MTH5118 Probability II. Problem Sheet 7.

This coursework consists of two parts. **You are required to submit solutions to the second part only. You are strongly encouraged to solve all problems on this problem sheet.** *Please staple your coursework and post it in the Blue Box in the basement of the Maths building by* **10:30** *on Thursday, 26th November 2009.*

To solve the problems below remind yourself (Introduction to Statistics) the definition of the joint p.d.f. $f_{X,Y}(x,y)$ of two random variables X, Y and that $f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy$, $f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx$. (To be discussed in the lecture on Monday.)

Part 1

1. X has p.d.f. $f_X(x) = 2\theta x e^{-\theta x^2}$ for $x > 0$ and $f_X(x)$ is zero elsewhere. Let $Y = X^2$. Note that this is a one to one transformation for the range of X for which the p.d.f. is non-zero. Use the standard transformation of variables result to obtain the p.d.f. for Y.

2. $X \sim N(0,1)$. Let $Y = |X|$. Use equivalent events to write the c.d.f. for Y in terms of the c.d.f. for X and hence obtain the p.d.f. for Y. (Note that $F_Y(y) = 0$ for $y \le 0$.)

3. Let X and Y have joint p.d.f. $f_{X,Y}(x,y) = C$ for $0 < x < 2$ and $0 < y < 2$ and $f_{X,Y}(x,y) = 0$ elsewhere. Find C and obtain the marginal p.d.f.'s for X and Y, $f_X(x)$ and $f_Y(y)$.

Find $P(X+Y \le z)$ for (i) $0 < z \le 2$; (ii) $2 < z < 4$ and state the value of $P(X+Y \le z)$ if (iii) $z \le 0$; (iv) $z \ge 4$. Hence obtain the p.d.f. for $Z = X + Y$.

4. Let X and Y have joint p.d.f. $f_{X,Y}(x,y) = C$ for $0 < x < 2y < 2$ and $f_{X,Y}(x,y) = 0$ elsewhere. Find the marginal p.d.f.'s for X and Y, $f_X(x)$ and $f_Y(y)$ and obtain C.

Find $P(2Y - X \le z)$, where $0 < z < 2$. State the value of $P(2Y - X \le z)$ if (i) $z \le 0$; (ii) $z \ge 2$. Hence obtain the p.d.f. for $Z = 2Y - X$.

Part 2

5. Random variables X and Y have joint p.d.f. $f_{X,Y}(x,y) = C(x^2 + xy)$ for $0 < x < 1$, $0 < y < 1$ and $f_{X,Y}(x,y) = 0$ elsewhere.

Find the marginal p.d.f.'s $f_X(x)$ and $f_Y(y)$ in terms of C. Find C.

6. Random variables X and Y have joint p.d.f. $f_{X,Y}(x,y) = Ce^{-(x+y)}$ for $0 < x < y < \infty$ and $f_{X,Y}(x,y) = 0$ elsewhere.

Obtain the marginal p.d.f.'s $f_X(x)$ and $f_Y(y)$ in terms of C. Find C.