Probability 2 - REQUIREMENTS FOR TEST.

1. P.g.f.:

(a) Know p.g.f. for standard discrete distributions.

(b) Know how to find $P(X = k)$ by differentiating the p.g.f.

(c) Know how to find $E[X]$ and $Var(X)$ by differentiating the p.g.f.

(d) Know that the p.g.f. of the sum of independent r.v.'s is the product of the individual p.g.f.'s.

2. Know the Theorem of Total Probability and be able to do simple examples.

Know the gambler's ruin result: $r_k \equiv r_k(M,N) = \frac{(\frac{q}{p})^k - (\frac{q}{p})^M}{(\frac{q}{p})^N - (\frac{q}{p})^M}$ if $p \neq q$ and $r_k = \frac{k-M}{N-M}$ if $p = q = \frac{1}{2}$. Be able to use the results in simple examples.

3. Know the Theorem of Total Probability for Expectations. Be able to do simple examples (NOT the expected duration of game for gambler's ruin)

4. Branching processes (BP).

(a) Know the definition of the BP Y_n with generating r.v. X.

(b) Be able to find the p.g.f. for Y_n for small n in simple examples using $G_{n+1}(t) = G(G_n(t))$

(c) Be able to find $\theta_n = P(Y_n = 0)$ for small n in simple examples using $\theta_{n+1} = G(\theta_n)$.

(d) Be able to find the probability of eventual extinction θ by solving $G(t) = t$.

5. Know results on conditional distributions and conditional expectations and be able to use this for simple examples.

In particular know:

$E[Y] = E[E[Y|X]]$ and $Var(Y) = E[Var(Y|X)] + Var(E[Y|X])$

Know that the following result for random sums follows from these formulae. If $S = \sum_{j=1}^{N} X_j$ where the $X's$ are i.i.d. with common mean a and common variance σ^2 and N is a random variable which is independent of the $X's$, then:

$E[S] = aE[N]$ and $Var(S) = \sigma^2 E[N] + a^2 Var(N)$