

## Probability III – 2008/09

### Exercise Sheet 3

*Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Return the assignment by 17:00 on Thursday, 5 February*

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The problems on this Exercise Sheet are very similar to those solved in the previous CW and to those discussed in the notes/lectures (First step analysis, Extended example). I suggested that you read the notes and the solutions to CW2 and make use of them.

1. A fair coin is flipped repeatedly until either HHH or HTT shows up for the first time (as usual H stands for “heads” and T for “tails”). Every time when a sequence TTT shows up, you are given £1. Every time HTH shows up, you pay £1. Determine the mean value of your gain. Is this a fair game? [35]

You should use the MC described in solution 1 to a problem with a similar setting. The relevant equations were discussed in lectures and in the notes concerned with the first step analysis (see example 4 there).

2. A fair 6-sided die is rolled until the sum of three consecutive rolls is 4 for the first time. What is the mean number of rolls in this game? Set up a MC with no more than 9 states which allows one to solve this problem and write down its transition matrix. (You are supposed to describe the MC but are not asked to provide the calculation.) [15]

3. A fair 6-sided die is rolled repeatedly until the sum of two consecutive results is 11 or 12. [50]

- (a) What is the expected number of rolls in this game?
- (b) What is the probability that the game will end when the sum of two last results is 12?
- (c) Which of the two numbers is observed more times on average in this game, 5 or 6?

To solve this problem set up a Markov chain  $X_n$  which is similar to the one in problem 3 from CW2.