. Is this a topological property?